Oliver Penrose
a symposium in celebration of his work and achievements

Tuesday 7 December 2021

Abstracts

Viscoelasticity and monodromy
John Ball, Heriot-Watt University
Abstract: The talk will discuss how to verify via monodromy methods a nondegeneracy condition that allows one to prove convergence to equilibrium in models of one-dimensional viscoelasticity allowing for phase changes. This is joint work with Yasemin Sengul and Inna Capdeboscq.

Surface tension and tissue growth
Peter Fratzl, Max Planck Institute of Colloids and Interfaces, Potsdam
(with F. Dieter Fischer, John W.C. Dunlop)
Abstract: Growing biological tissues can be considered as fluids at sufficiently long time scales and their shape development is strongly influenced by surface tension [1]. We present simple mathematical models that account for the role of surface tension in describing the development of tissue shapes and the kinetics of tissue growth [2-4]. In recent experimental work, it was discovered that the effective surface tension is increased by cells contracting in near-surface regions [5-6]. Under certain conditions, this leads to chiral arrangements of contracting actin filaments within these cells [6] that make surface tension effectively anisotropic. We show that a modification of Young-Laplace's law can be used to describe the shape development in fluid-like tissues with anisotropic surface tension [7].

[1] D'Arcy Thompson's 'on Growth and form': From soap bubbles to tissue self-organization. Carl-Philipp Heisenberg, Mechanisms of Development 145:32-37 (2017).
[2] Modelling the role of surface stress on the kinetics of tissue growth in confined geometries. E. Gamsjäger, C. M. Bidan, F. D. Fischer, P. Fratzl, J. W. C. Dunlop.
Acta Biomaterialia 9, 5531–5543 (2013).
[3] Tissue growth controlled by geometric boundary conditions : a simple model recapitulating aspects of callus formation and bone healing. F. D. Fischer, G. A. Zickler, J. W. C. Dunlop, P. Fratzl. Journal of the Royal Society Interface 12, 20150108 (2015).
[4] The emergence of complexity from a simple model for tissue growth
J. W. C. Dunlop, G. A. Zickler, R. Weinkamer, F. D. Fischer, P. Fratzl.
Journal of Statistical Physics 180, 459–473 (2020).
[5] Tensile forces drive a reversible fibroblast-to-myofibroblast transition during tissue growth in engineered clefts. P. Kollmannsberger, C. M. Bidan, J. W. C. Dunlop, P. Fratzl, V. Vogel. Science Advances 4, eaao4881 (2018).
[6] Surface tension determines tissue shape and growth kinetics. S. Ehrig, B. Schamberger, C. M. Bidan, A. West, C. Jacobi, K. Lam, P. Kollmannsberger, A. Petersen, P. Tomancak, K. Kommareddy, F. D. Fischer, P. Fratzl, J. W. C. Dunlop. Science Advances 5, eaav9394 (2019)
[7] Anisotropic Young-Laplace-equation provides insight into tissue growth.
Peter Fratzl, F. Dieter Fischer, Gerald A. Zickler, John W.C. Dunlop, to be published.

Statistical mechanical ensembles and typical behavior of macroscopic systems
Joel L Lebowitz, Rutgers University
Abstract: Statistical mechanics is able to predict, with great certainty, behavior of individual macroscopic systems, both in equilibrium and out of it. I will relate this to the fact that this behavior is typical for systems represented by the usual microcanonical ensemble or those derived from it by constraining the microstates to regions corresponding to nonequilibrium macrostates both classically and quantum mechanically. These take probability to be proportional to phase space volume in classical systems and to "number of wave functions" in quantum systems.

Coverage and connectivity in stochastic geometry

Mathew Penrose, University of Bath

Consider a random uniform sample of size n over a bounded region A in R^d, $d \geq 2$, having a smooth boundary. The coverage threshold T_n is the smallest r such that the union Z of Euclidean balls of radius r centred on the sample points covers A. The connectivity threshold K_n is twice the smallest r required for Z to be connected. These thresholds are random variables determined by the sample, and are of interest, for example, in wireless communications, set estimation, and topological data analysis.

We discuss results on the large-n limiting distributions of T_n and K_n. For example, when $d=2$, and A has unit area.

The corresponding result in higher dimensions is more complicated, essentially because the `most isolated' point of the sample is likely to lie near the boundary of A. Similarly, the limiting behaviour of T_n is determined by boundary effects when $d \geq 3$.

Some of the work described here is joint work with Xiaochuan Yang.

Invariance Principle for the Random Lorentz Gas - Beyond the Boltzmann-Grad Limit

Bálint Tóth, University of Bristol and Rényi Institute Budapest

Invariance principle is proved for a random Lorentz-gas particle in 3 dimensions under the Boltzmann-Grad limit and simultaneous diffusive scaling. That is, for the trajectory of a point-like particle moving among infinite-mass, hard-core, spherical scatterers of radius r, placed according to a Poisson point process of density ϱ, in the limit $\varrho \to \infty$, $r \to 0$, $\varrho r^{2} \to 1$, up to time scales of order $T = o((r |\log r|)^{-2})$. This represents the first significant progress towards solving this problem in mathematically rigorous classical nonequilibrium statistical physics, since the ground-breaking work of Gallavotti (1969), Spohn (1978) and Boldrighini-Bunimovich-Sinai (1983). The novelty is that the diffusive scaling of particle trajectory and the kinetic (Boltzmann-Grad) limits are taken simultaneously. The main ingredients are a coupling of the mechanical trajectory with the Markovian random flight process, and probabilistic and geometric controls on the efficiency of this coupling.