Transport, diffusion and mean field games

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We discuss dynamical optimal transport problems with additional density dependent costs, such as the entropy functional. Those terms penalize congestion effects and enhance some form of dissipation with respect to the classical transport of Wasserstein geodesics. This is to be observed through possibly different properties of the optimality system satisfied by minimal curves, which is an instance of mean field game system coupling Hamilton-Jacobi and continuity equations. At the crossroad between transport and diffusion, we will describe qualitative and quantitative properties of minimizers by exploiting different tools, which include quasilinear elliptic equations, displacement convexity arguments and free boundary formation due to finite speed of propagation.