In a nutshell, Proof Complexity is about the size of proofs: given a mathematical theorem, how long is its shortest proof? However, this simple question is intricately linked to fundamental concepts in both mathematical logic and computational complexity. In particular, there is a beautiful three-way correspondence between (weak) theories of arithmetic, complexity classes, and propositional proof systems. This allows us to reframe purely logical problems into algorithmic ones and vice versa. In this talk I will survey some of the key ideas behind Bounded Arithmetic and Proof Complexity, and explain some of the results and approaches that motivate me in these areas.