

Optimal transport in a nutshell

Abstract: Optimal transportation (OT) theory dates back to the works of the French geometer Gaspard Monge, who considered the following problem in 1781: Find the optimal transportation map given an initial and target distribution of mass and a transport cost, which depends on the initial and final position. Monge himself was not able to answer the question of whether such a minimiser exists and how it can be characterised in general. It took until 1940 when the Russian mathematician Leonid Kantorovich made significant progress by considering a relaxed formulation. Since then OT has developed into a well-established framework, providing powerful and versatile analytical and computational tools. It has made its ways into applications in economics, traffic and network theory, imaging and more recently data science.

In this mini-course I will give an overview on the theory of optimal transport and introduce the most important numerical techniques to compute optimal transportation maps efficiently. In the first two lectures I will discuss the original Monge and Kantorovich formulation of OT, as well as the properties of solutions. I will then continue with geodesics in Wasserstein spaces as well as the Benamou and Brenier formulation of OT before concluding with a very brief overview on computational OT.