Geometric Analysis

28 May – 1 June 2018

International Centre for Mathematical Sciences, Edinburgh

Abstracts

Buzano, Reto
Geometric convergence results for closed minimal surfaces via bubbling analysis
TBA

Cabezas-Rivas, Esther
Brownian motion on Perelman's almost Ricci-flat manifold
We study Brownian motion and stochastic parallel transport on Perelman's almost Ricci-flat manifold, whose dimension depends on a parameter N unbounded from above. By taking suitable projections we construct sequences of space-time Brownian motion and stochastic parallel transport whose limit as $N \to \infty$ are the corresponding objects for the Ricci flow. In order to make precise this process of passing to the limit, we study the martingale problems for the Laplace operator on Perelman's manifold and for the horizontal Laplacian on the corresponding orthonormal frame bundle. As an application, we see how the characterizations of two-sided bounds on the Ricci curvature established by A. Naber applied to Perelman's manifold lead to the inequalities that characterize solutions of the Ricci flow discovered by Naber and Haslhofer. This is joint work with Robert Haslhofer.

Chodosh, Otis
Allen–Cahn min-max in 3-manifolds
The Allen–Cahn equation is a semi-linear PDE that produces minimal surfaces via a certain singular limit. We will describe recent work proving index, multiplicity, and curvature estimates in the context of an Allen–Cahn min-max construction in a 3-manifold. Our results imply, for example, that in a 3-manifold with a generic metric, for every positive integer p, there is an embedded, two-sided minimal surface of Morse index p. This is joint with Christos Mantoulidis.

Colombo, Maria
Direct epiperimetric inequalities for the classical and thin obstacle problem
We study the regularity of the regular and of the singular set of the some free boundary problems in any dimension. At regular points, one approach to the regularity is given by the epiperimetric inequality of Weiss. In his paper, Weiss uses a contradiction argument and he asks the question if such epiperimetric inequality can be proved in a direct way (namely, exhibiting explicit competitors), which would have significant implications on the regularity of the free boundary in dimension $d > 2$. We answer positively the question of Weiss, proving at regular points the epiperimetric inequality in a direct way. More significantly we introduce a new tool, which we call logarithmic epiperimetric inequality, which works also at singular points. It allows to study the regularity of the singular set and yields an explicit logarithmic modulus of continuity on the C^2 regularity, thus improving the known regularity and providing a fully alternative method. The talk is based on joint work with Luca Spolaor and Bozhidar Velichkov.

Del Pino, Manuel
Vortex dynamics in Euler flows and the Liouville equation
We consider the two-dimensional Euler flow for an incompressible fluid confined to a smooth domain. We construct smooth solutions with concentrated vorticities around k points which evolve according to the Hamiltonian system for the Kirkhoff-Routh energy. The profile around each point resembles a scaled finite mass solution of Liouville's equation. This is joint work with Juan Davila, Monica Musso and Juncheng Wei.

Fraser, Ailana
Shape optimization for an eigenvalue problem on manifolds with boundary
When we choose a metric on a manifold we determine the spectrum of the Laplace operator. Thus an eigenvalue may be considered as a functional on the space of metrics. For example the first eigenvalue would be the fundamental vibrational frequency. In some cases the normalized eigenvalues are bounded independent of the metric. In such cases it makes sense to attempt to find critical points in the space of metrics. For surfaces, the critical metrics turn out to be the induced metrics on certain special classes of minimal (mean curvature zero) surfaces in spheres and Euclidean balls. The eigenvalue extremal problem is thus related to other questions arising in the theory of minimal surfaces. In this talk we will give an overview of progress that has been made for surfaces with boundary, and contrast this with some recent results in higher dimensions. This is joint work with R. Schoen.

Huisken, Gerhard
Inverse mean curvature flow of entire graphs
The lecture describes joint work with P. Daskalopoulos on the motion of mean convex entire graphs in direction of their inverse mean curvature. We prove longtime existence for mean convex, star-shaped initial data of superlinear growth and prove, in contrast, uniform finite time blow-up for convex initial data in the critical case of linear growth.

Ketover, Dan
On the existence of minimal surfaces isotopic to a Heegaard surface
When can a surface in a three-manifold be isotoped to be a minimal surface? A Heegaard splitting gives a natural way to sweep-out a 3-manifold and one can use the min-max theory of Simon-Smith to try to obtain an index 1 minimal surface in the isotopy class of the Heegaard surface. The difficulty is that the min-max limit could occur with multiplicity or degenerate to something of lower genus. In the 80s Pitts and Rubinstein sketched an argument that if the Heegaard surface were assumed strongly irreducible (as introduced by Casson and Gordon) it should roughly speaking always be isotopic to an index 1 minimal surface. I will describe joint work with Liokumovich and Song where we fill in what had been missing in their sketch to give a full proof of their claim. It has a number of applications in three-manifold topology which I will explain.

Klainerman, Sergiu
On the nonlinear stability of black holes

Luk, Jonathan
Strong cosmic censorship in general relativity
A fascinating feature of the explicit Reissner–Nordström and Kerr black hole solutions to the Einstein equations in general relativity is the existence of smooth Cauchy horizons. Beyond the Cauchy horizons, classical determinism breaks down. The celebrated strong cosmic censorship conjecture of Penrose nonetheless states that this breakdown of determinism is non-generic. I will review the strong cosmic censorship conjecture and discuss some recent mathematical progress.

Mantoulidis, Christos
Allen–Cahn min-max in 3-manifolds
The Allen–Cahn equation is a semi-linear PDE that produces minimal surfaces via a certain singular limit. We will describe recent work proving index, multiplicity, and curvature estimates in the context of an Allen–Cahn min-max construction in a 3-manifold. Our results imply, for example, that in a 3-manifold with a generic metric, for every positive integer p, there is an embedded, two-sided minimal surface of Morse index p. This is joint with Otis Chodosh.

Matthiesen, Henrik
The systole of large genus minimal surfaces in positive Ricci curvature
There are many results on the space of minimal surfaces of bounded genus or index in ambient three manifolds having some positivity condition on the curvature, e.g. positive Ricci or scalar curvature. In contrast, there are only few results describing asymptotic properties of a sequence of minimal surfaces with unbounded genus. We show that for such a sequence the systole, i.e. the length of a shortest non-contractible curve, has to tend to zero, if the ambient manifold has positive Ricci curvature. This is joint work with Anna Siffert.

Riviere, Tristan
Min-max minimal surfaces in arbitrary co-dimensions and the multiplicity one conjecture
to follow

Schoen, Richard
The positive mass theorem revisited
We will introduce the positive mass theorem which is a problem originating in general relativity, and which turns out to be connected to important mathematical questions including the study of metrics of constant scalar curvature and the stability of minimal hypersurface singularities. We will then give a general description of our recent work with S. T. Yau on resolving the theorem on high dimensional non-spin manifolds.

Song, Antoine
Equidistribution of minimal hypersurfaces for generic metrics
After explaining how Irie, Marques and Neves proved the denseness of minimal hypersurfaces for generic metrics using the Weyl law, I will show how to quantify their method and prove the equidistribution of a sequence of minimal hypersurfaces for generic metrics. This is joint work with Marques and Neves.

Sun, Song
Collapsing of hyper-Kähler metrics on K3 surfaces
I will talk about a new glueing construction, joint with Hans-Joachim Hein, Jeff Viaclovsky and Ruobing Zhang, of a family of hyper-Kähler metrics on K3 surfaces that shows a multiple scale of collapse behaviour.

Valtorta, Daniele
Energy Identity for Stationary Yang Mills connections
In this seminar we present some new results about Yang-Mills connections proved in collaboration with Aaron Naber. In particular, we prove that given a stationary Yang Mills connection with finite energy, there is a uniform L^1 bound for its hessian in the interior of the domain. In turn, we also prove the energy quantization conjecture. The results are based on the quantitative stratification technique and summability estimates for the hessian on annular regions carried out using harmonic gauges. The results are available at arXiv:1610.02898 .

Wang, Lu
Asymptotic structure of self-shrinkers of mean curvature flow
Self-shrinkers are a special class of solutions of mean curvature flow, in which a later time slice is a scaled-down copy of earlier ones. They are singularity models of the flow. In this talk, we will show each end of a non-compact self-shrinker in 3-Euclidean space of finite topology is smoothly asymptotic to a cone with isolated singularities or a round cylinder.

Wickramasekera, Neshan
Regularity of minimal and CMC stable hypersurfaces
to follow