Topological Rigidity and low-dimensional topology

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An aspherical manifold is a manifold whose universal cover is contractible, i.e. a $K(\pi, 1)$-manifold. Borel conjectured that any two closed aspherical manifolds with isomorphic fundamental groups are homeomorphic, in fact, that the structure set of a closed aspherical manifold is trivial. Wall asked if a $K(\pi, 1)$-space which satisfies Poincare duality is homotopy equivalent to a closed manifold.

This talk will survey the corresponding conjectures for compact aspherical manifolds with boundary, their relationship with low-dimensional topology, and eventually focus on the questions: What closed 3-manifolds are the boundary of a compact aspherical 4-manifold? What are the fundamental groups of compact aspherical 4-manifolds? We give complete answers when the fundamental group is known to be "good," when the fundamental group is elementary amenable. This is joint work with Jonathan Hillman.