Mean Field Games & minicourse

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Abstracts

Acciaio, Beatrice

Dynamic equilibrium via causal optimal transport

We study equilibrium problems in a dynamic setting, where agents are characterized by their types, and need to decide on their actions in time. Each agent faces a cost that depends on its own type and actions, as well as on the other agents' actions in a mean-field way. We use tools from dynamic optimal transportation of non-anticipative nature in order to get a characterization of competitive and cooperative equilibria, along with existence and uniqueness results.

The talk is based on an ongoing project with Julio Backhoff-Veraguas.

Aid, Rene

A mean-field of market exit.

We consider an attrition game with an infinite number of players representing a market where firms experience a downward pressure on price. We provide the PDEs giving the equilibrium density of players, show numerical results and provide economical interpretation of the dynamic of the system. Joint work with Charles Bertucci, Roxana Dumitrescu and Peter Tankov.

Alasseur, Clémence

An Extended Mean Field Game for flexibilities optimisation in smart grids

We consider a stylized model for a power system with distributed local power generation and flexibilities such as storage. This system is modeled as connection between a large number of nodes, where each node is characterized by a local electricity deman, has a local electricity production (e.g. photovoltaic panels), and manages a local flexibility such as a storage device. Depending on its instantaneous consumption and production rate as well as its flexibility management decision, each node may either buy or sell electricity, impacting the electricity spot price. The objective at each node is to minimize energy and flexibility costs by optimally controlling the flexibility. In a non-cooperative game setting, we are led to the analysis of a non-zero sum stochastic game with N players where the interaction takes place through the spot price mechanism. For an infinite number of agents, our model corresponds to an Extended Mean-Field Game (EMFG). We are able to compare this solution to the optimal strategy of a central planner and in a linear quadratic setting, we obtain and explicit solution to the EMFG and we show that it provides an approximate Nash-equilibrium for N-player game. We present extension of previous works considering jumps in the costs in order to represent the activation of demand-response contract.

Bertucci, Charles

Variational inequalities in Mean Field Games, optimal stopping and beyond

In this talk we present a quite general framework to express a MFG equilibrium of an optimal stopping game with variational inequalities (in the absence of common noise). We then show how this framework can be used in other types of games. Finally, we present a numerical method based on this framework to compute solutions of MFG system in non-potential cases.

Cardaliaguet, Pierre

Some aspects of the convergence problem in mean field games

In this talk, I will discuss the question of the convergence of N-player stochastic differential games to mean field game (MFG). I will first recall the so-called Folk's Theorem, which characterizes the Nash equilibria in memory strategies in N-player differential games and will explain that these equilibria do not converge to MFG equilibria in general. In memory strategies, players observes the controls of the other players.

In a recent example with C. Rainer (Brest), we show that, for ergodic games, the convergence does not even hold for a restrictive class of memory strategies where players observe the trajectories of the other players. This example is in sharp contrast with recent results by D. Lacker for finite horizon problems. Finally I will present positive convergence results towards MFG systems with a local coupling.

Carmona, Rene

Mean Field Games with finite states in the weak formulation, and application to contract theory.

Models for Mean Field Games (MFGs) with finite state spaces are typically introduced using controlled Markov chains and studied through the solutions of Hamilton-Jacobi-Belman and Fokker-Planck equations. We introduce the weak formulation based on change of measure techniques for stochastic integral equations and prove existence and uniqueness in this setting. We then apply these results to a contract theory problem in which a principal faces a field of agents interacting in a mean field manner. We reduce the problem to the optimal control of dynamics of the McKean-Vlasov type, and we solve this problem explicitly in a special case reminiscent of the linear - quadratic mean field game models. We conclude with a numerical example of epidemic containment.

Campi, Luciano

N-player games and mean-field games with smooth dependence on past absorptions

Mean-field games with absorption is a class of games, that have been introduced in Campi and Fischer (2018) and that can be viewed as natural limits of symmetric stochastic differential games with a large number of players who, interacting through a mean-field, leave the game as soon as their private states hit some given boundary. In this talk, we push the study of such games further, extending their scope along two main directions. First, a direct dependence on past absorptions has been introduced in the drift of players' state dynamics. Second, the boundedness of coefficients and costs has been considerably relaxed including drift and costs with linear growth. Therefore, the mean-field interaction among the players takes place in two ways: via the empirical sub-probability measure of the surviving players and through a process representing the fraction of past absorptions over time. Moreover, relaxing the boundedness of the coefficients allows for more realistic dynamics for players' private states. We prove existence of solutions of the mean-field game in strict as well as relaxed feedback form. Finally, we show that such solutions induce approximate Nash equilibria for the N-player game with vanishing error in the mean-field limit as N goes to infinity. This is based on a joint work with Maddalena Ghio and Giulia Livieri (SNS Pisa).

De Santis, Davide

Nonzero-sum stochastic differential games between an impulse controller and a stopper

We study a two-player nonzero-sum stochastic differential game where one player controls the state variable via additive impulses while the other player can stop the game at any time. The main goal of this work is characterize Nash equilibria through a verification theorem, which identifies a new system of quasi-variational inequalities whose solution gives equilibrium payoffs with the correspondent strategies. Moreover, we apply the verification theorem to a game with a one-dimensional state variable, evolving as a scaled Brownian motion, and with linear payoff and costs for both players. Two types of Nash equilibrium are fully characterized, i.e. semi-explicit expressions for the equilibrium strategies and associated payoffs are provided. Both equilibria are of threshold type: in one equilibrium the intervention regions of the players are separated, while in the other one they can overlap producing a situation where the first player induces her competitor to stop the game. Finally, we prove some asymptotic results with respect to the intervention costs.

Delarue, Francois

Stochastic forcing for mean field games with a finite state space

We address restoration of uniqueness for mean field games with a finite state space using a suitable stochastic forcing

Dumitrescu, Roxana

Mean-field games of optimal stopping: a relaxed solution approach

We consider the mean-field game where each agent determines the optimal time to exit the game by solving anoptimal stopping problem with reward function depending on the density of the state processes of agents still present in thegame. We place ourselves in the framework of relaxed optimal stopping, which amounts to looking for the optimal occupationmeasure of the stopper rather than the optimal stopping time. This framework allows us to prove the existence of the relaxedNash equilibrium and the uniqueness of the associated value of the representative agent under mild assumptions. Further, weprove a rigorous relation between relaxed Nash equilibria and the notion of mixed solutions introduced in earlier works on thesubject, and provide a criterion, under which the optimal strategies are pure strategies, that is, behave in a similar way tostopping times. Finally, we present a numerical method for computing the equilibrium in the case of potential games and showits convergence (joint work with Géraldine Bouveret and Peter Tankov)

Guo, Xin

Learning MFG

Motivated by the Ad auction problem for advertisers, we consider the general problem of simultaneous learning and decision-making in a stochastic game setting with a large population. We formulate this type of games with unknown rewards and dynamics as a generalized mean-field-game (GMFG), with incorporation of action distributions. We first analyze the existence of the solution to this GMFG, and show that naively combining Q-learning with the three-step fixed- point approach in classical MFGs yields unstable algorithms. We then propose an alternating approximating Q-learning algorithm and establish its convergence property. The numerical performance of this new algorithm on the repeated Ad auction problem shows superior computational efficiency.

Based on joint work with Anran Hu, Renyuan Xu of UC Berkeley and Junzi Zhang of Stanford.

Horst, Ulrich

Mean-Field Leader-Follower Games with Terminal State Constraint

We analyze linear McKean-Vlasov forward-backward SDEs arising in leader-follower games with mean-field type control and terminal state constraints on the state process. We establish an existence and uniqueness of solutions result for such systems in time-weighted spaces as well as a {convergence} result of the solutions with respect to certain perturbations of the drivers of both the forward and the backward component. The general results are used to solve a novel single-player model of portfolio liquidation under market impact with expectations feedback as well as a novel Stackelberg game of optimal portfolio liquidation with asymmetrically informed players. The talk is based on joint work with Guanxing Fu.

Hu, Kaitong

Principal-Agent problem with many principals

We study the principal-agent problem when there are many principals and the agent is allowed to switch from one principal to another. The switching process is described by a Poisson jump process whose intensity is controlled by the Agent. We first solve the agent's problem using dynamic programming and then we use the BSDE representation to transform this Stackelberg game problem into a control problem. Finally, we show by proving a theorem of backward propagation of chaos that the mean field version of the principal-agent problem with switching gives an approximation of the problem when the number of the principals is big.

Jaimungal, Sebastian

Mean-Field Games with Differing Beliefs for Algorithmic Trading

Even when confronted with the same data, agents often disagree on a model of the real-world. Here, we address the question of how interacting heterogenous agents, who disagree on what model the real-world follows, optimize their trading actions. The market has latent factors that drive prices, and agents account for the permanent impact they have on prices. This leads to a large stochastic game, where each agents' performance criteria is computed under a different probability measure. We analyse the mean-field game (MFG) limit of the stochastic game and show that the Nash equilibria is given by the solution to a non-standard vector-valued forward-backward stochastic differential equation. Under some mild assumptions, we construct the solution in terms of expectations of the filtered states. We prove the MFG strategy forms an epsilon-Nash equilibrium for the finite player game. Lastly, we present a least-squares Monte Carlo based algorithm for computing the optimal control and illustrate the results through simulation in market where agents disagree on the model.

[This is joint work with Philippe Casgrain. U. Toronto]

Kalise, Dante

Proximal methods for stationary Mean Field Games with local couplings

We address the numerical approximation of Mean Field Games with local couplings. For power-like Hamiltonians, we consider both unconstrained and constrained stationary systems with density constraints in order to model hard congestion effects. For finite difference discretizations of the Mean Field Game system, we follow a variational approach. We prove that the aforementioned schemes can be obtained as the optimality system of suitably defined optimization problems. In order to prove the existence of solutions of the scheme with a variational argument, the monotonicity of the coupling term is not used, which allow us to recover general existence results. Next, assuming next that the coupling term is monotone, the variational problem is cast as a convex optimization problem for which we study and compare several proximal type methods. These algorithms have several interesting features, such as global convergence and stability with respect to the viscosity parameter, which can eventually be zero. We assess the performance of the methods via numerical experiments.

Lacker, Daniel

Title: On the convergence of closed-loop Nash equilibria to the mean field game limit

This talk is about recent progress on the mean field game (MFG) convergence problem: In what sense do the Nash equilibria of n-player stochastic differential games converge to the mean field game? Previous work on this problem took two forms. First, when the n-player equilibria are open-loop, compactness arguments permit a characterization of all limit points of n-player equilibria as weak MFG equilibria, which contain additional randomness compared to the standard (strong) equilibrium concept. On the other hand, when the n-player equilibria are closed-loop, the convergence to the MFG equilibrium is known only when the MFG equilibrium is unique and the associated "master equation" is solvable and sufficiently smooth. We adapt the compactness arguments to the closed-loop case, proving a convergence theorem that holds even when the MFG equilibrium is non-unique.

Lehalle, Charles-Albert

Mean Field Games of Portfolio Trading

This talk is based on "A Mean Field Game of Portfolio Trading and Its Consequences On Perceived Correlations", co authored with Charafeddine Mouzouni. It starts by extending a previous paper ("Mean Field Game of Controls and An Application To Trade Crowding", with Pierre Cadaliaguet) to portfolios of correlated instruments. This leads to several original contributions: first that hedging strategies naturally stem from optimal liquidation schemes on portfolios. Second we show the influence of trading flows on naive estimates of intraday volatility and correlations. Focussing on this important relation, we exhibit a closed form formula expressing standard estimates of correlations as a function of the underlying correlations and the initial imbalance of large orders, via the optimal flows of our mean field game between traders. To support our theoretical findings, we use a real dataset of 176 US stocks from January to December 2014 sampled every 5 minutes to analyze the influence of the daily flows on the observed correlations. Finally, we propose a toy model based approach to calibrate our MFG model on data.

Mastrolia, Thibaut

Principal-mean field Agents problem and optimal energy demand response.

In this talk we present a Principal-Agent problem by considering a pool of agents monitored by one Principal. We show that the problem of the agents under fixed incentive policy is reduced to solve a mean field game and we rewrite the Principal's problem as a control problem of McKean−Vlasov SDEs. We apply our investigation to an optimal remuneration problem of correlated consumers in electricity demand. Based on joint works with Romuald Elie, Emma Hubert and Dylan Possamaï.

Miller, Enzo

Linear-quadratic McKean-Vlasov games with random coefficients

We propose a simple approach for solving linear-quadratic mean-field stochastic games. We study both finite-horizon and infinite-horizon problems, and allow some coefficients to be stochastic. Our method is based on an extension of the martingale formulation for verification theorems in control theory in addition to a fixed point problem in the space of controls. The optimal controls of each player involve the solutions to systems of Riccati ordinary differential equations and to linear mean-field backward stochastic differential equations; existence and uniqueness conditions are provided for such systems.

Nutz, Marcel

Convergence to the Mean Field Game Limit: A Case Study

Mean field games are generally interpreted as approximations to n-player games with large n. Indeed, n-player Nash equilibria are known to converge to their mean field counterpart when the latter is unique. In this talk we study a specific stochastic game where both the finite and infinite player versions naturally admit multiple equilibria. It turns out that mean field equilibria satisfying a transversality condition are indeed limits of n-player equilibria, but we also find a complementary class of equilibria that are not limits, thus questioning their interpretation as large n equilibria. (Joint work with Jaime San Martin and Xiaowei Tan)

Pham, Huyen

Some machine learning schemes for high-dimensional nonlinear PDEs

We propose new machine learning schemes for solving high dimensional nonlinear partial differential equations (PDEs). Relying on the classical backward stochastic differential equation (BSDE) representation of PDEs, our algorithms estimate simultaneously the solution and its gradient by deep neural networks.
In contrast with the recent approach in \cite{weinan2017deep}, these approximations are performed at each time step from the minimization of loss functions defined recursively by backward induction. The methodology is extended to variational inequalities arising in optimal stopping problems.
We analyze the convergence of the deep learning schemes and provide error estimates in terms of the universal approximation of neural networks.
Numerical results show that our algorithms give very good results till dimension 50 (and certainly above), for both PDEs and variational inequalities problems.
For the PDEs resolution, our results are very similar to those obtained by the recent method in \cite{weinan2017deep} when the latter converges to the right solution or does not diverge. Numerical tests indicate that the proposed methods are not stuck in poor local minima as it can be the case with the algorithm designed in \cite{weinan2017deep}, and no divergence is experienced. The only limitation seems to be due to the inability of the considered deep neural networks to represent a solution with a too complex structure in high dimension.
Based on joint work with Côme Huré (LPSM, Paris Diderot) and Xavier Warin (EDF and FiME).

Ren, Zhenjie

Mean-field Langevin dynamic and its application to neuron network

The scientific computation society and the emerging machine learning community are well aware of the use of Langevin dynamic for approximating the minimum of a convex function in real space, in particular of high dimension. In this talk, we shall see how to adapt this idea to the space of measures, and show that the invariant measures of some mean-field Langevin dynamics do approximate the minimum of a given convex function in the measure space. One of the applications (and motivations) of this study is to provide algorithms searching for the global minimizer of neuron network. At the end, we will discuss the open questions still puzzling us.

Yam, Phillip

Calculus on Space of Random Variables and Mean Field Theory

Mean field games and mean field type control problems, which aims at searching for the Nash equilibrium of differential games among a large number of players, have become so vital to disciplines ranging from engineering to economics and finance. In this talk, we shall introduce a new calculus technique to study the (infinite- dimensional) forward-backward stochastic differential equations of McKean-Vlasov type arisen from mean field games and mean field type problems. More specifically, after explaining this Hilbert space framework of random variables, we shall provide a sketch of proof of the global unique existence of the solution to mean field systems when the mean field effect is not too large.