Modern Applied Probability. A workshop in celebration of Sergey Foss' 65th birthday
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Abstracts

Venkat Anantharam
Asmussen-Sigman duality for monotone recursions on graphs and unimodular random rooted graphs. Dedicated to Sergey Foss on the occasion of his 65th birthday

We start by viewing Asmussen-Sigman duality as a duality between two nonnegative real interval-valued evolutions running in opposite directions in time, driven by a parameterized family of monotonically increasing relations in the product of two copies of the nonnegative real numbers. When the parameters form an i.i.d. sequence this viewpoint is compatible with the Lindley equation with its well-known dual, and is compatible with Siegmund duality. When the parameters form a stationary sequence, this viewpoint is compatible with the Loynes scheme and with more general monotone stochastic recursions with state space the nonnegative real numbers, with their duality theory.

We then demonstrate how this pathwise duality can be naturally extended to a pathwise duality for nonnegative real interval-valued processes indexed by the directed edges of graphs with finite or countably infinite vertex sets. The vertices are marked by the parameters and a permutation on the incident edges. We also develop this duality for unimodular random rooted marked graphs with such vertex marks, which is the natural analog for stationary stochastic processes that are agnostic to the direction of time. The structure of the evolution is most transparent when one looks at the universal cover.

Søren Asmussen
Insurance premium control and game equlibriums in some Bayesian contexts
A standard model in non-life insurance is that the Poisson rates at which customers in a portfolio generate claims form an i.i.d. sample from some distribution. Based on this, we determine the portfolio size for a given premium in a market with only one company when customers use an utility-based criterion for choosing whether to insure or not. This is then applied to ruin probability minimization. Similar ideas are used for the competition between two companies and applied to determine Nash- or Stackelberg equilibrium premium strategies. The market may either be frictional with Bayesian preferences or involve product differentiation. The talk is based on joint work with Bent Jesper Christensen, Michael Taksar and Julie Thøgersen.

Francois Baccelli
Wireless birth and death processes
This talk features networks of coupled processor sharing queues in the Euclidean space, where customers arrive according to independent Poisson point processes at every queue, are served, and then leave the network. The coupling is through service rates. In any given queue, this rate is inversely proportional the interference seen by this queue, which is determined by the load in neighbouring queues, attenuated by some distance-based path-loss function. The main focus is on the infinite grid network and translation invariant path-loss case.

The model is a discrete version of a spatial birth and death process where customers arrive to the Euclidean space according to Poisson rain and leave it when they have transferred an exponential file, assuming that the instantaneous rate of each transfer is determined through information theory by the signal to interference and noise ratio experienced by the user.

The stability condition is identified. The minimal stationary regime is built using coupling from the past techniques. The mean queue size of this minimal stationary regime is determined in closed form using the rate conservation principle of Palm calculus. When the stability condition holds, for all bounded initial conditions, there is weak convergence to this minimal stationary regime; however, there exist translation invariant initial conditions for which all queue sizes converge to infinity.

Joint work with S. Foss and A. Sankararaman

Onno Boxma
On two classes of reflected stochastic processes
We introduce two general classes of reflected stochastic processes, INGAR+ and GAR+, INGAR+ being a generalization of the integer-valued INAR(1) process and GAR+ a generalization of the AR(1) process. The two processes are shown to be connected via a duality relation. We present a detailed analysis of the transient and steady-state behavior of the INGAR+ process, and then use the duality to obtain the transient and steady-state behavior of the GAR+ process.
Joint work with Andreas Lopker and Michel Mandjes.

Mikhail Chebunin
Spatially decentralized protocols in random multiple access networks
We analyse a spatial ALOHA-type random multiple-access protocol in a stochastic network with local interactions. We obtain a sufficient condition for the stability of this protocol. We discuss the necessary condition. Furthermore, for the underlying Markov chain, we estimate the rate of convergence to the stationary distribution. Finally, we compare various ALOHA-type protocols in a symmetric network.

Denis Denisov
First passage times for random walks over moving boundaries.
We consider one-dimensional random walks over a moving boundary. We will discuss the asymptotic behaviour of the first exit times below the moving boundary.
This talk is based on joint works with V. Wachtel and A. Sakhanenko.

Sergey Foss
TBA

Istvan Gyongy
On Itô's formula for jump processes
First we revisit the well-known Itô formula for processes with Poisson random measures. Then we present a suitable generalisation of it to prove energy equalities in L_p spaces for stochastic integro-differential equations arising in filtering theory. This result we use to obtain existence and uniqueness theorems for filtering equations of jump diffusions. The talk is based on a recent joint work with Sizhou Wu.

Jennie Hansen
Structural transition in random mappings via urn schemes
Random mapping models have been studied by various authors since the 1950's and have applications in modelling epidemic processes, the analysis of cryptographic systems (e.g. DES) and of Pollard's algorithm, and random number generation. In this talk we use urn schemes and some related results for random mappings with exchangeable in-degrees to investigate the distributions of variables (e.g. number of connected components, size of a 'typical' component, the normalised order statistics of the component sizes) associated with the structure of the random directed graph which represents a mapping. In particular, we consider how the structure of these random directed graphs is affected by imposing ever greater restrictions on the in-degree sequence of vertices of random directed graphs. The results presented are based a calculus for random mappings with exchangeable in-degrees that has been developed in a series of papers by Hansen and Jerzy Jaworski, (Adam Mickiewicz University), who was supported by the Marie Curie Intra-European Fellowship No. 236845 (RANDOMAPP) within the 7th European Community Framework Programme.

Takis Konstantopoulos
Probabilistic and combinatorial analysis of a simple computer security model
We consider a simple stochastic model of bad guys attacking good computers who must defend themselves by frequent inoculations. We discuss queueing network interpretations and derive exact and asymptotic formulas for distributions of quantities of interest.

Dmitry Korshunov

Strong Law of Large Numbers for Function of Local Times of Transient Random Walk in Z^d

For an arbitrary transient random walk S_n on Z^d, $\mathrm{d}\geq 1$, we prove a strong law of large numbers for spatial sum of a function of its local times, $\mathrm{f}(\mathrm{l}(\mathrm{n},\mathrm{x}))$. Particular cases are given by strong laws of large numbers for the number

(a) of visited sites (first time considered by Dvoretzky and Erdös (1951), which corresponds to a function $\mathrm{f}(\mathrm{i})=\mathrm{I}\{\mathrm{i}\geq 1\}$;

(b) of α-fold self-intersections of the random walk (studied by Becker and König (2009)), which corresponds to $\mathrm{f}(\mathrm{i})=\mathrm{i}^{\alpha}$;

(c) of sites visited by the random walk exactly j times (considered by Erdös and Taylor (1960) and by Pitt (1974)), where $\mathrm{f}(\mathrm{i})=\mathrm{I}\{\mathrm{i}=\mathrm{j}\}$.

Artem Kovalevskii

An elementary probabilistic model of literary texts and Poissonization

Bahadur (1960) studied an infinite urn scheme in which probabilities decrease according to a power law. He found asymptotics for growth of the expectation of the number of nonempty urns and proved the Law of Large Numbers. Note that the decrease in the probabilities of words according to a power law for natural language texts is known as Zipf's law (1936), and the growth of the number of different words in accordance with a power law in linguistics is known as the Heaps law (1978). Karlin (1967) proposed Poissonization of the Bahadur scheme, that is, the length of the text has a Poisson distribution and does not depend on the urn choosing process. Karlin proved the Central Limit Theorem for the number of nonempty urns using this technique. Barbour (2009) found a more accurate Poisson approximation for the number of nonempty urns. Chebunin and Kovalevskii (2016) proved the Functional Central Limit Theorem using the Karlin method.

We want to apply FCLT to analyze homogeneity of natural language texts. To do this, we need an estimate of the Zipf parameter such that asymptotics of its joint distribution with the process of the number of non-empty urns can be found. We have proposed a class of such estimates. We proved their strong consistency. Using such an estimate, we construct a process which asymptotic behavior is known for the case of homogeneous text. The homogeneity of literary texts is analyzed with a statistical test of omega-square type.

The research is joint with Mikhail Chebunin. The research was supported by RFBR grants 17-01-00683, 19-51-15001.

Guenter Last

Doubly stable queueing systems in space

In a seminal paper Gale and Shapley (1962) introduced stable matchings in a finite and deterministic setting. Following closely Holroyd, Pemantle, Peres and Schramm (2009), we shall first discuss a few basic properties of stable matchings between two point processes. In the second part of the talk we shall consider a stable matching between the cubic lattice and a stationary Poisson process (or a determinantal point process) with higher intensity. Then all lattice points will be matched, while the matched Poisson points can be interpreted as departure process of a stable spatial queue. In fact, a one-sided version of the stable matching on the line can be interpreted as a queueing system with a LIFO rule. We have not seen the two-sided version (on the line) in the queueing literature. The matched Poisson points form a stationary and ergodic (under lattice shifts) point process with unit intensity, that has many remarkable properties. If the intensity of the Poisson process is close to one, then it very much resembles a Poisson process, while for large intensities it approaches the lattice. Moreover, the matched points form a hyperuniform and number rigid point process, in sharp contrast to a Poisson process. Still the pair correlation decays exponentially fast. The talk is based on joint work with M. Klatt and D. Yogeshwaran.

Nelly Litvak
Local weak convergence for PageRank
PageRank is the algorithm originally proposed by Google for ranking pages in the World-Wide Web. In this work we investigate the existence of an asymptotic PageRank distribution, when the graph size goes to infinity, using the recent notion of local weak convergence for sparse graphs. We start from the definition of local weak convergence for sequences of (random) undirected graphs, and extend this notion to directed graphs. Then we use this to prove the existence of an asymptotic PageRank distribution. As a result, the limiting distribution of PageRank can be computed directly as a function of the limiting object. We apply our results to the directed configuration model and continuous-time branching processes trees, as well as to preferential attachment models. This is a joint work with Remco van der Hofstad and Alessandro Garavaglia.

Masakiyo Miyazawa
A martingale view of Blackwell's renewal theorem and its extensions to a general counting process
Martingales constitute a basic tool in stochastic analysis. We consider their application to counting processes. Using this tool, we revisit a renewal theorem and its extensions for various counting processes. For this, we first consider a renewal process as a pilot example, deriving a new semimartingale representation that differs from the standard decomposition via the stochastic intensity function. We then revisit Blackwell's renewal theorem, its refinements and extensions. Based on these observations, we extend the semimartingale representation to a general counting process, and give conditions under which asymptotic behaviour similar to Blackwell's renewal theorem holds. Various scenarios are considered including a modulated renewal process and a point process with stationary interarrival times. This is joint work with Daryl J. Daley of The University of Melbourne.

Ilya Molchanov
Scale invariant stochastic processes arising from general iterative schemes
It is well understood that sequences of backward iterations of random Lipschitz functions converge almost surely under suitable conditions on the Lipschitz constants. We suggest a sieving scheme based on considering Poisson processes marked by Lipschitz functions in order to couple these iterative constructions and to come up with a scale invariant limiting process. The conditions on the uniform convergence to this process are found; it is shown that this process is cadlag and of a finite total variation. Several examples arising from Bernoulli convolutions, general perpetuities, maximum schemes and continued fractions are presented. Ilya Molchanov (University of Bern) joint work with Alexander Marynych (Kiev University).

Thomas Mountford
Many greedy servers in a Poisson environment
We consider as a state space a large number, N, of half lines meeting at the origin. On each half line we place a rate one Poisson process of dust particles. Initially we have N^{α} greedy servers that independently jump according to rate one Poisson processes to the nearest dust particle. Each server must eventually "choose" a halfline on which to stay. We investigate for which alpha do there exist halflines chosen by more than one server.
Joint work with Sergey Foss

Zbigniew Palmowski
Persistence of heavy-tailed sample averages occurs by infinitely many jumps
We consider the sample average of a centered random walk in Rd with regularly varying step size distribution. For the first exit time from a compact convex set A not containing the origin, we show that its tail is of lognormal type. Moreover, we show that the typical way for a large exit time to occur is by having a number of jumps growing logarithmically in the scaling parameter.
This is based on joint work [1] with Ayan Bhattacharya and Bert Zwart.
[1] A. Bhattacharya, Z. Palmowski, B. Zwart (2019)
Persistence of heavy-tailed sample averages occurs by infnitely many jumps,
submitted for publication, https://arxiv.org/abs/1902.09922.

Kavita Ramanan
TBA

Tomasz Rolski

Remarks on the concept of scale functions.

By a scale function $W^{(q)}$ for a spectrally negative Levy process $X(t)$, defined by (γ, σ^2, Π) with exponent function $\psi(\theta)$ we mean a function, being zero for negative and

$$\int_0^\infty e^{-sx} W^{(q)}(x)\, dx = \frac{1}{\psi(s)-q},$$

for enough big s. It serves to state different fluctuation identities. In [1] the concept was generalised for a larger class of processes $U(t)$, which are the unique solution of SDE

$$dU(t) = dX(t) - \phi(U(t)),$$

where ϕ fulfills some conditions:

1. The function ϕ is non-decreasing and for fixed $d' \in R$, $\phi(x) = 0$ for $x \le d'$. Furthermore ϕ is either locally Lipschitz continuous or ϕ is of the form

$$\phi_k(x) = \sum_{j=1}^k \delta_j 1_{\{x \ge b_j\}}, \tag{1}$$

In the bounded variation case, we assume that $\phi(x) < c$ for all $x \in R$.

For such the processes we find that the scale function $w^{(q)}$ is the solution of the following integro-differentional equation

$$w^{(q)}(x; d) = W^{(q)}(x - d) + \int_d^x W^{(q)}(x - y)\phi(y) w^{(q)\prime}(y; d)\, dy, \tag{2}$$

Define now an operator

$$A_{(\gamma,\sigma,\Pi)} f(x)$$

$$= (\gamma - \phi(x)) f'(x) + \frac{\sigma^2}{2} f''(x) + \int_0^\infty (f(x - y) - f(x) + f'(x) y 1_{y \le 1})\, \Pi(dy).$$

In the talk we will discuss relations and appearing problems between the solution of

$$Aw^{(q)} = qw^{(q)},$$

and solutions of the integro-differential equation (2).

References I. Czarna, J.-L. Pérez, T. Rolski and K. Yamazaki Fluctuation theory for Level-dependent Lévy risk processes. SPA.

Seva Shneer

Stability conditions for a discrete-time decentralised medium access algorithm

We consider a stochastic queueing system modelling the behaviour of a wireless network with nodes employing a discrete-time version of the standard decentralised medium access algorithm. The system is unsaturated -- each node receives an exogenous flow of packets at the rate λ packets per time slot. Each packet takes one slot to transmit, but neighboring nodes cannot transmit simultaneously. The algorithm we study is standard in that a node with empty queue does not compete for medium access and the access procedure by a node does not depend on its queue length, as long as it is non-zero. Two system topologies are considered, with nodes arranged in a circle and in a line. We prove that, for either topology, the system is stochastically stable under condition λ<2/5. This result is intuitive for the circle topology as the throughput each node receives in a saturated system (with infinite queues) is equal to the so-called parking constant, which is larger

than 2/5. (The latter fact, however, does not help to prove our result.) The result is not intuitive at all for the line topology as in a saturated system some nodes receive a throughput lower than 2/5.
This is a joint work with Sasha Stolyar (UIUC).

Sasha Stolyar
Join-Idle-Queue with Service Elasticity: Stability and Asymptotics
A parallel server system with n identical servers is considered. The service is "elastic" in that active servers are turned off after some timeout if they stay idle, and need a "warm up" time to become active again. The load distribution strategy includes Join-Idle-Queue routing, under which an arriving customer is sent to an idle active server, if any, plus a simple algorithm for activation/deactivation of servers. Under Markov assumptions and strictly subcritical load, we prove that the system is (a) stochastically stable for large n and (b) the sequence of (appropriately scaled) stationary distributions concentrates at the optimal equilibrium point, where the servers are either busy or "off," and the probability of waiting vanishes.
Joint work with D. Mukherjee (Brown University)

Vitali Wachtel
Persistence of AR(1)-sequences}

Let ξ_k be independent, identically distributed random variables and let $a \in (0,1)$ be a fixed constant.

An AR(1) sequence is defined by $X_n = aX_{n-1} + \xi_n, \quad n \geq 1,$ where the starting point X_0 of this process may be either deterministic or distributed according to any probabilistic measure ν. We are interested in the tail behaviour of the stopping time

$$T_0 := \min\{k \geq 1 : \ X_k \leq 0\}.$$

We find minimal moment conditions on the innovations, under which one has

$$\mathbf{P}_x(T_0 > n) \sim V(x)e^{-\lambda n}, \ n \to \infty,$$

where λ is a positive number.

Bert Zwart
Cascading failures in power grids and heavy tails
Power grids occasionally suffer from blackouts which occur through cascading failures. Such cascades are very complex, partly due to the physics of power flow. Data, however, reveal that the total size of a blackout has a regularly varying tail. A key question is how this tail behaviour emerges. In this talk, I will make the case for an explanation that contradicts the folklore in the physics and engineering literature. The case will be made with actual data, a detailed case study, and a mathematical model, including a theorem.

Sergei Zuyev
Selfdecomposable point processes and limit theorems for superpositions
Selfdecomposable point processes constitute the class of point processes arising as a limit of superpositions of independent point processes. They are a subclass of infinitely divisible processes (the limits in uniform asymptotic negligible array schemes) and contain stable processes (the limits of superposition of iid point processes). We characterise selfdecomposable processes for the most general scheme involving independent branching operation on points, in particular, thinning.