We use global results about functions that are meromorphic in regions of the plane to find individual solutions of differential, difference and delay-differential equations whose only movable singularities are poles. We also allow for simple global branching. In this way we can find or describe subsets of solutions of equations that are in general non-integrable.