Generalised higher order Freud polynomials

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Abstract: Gabor Szegő pioneered much of what is known in the theory of orthogonal polynomials on finite intervals but did not carry his ideas over to infinite intervals, despite there being significant differences. In the second half of the 20th century, starting with the work of Géza Freud on orthogonal polynomials on $\mathbb{R}$, the study of Freud-type polynomials and their generalisations flourished. In this talk I will present some recent results on polynomials orthogonal with respect to higher order Freud weights, in particular the quartic and sextic weights. I will show that, for generalised quartic Freud weights, the coefficients in the three-term recurrence relation are expressed in terms of Wronskians of parabolic cylinder functions that arise in the description of special function solutions of the fourth Painlevé equation and, for the generalised sextic Freud weight, in terms of Wronskians of generalised hypergeometric functions. I will derive a differential equation and mixed recurrence relation satisfied by generalised sextic Freud polynomials and use these to study the zeros and recurrence coefficients of the polynomials. Further, I will derive a nonlinear discrete equation as well as a sytem of differential equations satisfied by the recurrence coefficients and use these to investigate their asymptotic behaviour.

References

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