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Identifying Painlevé equations related to orthogonal polynomials: the geometric approach

Abstract: Painlevé equations, both differential and discrete, appear frequently in the study of orthogonal polynomials. For semi-classical orthogonal polynomials, this is often through a system of differential or discrete equations satisfied by the recurrence coefficients being transformable to the standard form of one of the Painlevé equations. However, given such a system, it can be a very nontrivial problem to determine whether it is related to a Painlevé equation and, if so, determining which type and transforming it to a standard form. Fortunately, the geometric theory of Painlevé equations provides an elegant and canonical way of doing precisely this. In this talk we outline this procedure and present a number of examples of it being applied to systems from orthogonal polynomials. We also discuss some further insights this geometric approach can lead to, beyond relating the differential or discrete systems to standard forms of Painlevé equations. These include determinantal expressions for the recurrence coefficients, Hamiltonian forms of the differential equations governing them, and transformations between differential systems coming from seemingly unrelated weights. This is based on collaborative work with Anton Dzhamay (University of Northern Colorado) and Galina Filipuk (University of Warsaw).