Motivated by the desire to automate classification of neuron morphologies, we designed a topological signature, the Topological Morphology Descriptor (TMD), that assigns a barcode to any geometric tree (i.e, any finite binary tree embedded in R^3). We showed that the TMD effectively determines the reliability of clusterings of random and neuronal trees. Moreover, using the TMD we performed an objective, stable classification of pyramidal cells in the rat neocortex, based only on the shape of their dendrites. We have also reverse-engineered the TMD, in order to digitally synthesize dendrites, to compensate for the dearth of available biological reconstructions. The algorithm we developed, called Topological Neuron Synthesis (TNS), stochastically generates a geometric tree from a barcode, in a biologically grounded manner. The synthesized cortical dendrites are statistically indistinguishable from the corresponding reconstructed dendrites in terms of morpho-electrical properties and the networks they form. We synthesized cortical networks of structurally altered dendrites, revealing principles linking branching properties to the structure of large-scale networks. In this talk I will provide an overview of the TMD and the TNS and then present recent theoretical and computational results concerning their behavior and properties, in which symmetric groups play a key role.

This is joint work with Adélie Garin and Lida Kanari, building on earlier collaborations led by Lida Kanari.