LMS Prospects - September 10th 2021

LOGIC

Syntax

$\varphi = \quad \forall x \, \forall y \quad x \cdot y = y \cdot x$

Semantics / Models / Structure

$(\mathbb{Z}, \cdot, 1) \models \varphi$

$(S_n, \circ, id) \not\models \varphi$

Thm (Completeness Thm, Gödel)

$\forall \, \mathcal{M}$ structure for T

$T \vdash \varphi \quad \Longleftrightarrow \quad \mathcal{M} \models \varphi$

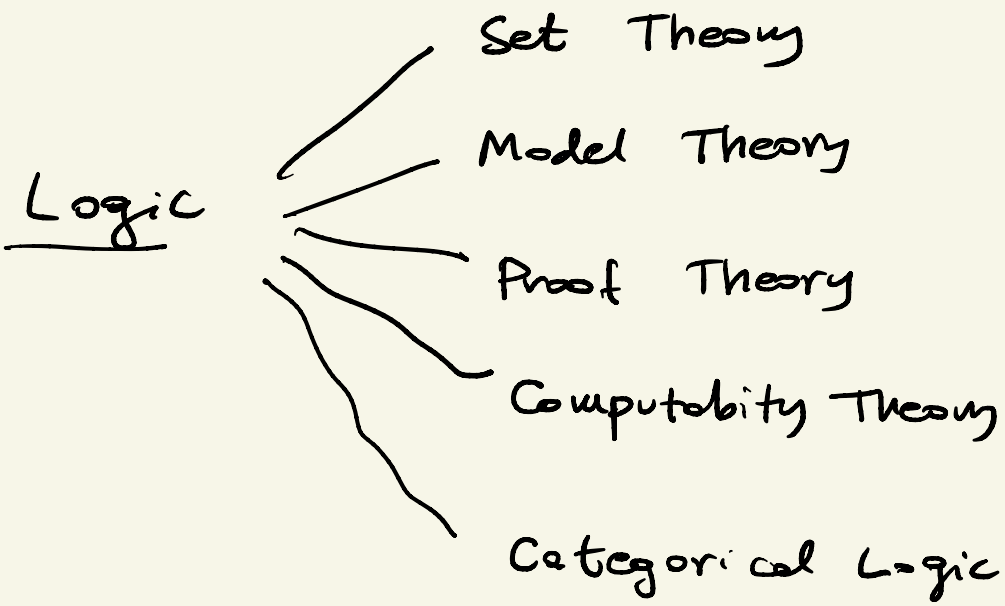
"T proves φ" — "φ valid in $\mathcal{M}$"

Application

Thm (Independence of CH) $ZF \not\vdash CH$, $ZF \not\vdash \neg CH$

ZF: Zermelo–Fraenkel set theory

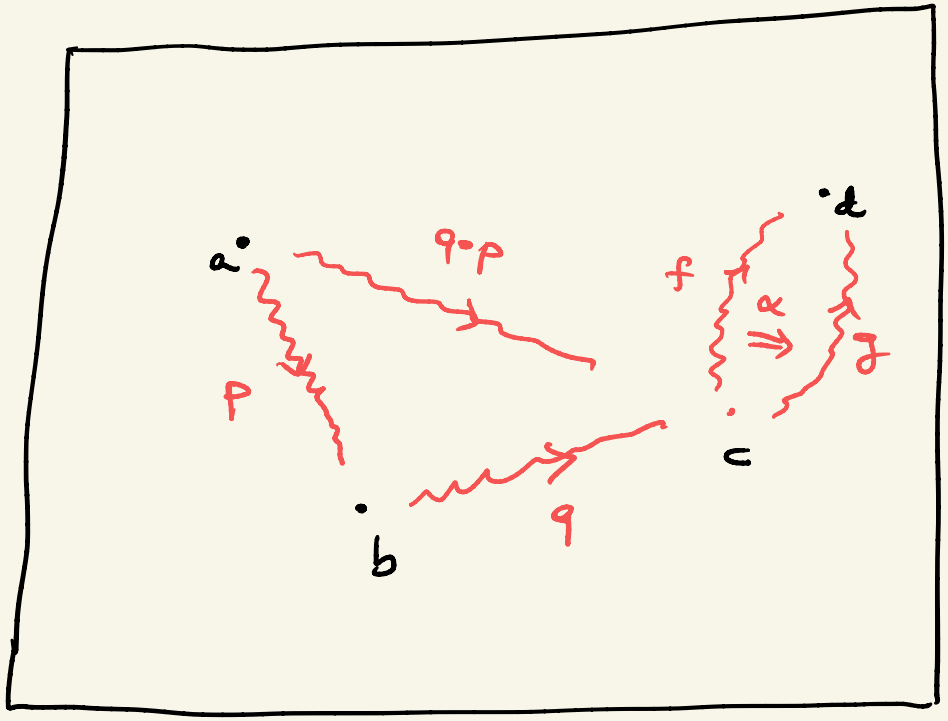
CH: continuum hypothesis.



Theorem (van den Berg & Garner, Lumsdaine).

Every type A of Martin-Löf type theory admits the structure of a weak ω-groupoid, given by the identity types of A.

A



p is an element of $\mathrm{Id}_A(a,b)$

α is an element of $\mathrm{Id}_{\mathrm{Id}_A(c,d)}(f,g)$

$\Rightarrow$ Homotopy Type Theory, Univalent Foundations (Voevodsky).

Where can I do a PhD in Logic?

- Oxford (model theory, set theory)
- Cambridge (categorical logic)
- Bristol (set theory)
- Imperial College (model theory)
- QMUL (model theory)
- Manchester (model theory)
- East Anglia (model theory, set theory)
- Leeds (set theory, model theory, proof theory, computability theory, categorical logic)

+ lots of Logic in Computer Science departments!