On a family of Jacobi type polynomials as eigenfunctions of 2×2 hypergeometric operators: Structural formulas.

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In the last few years, the search for examples of matrix-valued orthogonal polynomials that are common eigenfunctions of a second order differential operator has received a lot of attention after the seminal work of A. Durán in [3].

The theory of matrix-valued orthogonal polynomials was started by Krein in 1949 in connection with spectral analysis and moment problems [8]. Nevertheless, the first examples of orthogonal matrix polynomials satisfying this extra property and non reducible to scalar case, appeared more recently in the work of F. A. Grünbaum, I. Pacharoni and A. Tirao [5, 6, 7] and that of A. Durán and F. A. Grünbaum [4].

Recently, in [2], a new family of matrix-valued orthogonal polynomials of size 2×2 was introduced, which are common eigenfunctions of a differential operator of *hipergeometric type* (in the sense defined by A. Tirao in [9]), with diagonal matrix eigenvalues.

As the case of classical orthogonal polynomials, the families of matrix-valued orthogonal polynomials satisfy many formal properties such as structural formulas, which have been very useful to compute explicitly the orthogonal polynomials related with several of these families.

We give some structural formulas for the family of matrix-valued orthogonal polynomials introduced in [2]. In particular, we give a Rodrigues formula, which allows us to write this family of polynomials explicitly in terms of the classical Jacobi polynomials. We also obtain a Pearson equation, which allows us to prove that the sequence of derivatives of k order, $k \geq 1$, of the orthogonal polynomials is also orthogonal.

This is a joint work with Celeste Calderón from Universidad Nacional de Cuyo, Mendoza, Argentina [1].

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