Abstract

This talk explores the existence and classification of (G-invariant) generalized spin structures on homogeneous and cohomogeneity one manifolds. These structures provide a unifying framework for studying spin, $\mathrm{Spin}^{\mathbb{C}}$, and $\mathrm{Spin}^{\mathbb{H}}$ geometries.

We discuss how the existence of such structures can be characterized in representation-theoretic terms. These ideas offer a systematic approach to understanding invariant spinors and their relation to special geometric structures within a unified, equivariant context. In the case of homogeneous spaces, finding invariant spinors can be explicitly done by computing the invariant subspace for the composition $H \to Spin(n) \to GL(V)$ where V is the Spin representation; this translates the problem into representation theory, and more specifically invariant theory. The elements of this subspace will be exactly the invariant sections or ”special spinors” we are looking for. Conversely the geometric properties of the homogeneous space will be characterized by the existence of such spinors.

In the cohomogeneity one setting, we show how the existence and classification of (G-invariant) spin structures reduces to data encoded in the geometry of the principal orbit. This is based on joint work with Ilka Agricola, Diego Artacho, Jordan Hofmann and Jordi Daura Serrano.