Understanding structure and function in a natural neural network is a formidable task that has been occupying the attention of scientists and mathematicians for many decades. Algebraic topology entered that scene relatively recently, equipped with its own tools as well as those of adjacent subjects such as graph theory and combinatorics. In this talk I will review some of the work I've done with collaborators where we employ various combinatorial constructions to networks emerging from digital reconstructions of sections of brain tissue developed by the Blue Brain Project. I will demonstrate how these combinatorial objects, analysed by methods of algebraic topology, reveal interesting structural features of a neural network, as well as hold the potential to inform on its functionality.