Quantum Homogeneous Spaces

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Abstracts

Yuki Arano
Rokhlin actions of tensor categories
We introduce and classify actions of tensor categories on operator algebras as a generalization of quantum group actions. We explain a relation to the theory of subfactors.

Julien Bichon
Graded twisting of comodule algebras
We will present the graded twisting operation for Hopf algebras and their comodule algebras, and indicate how it can be used in various situations, in particular in invariant theory, in the study of quantum groups actions on quantum planes. Talk based on joint work with Sergey Neshveyev and Makoto Yamashita.

Pierre Bieliavsky
Noncommutative smooth surfaces in higher genera

We will present a notion of noncommutative Riemann surface in every higher gender, generalising the example of the NC torus in its smooth version: the whole space of smooth functions is stable under the deformed products for every value of the deformation parameter.

Alexandru Chirvasitu
Quantum homogeneous spaces for compact quantum groups
In noncommutative geometry cosemisimple Hopf algebras are treated analogous to the algebras $O(G)$ of regular functions on linearly reductive affine algebraic groups G. In this context, a homogeneous space of the “quantum group” attached to a Hopf algebra H would be a coideal subalgebra of A. Similarly, a quantum subgroup would be a quotient Hopf algebra $H \to C$.

Classically, the quotient stack of an algebraic group G by a closed subgroup N is an affine scheme precisely when $O(G)$ is faithfully flat over the coideal subalgebra $O(G/N)$ consisting of functions constant along the cosets of N. For this reason, faithful flatness is the technical condition that encapsulates the vague notion that homogeneous spaces are “well behaved”.

For Hopf algebras H equipped with enough structure to render them analogous to function algebras on compact quantum groups it turns out that inclusions of right coideal $*$-subalgebras are automatically faithfully flat. In view of the previous paragraph, this confirms the intuition that quotients of semisimple affine algebraic groups by semisimple closed subgroups are affine schemes.

Miguel Couto
Commutative-by-finite Hopf algebras and their Finite Dual
I will talk about Hopf algebras, more specifically a particular class of Hopf algebras which are "close" to being commutative; some of their properties and examples will be mentioned. Furthermore, I will show some results on the finite duals of these Hopf algebras, some of its properties, decompositions and maybe some interesting Hopf subalgebras.

Kenny de Commer

Quantum groups, quantum flag manifolds and quantum symmetric spaces

We will present a short survey on quantum homogeneous spaces for quantized compact semisimple Lie groups, focusing attention on quantum flag manifolds and quantum symmetric spaces. We will give an overview of the various approaches which have been used to clarify their structure, ranging from pure algebra to operator algebras, non-commutative geometry, Poisson geometry, tensor categories and integrable systems. We end with some open problems.

Liam Dobson

Factorisation of quasi K-matrices for quantum symmetric pairs

The theory of quantum symmetric pairs provides a universal K-matrix which is an analogue of the universal R-matrix for quantum groups. The main ingredient in the construction of the universal K-matrix is a quasi K-matrix which has so far only been constructed recursively. In this talk, I will recall how the quasi K-matrix is constructed and show for certain classes of examples that it can be explicitly factored into a product of quasi K-matrices for Satake diagrams of rank one. This factorisation depends on the restricted Weyl group of the underlying symmetric Lie algebra in the same way as the factorisation of the R-matrix depends of the Weyl group of the Lie algebra.

Debashish Goswami

Quantum symmetry of classical spaces

I review the state-of-the-art development towards a possible proof of the conjecture made by me some years ago, which states that there can be no faithful smooth (co)action by a genuine compact quantum group on the space of smooth functions on a compact connected smooth manifold. After some brief history and review of initial attempts for resolution of this conjecture, including the case for quantum isometry (solved in my joint paper with Soulamya Joardar, to appear in GAFA (2018)), I'll discuss very recent progress on this direction.

Pawel Kasprzak

Contractive idempotent functionals on locally compact quantum groups

A one to one correspondence between shifts of group-like projections on a locally compact quantum group G preserved by the scaling group and contractive idempotent functionals on the dual of G is established. This is a generalization of the Illie–Spronk's correspondence between contractive idempotents in the Fourier-Stieltjes algebra of a locally compact group G and cosets of open subgroups of G. We also establish a one to one correspondence between non-degenerate, integrable, G-invariant ternary rings of operators preserved by the scaling group and contractive idempotent functionals on G.

Gail Letzer

Quantum symmetric pairs and their representations

In this talk, we present basic structure results for quantum symmetric pair (QSP) coideal subalgebras that lay the groundwork for a general theory of finite-dimensional representations. We first show that every QSP coideal subalgebra admits a quantum Cartan subalgebra which is a polynomial ring that specializes to its classical counterpart. We then explain how QSP coideal subalgebras can be enlarged in order to obtain triangular-like decompositions. These results are illustrated for a number of families of QSP coideal subalgebras and connections are made to cases where the finite-dimensional simple modules are well-understood.

Sara Malacarne

Martin boundary of the dual of a free unitary quantum group

Given a free unitary quantum group G different from the one defined by the unit 2 by 2 matrix, we show that the Martin boundary of $\hat{G}$ with respect to any finite range nondegenerate $\hat{G}$ and $\hat{G}$-invariant quantum random walk coincides with the topological boundary defined by Vander Vennet and Vaes. This can be thought as a quantum analogue of the fact that the Martin boundary of a free group coincides with its Gromov boundary. (Joint work with Sergey Neshveyev.)

Marco Matassa
*Coideal subalgebras and K-matrices in the *-algebra setting*
I will discuss some recent results on coideal subalgebras and universal K-matrices in the *-algebra setting. Joint work with Kenny De Commer.

Andrey Mudrov
Homogeneous vector bundles over quantum spheres
We present an equivariant quantization of vector bundles over a 2n-dimensional complex sphere as a conjugacy class of the orthogonal group $SO(2n+1)$. We realize them in two alternative ways: by linear maps between pseudo-parabolic Verma modules over $U_q(so(n+1))$ and as an induced module over a quantum symmetric pair. To that end, we prove complete reducibility of the tensor product of a finite dimensional $U_q(so(n+1))$-module V and a base pseudo-parabolic Verma module, into a direct sum of (pseudo-parabolic) highest weight submodules. By the alternative realization of the quantum sphere, we construct Gelfand–Zetlin reduction for the corresponding quantum symmetric pair, via a quantum version of Frobenius reciprocity

Manuel Jose Silva Martins
Quantum groups acting on the nodal cubic
Some singular curves in the plane have recently been shown to admit a quantum group action, which turns them into quantum homogeneous spaces. One concrete example is the nodal cubic. In this talk we understand the structure of the corresponding Hopf algebra in terms of small quantum groups at roots of 1 and use this to show that the resulting coalgebra Galois extension is cleft and minimal. We also discuss some ring-theoretic properties of the Hopf algebra and describe its group-likes and twisted primitives.

Jasper Stokman
Integrable structures arising from split symmetric pairs
I will introduce vector-valued Harish–Chandra series associated to split symmetric pairs and show that they produce common eigenfunctions for the quantum Hamiltonians of a new class of quantum integrable spin Calogero–Moser systems. For a natural subclass of vector-valued Harish-Chandra series, I will explain that they are also common eigenfunctions of a consistent system of first order differential equations. These first order differential equations are explicitly given in terms of folded versions of Felder's classical dynamical trigonometric r-matrix and a new universal classical dynamical k-matrix. If time permits I will shortly discuss the generalization to affine symmetric pairs. The talk is based on joint work with Nicolai Reshetikhin.

Angela Tabiri
Reducible and compact real form singular curves which are quantum homogeneous spaces
We construct a Hopf algebra $A(f,g)$ which contains the coordinate ring of a decomposable plane curve (a curve of the form $f(y)=g(x)$) as a right coideal subalgebra and is free over the coordinate ring of the curve. For singular plane curves, examples of $A(f,g)$ enable us to show that a reducible curve (for example the coordinate crossing) and a curve with compact real form (for example the lemniscate) can be quantum homogeneous spaces. We show that the Gelfand–Kirillov dimension of $A(f,g)$ depends on the degree of the plane curve and we give conditions for when these Hopf algebras are domains. Some well-known algebras occur as special cases of the Hopf algebras constructed.

Mariusz Tobolski
An equivariant pushout structure of Vaksman-Soibelman odd quantum spheres
Presenting the C^*-algebras $C(S_q^{2n+1})$ and $C(B_q^{2n}))$ of the Vaksman–Soibelman quantum $(2n+1)$-sphere and the Hong–Szymanski quantum $2n$-ball as graph C^*-algebras, we unravel a $U(1)$-equivariant pullback structure of the former.

As a main application, we prove that the generators of the even K-group of the quantum complex projective space CP_q^n are given by the Milnor connecting homomorphism applied to the (unique up to sign) generator of

$$K_1\left(C\left(S_q^{2n-1}\right)\right)$$

and by the generators of

$$K_0\left(C\left(CP_q^{n-1}\right)\right).$$

This talk is based on a joint work with Francesco D'Andrea and Piotr M. Hajac.

Ami Viselter
Convolution semigroups on quantum groups and non-commutative Dirichlet forms
The probabilistic notion of a Lévy process was generalized to the quantum setting of *-bialgebras in the late 80s, initiating a deep theory. We will discuss convolution semigroups of states on locally compact quantum groups, which are an analytic generalization of (the families of distributions of) Lévy processes. We are particularly interested in semigroups that are symmetric in a suitable sense. These are proved to be in one-to-one correspondence with KMS-symmetric Markov semigroups on the L^∞ algebra that satisfy a natural commutation condition, as well as with non-commutative Dirichlet forms on the L^2 space that satisfy a natural translation invariance condition. This Dirichlet forms machinery turns out to be a powerful tool for analyzing convolution semigroups as well as proving their existence. We will use it to derive geometric characterizations of the Haagerup Property and of Property (T) for locally compact quantum groups, unifying and extending earlier partial results. Time permitting, we will also show how examples of convolution semigroups can be obtained via a cocycle twisting procedure. Based on joint work with Adam Skalski.

Weiqiang Wang
Perspectives on quantum symmetric pairs
There are several significant constructions for quantum groups, which have connections and applications to related fields. A partial list includes Schur–Jimbo duality, BGG category O, canonical basis, R-matrices, quiver varieties, Hall algebras, modular representation theory (many of which were originated by Lusztig, after earlier work of Drinfeld and Jimbo). In recent years, it has emerged that many of these connections can be generalized to quantum symmetric pairs, and often the original quantum group constructions can be viewed in the context of quantum symmetric pairs of diagonal type (or the trivial pairs). In this talk we will survey some of these recent developments on quantum symmetric pairs, and then indicate some new directions which are being developed.

Moritz Weber
Quantum spaces arising from partitions of sets
When dealing with quantum symmetries, we may ask two basic questions:

(1) Given a quantum space - what is its quantum symmetry group?
(2) Conversely, given a quantum group - can we find a quantum space whose quantum symmetry group is precisely the given quantum group?

The second question was the starting point of our joint work with Stefan Jung in the context of Banica and Speicher's so called easy quantum groups. More precisely, we define partition quantum spaces given by universal C^*-algebras whose relations come from partitions of sets. In a way they arise as the first d columns of easy quantum groups; however, we define them as universal C^*-algebras rather than as C^*-subalgebras. We show that free unitary easy quantum groups act maximally on partition quantum spaces with d taking the values one or two. This talk is based on the arxiv preprint 1801.06376.

Tim Weelinck
Quantum symmetric pairs, low-dimensional topology and Hecke algebras
The universal K-matrix of M. Balagovic and S. Kolb is the quantum symmetric pair analogue of the universal R-matrix of the quantum group. It provides universal solutions to (twisted versions of) the reflection equation. We propose a topological interpretation of the twisted reflection equation in terms of the topology of $Z/2Z$-orbifolds. This leads us to consider an operad of $Z/2Z$-orbifold disks

which we show completely encodes the symmetries and structure of the universal K-matrix. By combining ideas from topological quantum field theory with our operadic perspective on quantum symmetric pairs we construct new invariants of $Z/2Z$-orbifold surfaces and representations of the orbifold surface braid groups. We conclude by explaining how our methods conjecturally reconstruct certain representations of Hecke algebras of type $C^\vee C_n$ that were previously defined by D. Jordan and X. Ma via a generators-and-relations method.

Anna Wysoczanska-Kula
On a a Lévy–Khinchine type decomposition on universal quantum groups and the related cohomological properties
Lévy–Khintchine formula provides a classification of convolution semigroups of probability measures, or equivalently, of Lévy processes on $\mathbb{R}$"n in terms of their genererator. It's generalization onto Lie groups is the Hunt formula. They both show how the generators of Lévy processes are combinations of continuous (or Gaussian) parts and jump parts. An analogous decomposition into maximal Gaussian and the remaining part was shown to be true on any commutative *-bialgebra and on the Brown–Glockner–von Waldenfels algebra (M. Schürmann), as well as on $U_q(2)$ (Schürmann, Skeide). Recently, the first example of a *-bialgebra which does not admit such a decomposition was found (Franz, Gelrhold, Thom).We shall discuss the problem of the existence of the decomposition on the universal compact quantum groups U_N" $+ (F)$ and O_N" $+ (F)$, and show that there exists a family of quantum groups that does not admit the Lévy–Khintchine type decomposition and which are neither commutative nor cocommutative. The method we use allows to describe the second cohomology group of universal compact quantum groups. This talk is based on the joint work with Biswarup Das, Uwe Franz and Adam Skalski.

Makoto Yamashita
Ribbon braided module categories, quantum symmetric pairs and Knizhnik–Zamolodchikov equations
The reflection equation (RE) in various incarnations gives quantum homogeneous spaces through Tannaka–Krein duality principle. In this talk, I explain a conjectural correspondence of the solutions to RE, on the one hand from the Knizhnik–Zamolodchikov scheme (Leibman, Golubeva–Leksin, Enriquez) and on the other from the universal K-matrix scheme (Kolb, Balagovic–Kolb) based on joint work with Kenny De Commer, Sergey Neshveyev, and Lars Tuset.

R**obert Yuncken**
Twisted spectral triples and pseudodifferential calculus on quantum projective spaces
This talk concerns the goal of incorporating quantum homogeneous spaces into Connes-style noncommutative geometry, which turns out to be more difficult than one might have hoped. For instance, amongst the quantum flag varieties, we have a local index formula only in the simplest case of the Podles sphere (Neshveyev–Tuset). Krahmer has procuced Dirac-type operators on all irreducible quantum flag varieties, and D'Andrea-Dabrowski showed that for the quantum projective spaces they have compact resolvent and so give spectral triples. This talk will concern the natural next step of proving regularity and constructing an associated pseudodifferential calculus. For this, we need to adapt Connes' abstract pseudodifferential calculus to a class of type III spectral triples. (Joint work with M. Matassa)

James Zhang
Quantum homogeneous spaces associated to Hopf operads
We introduce the notion of a quantum homogeneous space associated to a Hopf operad, and provide some examples.