Constructions and obstructions in birational geometry

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Abstracts

Nicolas Addington

Exoflops

The derived category of a complete intersection is equivalent to the category of matrix factorizations of a certain function on the total space of a vector bundle over the ambient space. The complete intersection is smooth if and only if the critical locus of the function is compact. I will present a construction through which a resolution of singularities of the complete intersection corresponds to a compactification of the critical locus of the function, which can be very interesting in examples. Kuznetsov and Lunts's categorical crepant resolution will make an appearance. (Some of you heard me talk about this phenomenon in 2015, but now I understand it much better. This is joint work with Paul Aspinwall and Ed Segal.)

Hamid Ahmadinezhad

Models of 3*-fold Mori fibre spaces*:

I will talk about a new approach to explicit classification of Mori fibre spaces in dimension three. Special attention is given to improving models of low degree del Pezzo surfaces over curves; a joint work with Maksym Fedorchuk and Igor Krylov.

Ekaterina Amerik

Contraction loci on hyperkaehler manifolds in families

This is joint work with Misha Verbitsky. Let X be an irreducible holomorphic symplectic manifold and z a Beauville-Bogomolov negative $(1,1)$-class. Using the ergodicity of the monodromy action, we prove some deformation-invariance statements for the locus covered by rational curves of class z in X.

Lev Borisov

Explicit equations of ball quotients.

I will talk about recent work on finding explicit equations of quotients of the two-dimensional complex ball, specifically a fake projective plane (joint with J. Keum) and Cartwright-Steger surface (joint with S. Yeung).

Gavin Brown

Flops and contraction algebras

I will describe an on-going project with Michael Wemyss (Glasgow) in which we construct smooth 3-fold flops from the ingredients of Donovan–Wemyss contraction algebras, which are certain finite-dimensional non-commutative algebras related to the flopping geometry, and their Calabi–Yau potentials. Our methods are enough to construct and distinguish certain flops that conventional invariants cannot.

Cinzia Casagrande

Fano 4-folds with rational fibrations

Smooth, complex Fano 4-folds are not classified, and we still lack a good understanding of their general properties. In the talk we will focus on Fano 4-folds with large second Betti number b_2, studied via birational geometry and the detailed study of their contractions and rational contractions. We recall that a contraction is a morphism with connected fibers onto a normal projective variety, and a rational contraction is given by a sequence of flips followed by a (regular) contraction.

The main result that we present is the following: let X be a Fano 4-fold having a rational contraction $X \to Y$ of fiber type (with $\dim Y > 0$). Then either X is a product of surfaces, or $b_2(X)$ is at most 17, or Y is P^1 or P^2.

Jean-Louis Colliot-Thélène

A few results on cubic surfaces and hypersurfaces

Two results on the generation of Chow groups of 1-cycles by lines; Chow-zero triviality of partially diagonal cubic hypersurfaces; stable rationality versus rationality for surfaces over a quasi-finite field.

Alessio Corti

Volume-preserving birational maps of PP3

After a short summary of the Sarkisov program for Mori fibred Calabi–Yau pairs, I report on joint work with Araujo and Massarenti on volume preserving birational maps of PP3.

Enrica Floris

On the b-Semiampleness conjecture

An lc-trivial fibration $f: (X, B) \to Y$ is a fibration such that the log-canonical divisor of the pair (X, B) is trivial along the fibres off. As in the case of the canonical bundle formula for elliptic fibrations, the log canonical divisor can be written as the sum of the pullback of three divisors: the canonical divisor of Y; a divisor, called discriminant, which contains informations on the singular fibres; a divisor, called moduli part that contains informations on the variation in moduli of the fibres. The moduli part is conjectured to be semiample. Ambro proved the conjecture when the base Y is a curve. In this talk we will explain how to prove that the restriction of the moduli part to a hypersurface is semiample assuming the conjecture in lower dimension. This is a joint work with Vladimir Lazić.

Sergey Gorchinskiy

Categorical measures for varieties with finite group actions

This talk is based on a common work with D. Bergh, M. Larsen, and V. Lunts. Given a variety with a finite group action, we compare categorical measures of the corresponding quotient stack and the extended quotient. Using weak factorization for orbifolds, we show that for a wide range of cases, these two measures coincide. This implies, in particular, a conjecture of Galkin and Shinder on categorical and motivic zeta-functions of varieties. We provide examples showing that, in general, these two measures are not equal.

Anne-Sophie Kaloghiros

Maximal intersection Calabi–Yau pairs

A Calabi–Yau (CY) pair (X, D) consists of a normal projective variety and a reduced anti-canonical integral divisor D on it. Such pairs come up in a variety of contexts; for example, cluster varieties are obtained by glueing CY pairs by crepant birational maps. Recent developments in mirror symmetry suggest that cluster varieties are "natural" mirror partners of Fano varieties with a toric degeneration, and that understanding the birational geometry of CY pairs is an important step in the study of mirror symmetry and moduli of Fano varieties. In this talk, I will discuss the birational geometry of CY pairs and some 3-fold examples.

Alexander Kuznetsov

Semiorthogonal decompositions of singular surfaces

It is well known that any smooth projective toric surface has a full exceptional collection. I will talk about a generalization of this fact for singular surfaces. First, if the class group of Weil divisors of the surface is torsion free (for instance, this holds for all weighted projective planes), I will construct a semiorthogonal decomposition of the derived category with components equivalent to derived categories of modules over certain local finite dimensional algebras. When the class group has torsion, a similar semiorthogonal decomposition will be constructed for an appropriately twisted derived category. Many of these results extend to non-necessarily toric rational surfaces. This is a joint work with Joseph Karmazyn and Evgeny Shinder.

Diane Maclagan

Tree compactifications of the moduli space of genus zero curves

The moduli space $\overline{M}_{0,n}$ of stable genus zero curves with n marked points can be constructed as the closure of $M_{0,n}$ inside a toric variety. The fan of the toric variety is moduli space of phylogenetic trees. I will discuss joint work with Dustin Cartwright to construct other compactifications of $M_{0,n}$ by varying the toric variety using variants of phylogenetic trees. These compactifications include many of the standard alternative compactifications of $M_{0,n}$.

John Ottem

The integral Hodge conjecture for curve classes

The Hodge conjecture predicts which rational cohomology classes on a smooth complex projective variety can be represented by linear combinations of complex subvarieties. The integral Hodge conjecture, the analogous conjecture for integral homology classes, is known to be false in general (the first counterexamples were given in dimension 7 by Atiyah and Hirzebruch). I'll give a short survey talk on this conjecture, and present some new counterexamples in dimension three. This is joint work with Olivier Benoist.

Rita Pardini

Higher dimensional Clifford-Severi equalities

Let X be a projective variety, let $a: X \to A$ be a morphism to an abelian variety with $\dim a(X) = \dim \mathrm{X}$. Given a line bundle L on X, we define its continuous rank $h^0_a(L)$ as the generic value of $h^0(L+P)$, where P is the pullback of an element of $\mathrm{Pic}^0(A)$. The slope $\lambda(L)$ is the ratio $\mathrm{vol}(L)/h^0_a(L)$; the Clifford-Severi inequalities are lower bounds for the slope that generalize at the same time the classical Severi inequality for surfaces of general type and Clifford's theorem for line bundles on a curve. I will describe a characterization of:

a) the triples (X, a, L) attaining the lower bound $\lambda(L) = n!$ (1st Clifford-Severi line)
b) the triples (X, a, L) with K_{X-L} pseff attaining the lower bound $\lambda(L) = 2n!$ (2nd Clifford-Severi line).

This is joint work with M.A. Barja and L. Stoppino.

Marta Pieropan

On rationally connected varieties over C_1 fields of characteristic 0

In the 1950s Lang studied the properties of C_1 fields, that is, fields over which every hypersurface of degree at most n in a projective space of dimension n has a rational point. Later he conjectured that every smooth proper rationally connected variety over a C_1 field has a rational point. The conjecture is proven for finite fields (Esnault) and function fields of curves over algebraically closed fields (Graber–Harris–de Jong–Starr), but it is still open for the maximal unramified extensions of p-

adic fields. I use birational geometry in characteristic 0 to reduce the conjecture to the problem of finding rational points on Fano varieties with terminal singularities.

Piotr Pokora

On the bounded negativity conjecture

The main aim of my talk is to present some recent developments on the bounded negativity conjecture. The conjecture states that for every smooth complex projective surface X there exists an integer $b(X)$ such that for all irreducible and reduced curves $C \subset X$ one has $C^2 \geq -b(X)$. There are some surfaces for which the BNC holds, for instance surfaces with the canonical divisor $-K_X$ being $\mathbb{Q}$-effective, but in general the question is open. It is notoriously difficult to predict where a certain blowing up of a given surface has bounded negativity, for instance it is not known whether the blowing up of the complex projective plane along 10 very general points has bounded negativity. In my talk I am going to discuss the current state of the art in that field especially focusing on the effective bounds on the self-intersection numbers of certain classes of curve on blow ups of surfaces with Kodaira dimension ≤ 0.

Elisa Postinghel

Mutations of Newton–Okounkov polytopes and classification of Fano manifolds

Mutations are certain operations of Laurent polynomials, or of their Newton polytopes, that induce deformations between the corresponding toric varieties, as showed by Ilten. Mutations of Fano polytopes are studied by Corti et al. in order to classify Fano manifolds up to deformation using the mirror symmetry approach. On the other hand, to a pair given by a projective algebraic variety and a line bundle, we can associate the so called Okounkov bodies, that are a generalisation of Newton polytopes. In this talk, I will show how to obtain mutations of Okounkov polytopes, and hence toric deformations, of certain Fanos. This is join work (in progress) with A. Laface and S. Urbinati.

Claire Voisin

Decomposition of the diagonal and nilpotence

The notion of decomposition of the diagonal of an algebraic variety appears in the work of Bloch and Srinivas, at least with rational coefficients. With integral coefficients, it has recently played an important role in the study of rationality questions. In this talk I will present this notion and discuss the relationships between the cohomological and Chow decompositions of the diagonal, the former being much better understood.

Susanna Zimmermann

Quotients of Cremona groups

I will explain how the Sarkisov program and relations between Sarkisov links allow us to construct a surjective homomorphism from the Cremona groups in higher dimension to a (non-trivial) abelian group, whose kernel contains an infinite number of normal subgroups of index 2.

Francesco Zucconi

Rationality of special subloci in the moduli space of spin curves

I will show how to use the birational geometry of some well-known Fano 3-folds to prove the rationality of the moduli space of spin hyperelliptic curves and of genus 4 spin curves with a unique trigonal series.