Why did mathematicians not embrace the vision of the QED manifesto?

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The QED Manifesto.

The QED Manifesto, in: A. Bundy (ed.), Automated Deduction. CADE–12, 12th International Conference on Automated Deduction, Nancy, France, June 26–July 1, 1994, Proceedings. Lecture Notes in Computer Science 814, Springer 1994, pp. 238–251.

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> *[The QED Manifesto] paints a future in which most of mathematics will be put in the computer, even the proofs —especially the proofs—in such a way that the computer will be able to check it for correctness. The QED manifesto describes the development of a system ... that mathematicians will adopt for this purpose. The future that the QED manifesto sketches has not happened. (Wiedijk, 2007).*

Why? (1)

The QED manifesto itself lists a number of reasons why the project might fail:

> *1. Too much code to be trusted; 2. Too strong a logic; 3. Too limited a logic; 4. Too unintelligible a logic; 5. Too unnatural a syntax; 6. Parochialism; 7. Too little extensibility; 8. Too little heuristic search support; 9. Too little care for rigour; 10. Complete absence of inter-operability; 11. Too little attention paid to ease of use.*

Why? (2)

Freek Wiedijk, The QED Manifesto Revisited, *Studies in Logic, Grammar and Rhetorik* 10(23): 2007, pp. 121–133.

> § *1. Why the QED manifesto has not been a success (yet). The most important reason is that only very few people are working on formalization of mathematics. [...] The other reason that there has not been much progress on the vision from the QED manifesto is that currently formalized mathematics does not resemble real mathematics at all. Formal proofs look like computer program source code. For people who do like reading program source code that is nice, but most mathematicians, the target audience of the QED manifesto, do not fall in that class.*

Why? (3)

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A system that implements the QED manifesto should be usable to people who are not aware of the existence of constructive mathematics. The possibility to do constructive mathematics with the system, if at all present, should be hidden to people who are not interested in it. ... [F]or logicians and other philosophers this is all very interesting, but classical mathematicians should not be bothered by these issues. This kind of fine structure of the axiomatics probably does not interest them.

Why? (4)

Marcos Cramer, The Naproche system: Proof-checking mathematical texts in controlled natural language, *Sprache und Datenverarbeitung* 38:1-2 (2014), pp. 9–33.

> *Formal mathematics is a branch of mathematics that aims at developing substantive parts of mathematics in a purely formal way. This is usually done with the help of computer programs, and the usual input language of formal mathematics systems have more resemblance with programming languages than with the natural language of mathematics. We think that this is one of the reasons why formal mathematics is not widely used by mathematicians outside the circles of the relatively small formal mathematics community.*

The traditional narrative.

Mathematicians are conservative luddites who resist change. Accepting formal mathematics would require them to write in a programming language and understand something about axiomatic frameworks; both are unacceptable to the majority of mathematicians.

In order to make formal proofs appealing to them, we need to re-package them in such a way that they are as similar as possible to ordinary (natural language) mathematical vernacular.

Claim.

The traditional narrative is too simplistic: mathematicians do embrace some innovations that resemble programming languages.

If it is not the format in which formal proofs are written that is the reason for the mathematicians' rejection, we need to ask: why don't the arguments in favour of formal mathematics convince the mathematicians; is there something fundamental that is being changed by the adoption of formal proofs that could explain the rejection?

These questions are closely linked to a number of questions that have been raised in *Philosophy of Mathematical Practice*.

Interactions between mathematics and formal mathematics.

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Hales's proof of the Kepler conjecture.

Thomas Hales, A proof of the Kepler conjecture, *Annals of Mathematics* 162:3 (2005), pp. 1065–1185.

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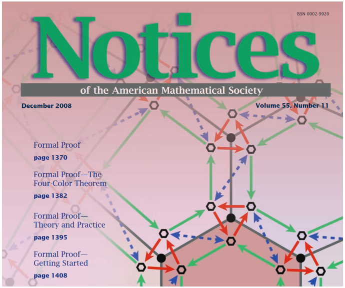
Issue of the Notices of the AMS devoted to formal proof.



Mathematics and programming.

Many or most mathematics undergraduate programmes contain courses on programming. Programming skills are expected of maths graduates, not just as a preparation for future programming jobs, but as a genuine mathematical research skill in some parts of mathematics.

LaTeX.

Pre-1980s. Mathematical typesetting is a highly specialised skill, usually not done by mathematicians.

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Like proof assistants, the use of LaTeX channels certain practices and has an effect on traditions and reader expectations.

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If the rejection cannot be explained solely by the reluctance of mathematicians to accept formal code and its effects on practice, we need to evaluate the arguments in favour of the use of proof assistants:

1. *Correctness check.* "To improve the actual precision, explicitness, and reliability of mathematics. (Harrison, 2008)."
2. *Support for refereeing.* "Supplementing, or even partly replacing, the process of peer review for mainstream mathematical papers with an objective and mechanizable criterion for the correctness of proofs. (Harrison, 2008)"

Links to philosophy of mathematical practice (1).

Epistemology of Mathematics. *Traditional view:* formal proofs are the truthmakers of mathematics; informal proof approximates formal proof ("points to it", "sketches it", etc.). Certainty of mathematics derives from the fact that formal proofs are checkable.

> *In Bourbaki's view, the foundations of mathematics are roped-off museum pieces to be silently appreciated, but not handled directly. There is an opposing view that regards the foundational enterprise as unfinished until it is realized in practice and written down in full. (Hales, 2008).*

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Eva Müller-Hill, *Die epistemische Rolle formalisierbarer mathematischer Beweise. Formalisierbarkeitsorientierte Konzeptionen mathematischen Wissens und mathematischer Rechtfertigung innerhalb einer sozio-empirisch informierten Erkenntnistheorie der Mathematik*, Ph.D. thesis, Rheinische Friedrich-Wilhelms-Universität Bonn (2011).

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Arthur Jaffe, Frank Quinn, "Theoretical mathematics": Toward a cultural synthesis of mathematics and theoretical physics. *Bulletin of the American Mathematical Society* 29 (1993), pp. 1–13.

Links to philosophy of mathematical practice (2).

The role of mathematical peer review. Is the main role of mathematical peer review the checking of the correctness of the claims?

Christian Geist, Benedikt Löwe, Bart Van Kerkhove, Peer review and knowledge by testimony in mathematics, in: Benedikt Löwe, Thomas Mller, PhiMSAMP, Philosophy of Mathematics: Sociological Aspects and Mathematical Practice, London 2010 [Texts in Philosophy 11], pp. 155–178.

(Annika Dreher, Aiso Heinze, Christian Greiffenhagen.)

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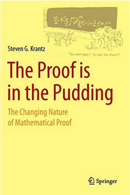

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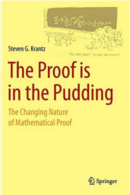

- Understand the fundamental features of mathematics or mathematical practice that do not match with some of the changes that increased use of proof assistants would entail.