We show that the expected solution operator of prototypical linear elliptic partial differential equations with random coefficients is well approximated by a computable sparse matrix. This result is based on a random localized orthogonal multiresolution decomposition of the solution space that allows both the sparse approximate inversion of the random operator represented in this basis as well as its stochastic averaging. The approximate expected solution operator can be interpreted in terms of classical Haar wavelets. When combined with a suitable sampling approach for the expectation, this construction leads to an efficient method for computing a sparse representation of the expected solution operator which is relevant for stochastic homogenization and uncertainty quantification.