Elastostatic cloaking, low frequency elastic wave transparency and neutral inclusions

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Cloaking of waves has been of interest for decades now although only in the last two decades have significant advances been made in achieving this in specific physical scenarios, including acoustics, electrodynamics and elastodynamics. The latter has proved significantly more difficult than the former two application areas and it transpires that a whole new theoretical framework (associated with so-called *Willis coupling and equations*) is required for invariance of the governing equations. In principle, an elastodynamic metamaterial cloak design should work at any frequency and for any wave type and it should conceal any object interior to it thus being independent of its nature and geometry, although in practice of course all of these properties are impossible. In this talk we take a step back and reconsider the theoretical underpinnings of *elastostatic* cloaking and the associated limit of low-frequency transparency from elastodynamics. We tie these concepts to that of the *neutral inclusion (NI)*, which is a coated inclusion with the coating designed so as to render the inclusion invisible to a *specific type of applied loading*. The specificity of the loading and the dependence of coating on the inclusion properties is what makes the NI significantly different to ideal (currently unachievable) metamaterial design, although NIs do have the benefit of being more achievable to design and also of huge potential use in numerous materials science applications.

Neutral inclusions have long been known to be possible for hydrostatic loading, given that the governing equations reduce to scalar form in that scenario as for thermal and electrical problems. However for shear-type loading this is not the case and although “imperfect boundary conditions” (difficult to achieve in practice) can yield neutrality [1], it has been thought, even until recently that finite thickness coatings cannot ensure neutrality [2]. In [3] however, we addressed this problem, employing the impedance matrix approach to the two-dimensional equations of elastostatics in order to obtain conditions on coating properties in order that they act neutrally for *both hydrostatic and in-plane shear loading*. The coating is found to require anisotropic properties in general.

In this talk we summarise [3], provide links to elastodynamic cloaking and low-frequency transparency, noting in particular (and perhaps non-intuitively) that *leading order* low-frequency transparency is *not* equivalent to an elastostatic cloak in general. We introduce the concepts of *weak* and *strong* neutral inclusions to distinguish between these different concepts and show that the generalised self-consistent method from the theory of micromechanics [4] can be considered as introducing the concept of the weak (energetically neutral) neutral inclusion.

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[2] Song, H.P., Song, K., Schiavone, P. and Gao, C.F., 2020. Design of a neutral elastic inhomogeneity via thermal expansion. *Acta Meccanica, 231, pp. 2867-2876*
[3] Norris, A.N. and Parnell, W.J., 2020. Static elastic cloaking, low-frequency elastic wave transparency and neutral inclusions. *Proceedings of the Royal Society A*, *476*(2240), p.20190725.
[4] Christensen, R.M. and Lo, K.H., 1979. Solutions for effective shear properties in three phase sphere and cylinder models. *Journal of the Mechanics and Physics of Solids*, *27*(4), pp.315-330.