Uniqueness of the filtering equations in the space of measures
Abstract

Étienne Pardoux

Aix Marseille Université

In this work, we consider a general filtering problem of the following form

$$X_t = X_0 + \int_0^t f(s, X_s, Y_s)ds + \int_0^t \sigma_1(s, X_s, Y_s)dV_s + \int_0^t \sigma_2(s, X_s, Y_s)dW_s,$$
$$Y_t = Y_0 + \int_0^t h(s, X_s, Y_s)ds + \int_0^t k(s, Y_s)dW_s,$$

where only the process Y is observed, and one is interested in computing the evolution of the conditional law of X_t given $\{Y_s,\ 0 \le s \le t\}$. Here V and W are two mutually independent Brownian motions, all processes being multidimensional. The novelty in our work is that the matrix k is not assumed to be invertible, which forces us to assume that h is of the form

$$h(t, x, y) = h_1(t, y) + k(t, y)h_2(t, x, y)\,.$$

The same kind of filtering problem has been considered in [4], in the particular case where $\sigma_2 \equiv 0$ (signal and observation noises are independent), and the observation process Y is one dimensional.

Our main result is a uniqueness result for the Zakai equation, in a class of measure valued solutions. Note that we do not assume that either of the matrices $\sigma_1\sigma_1^*$ and $\sigma_2\sigma_2^*$ is non degenerate. Our uniqueness argument is a duality argument. Generalizing the results of [2] and [3], we prove that a certain system of BSPDEs, which is dual to a proper version of the Zakai equation, has a smooth enough solution. This approach generalizes the arguments in sections 4.2 and 4.5 of [1], which are based on duality with a system of backward deterministic PDEs, and treat respectively the classical filtering problem with independent and correlated signal and observation noises.

This is joint work with Dan Crisan, Imperial College.

References

[1] A. Bensoussan, *Stochastic control of partially observable systems.* Cambridge University Press, Cambridge, 1992.

[2] K. Du, Q. Meng, A revisit of W_2^n-theory of super-parabolic backward stochastic partial differential equations in $\mathbb{R}^d$, *Stoch. Proc. and Applic.* **120**, 1996–2015, 2010.

[3] K. Du, S. Tang, Q. Zhang, $W^{m,p}$–solutions ($p \geq 2$) of linear degenerate backward stochastic partial differential equations in the whole space, *J. of Differential Equ.* **254**, 2877 –2904, 2013.

[4] D. J. Fotsa–Mbogne, E. Pardoux, Nonlinear filtering with degenerate noise, *Electron. Comm. Probab.* **22**, 44, pp. 1–14, 2017.