Lucas Mason-Brown, “What is a unipotent representation?”
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**Abstract**: Let $G$ be a connected reductive algebraic group, and let $G(\mathbb{F}_q)$ be its group of $\mathbb{F}_q$-rational points. Denote by $\mathrm{Irr}(G(\mathbb{F}_q))$ the set of (equivalence classes) of irreducible finite-dimensional representations. Deligne and Lusztig defined a finite subset

$$\mathrm{unip}(G(\mathbb{F}_q)) \subset \mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$$

of *unipotent representations*. These representations play a distinguished role in the representation theory of $G(\mathbb{F}_q)$. In particular, the classification of $\mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$ reduces to the classification of $\mathrm{unip}(G(\mathbb{F}_q))$.

Now replace $\mathbb{F}_q$ with a local field $k$ and replace $\mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$ with $\mathrm{Irr}_{\mathrm{u}}(G(k))$ (irreducible unitary representations). Vogan has predicted the existence of a finite subset

$$\mathrm{unip}(G(k)) \subset \mathrm{Irr}_{\mathrm{u}}(G(k))$$

which completes the following analogy

$$\mathrm{unip}(G(k)) \text{ is to } \mathrm{Irr}_{\mathrm{u}}(G(k)) \text{ as } \mathrm{unip}(G(\mathbb{F}_q)) \text{ is to } \mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$$

In this talk I will propose a definition of $\mathrm{unip}(G(k))$ when $k = \mathbb{C}$. The definition is geometric and case-free. The representations considered include all of Arthur's, but also many others. After sketching the definition and cataloging its properties, I will explain a classification of $\mathrm{unip}(G(\mathbb{C}))$, generalizing the well-known result of Barbasch-Vogan for Arthur's representations. Time permitting, I will discuss some speculations about the case of $k = \mathbb{R}$.

This talk is based on forthcoming joint work with Ivan Loseu and Dmitryo Matvieievskyi.