Analytical and combinatorial aspects of quantum information theory

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Abstracts

Abiad, Aida

On the k-independence number of graphs

In this talk I shall present recent spectral bounds on the k-independence number of a graph, which is the maximum size of a set of vertices at pairwise distance greater than k. The previous bounds known in the literature follow as a corollary of our main results. We show that for most cases our bounds outperform the previous known bounds. Some infinite families of graphs where the bounds are tight are also presented. Finally, as a byproduct, we derive some spectral lower bounds for the diameter of a graph. This is joint work with G. Coutinho and M.A. Fiol.

Bitan, Dor

Information-theoretically secure quantum gate computation and applications

Quantum computations are typically based on representing the classical bits 0 and 1 as the pure state qubits of the computational basis $\ket{0}$ and $\ket{1}$, utilizing quantum phenomena of superposition and entanglement. We explore the case of using randomized bases (instead of the typically used computational basis) to represent classical bits and perform an information-theoretically secure evaluation of quantum gates. We suggest an information-theoretically secure encryption scheme, based on randomized bases, and discuss its quantum homomorphic properties. We demonstrate the usefulness of using randomized bases in an efficient secure QKD protocol and other applications.

Boreland, Gareth

A sandwich theorem and a capacity bound for non-commutative graphs

We recall the properties of convex corners in Rn and the Sandwich Theorem of Grotschel, Lovasz and Schrijver concerning the vertex packing polytope, the theta convex body and the fractional vertex packing polytope of a graph. Generalising to the quantum case, we outline the concept of convex corners in Md, the set of d x d matrices, and establish a quantum version of the Sandwich Theorem for non-commutative graphs. This leads us to define quantum versions of a number of widely used graph parameters, including the Lovasz number and the fractional chromatic number. Another quantum generalisation of the Lovasz number will be shown to be an upper bound on the zero-error capacity of the quantum channel associated to a given non-commutative graph. If time permits, we discuss non-commutative graph entropy, a quantity generalising graph entropy as introduced by Korner. This is joint work with Ivan Todorov and Andreas Winter.

Briet, Jop

Arithmetic progressions in nonlocal games and quantum query algorithms

Szemerédi's Theorem is considered one of the most important results in combinatorics. One reason for this is the wide variety of proofs this result admits. Gowers's celebrated "higher-order Fourier-analytic" proof ingeniously expanded basic Fourier analysis over abelian groups used to prove the theorem for 3-term progressions. In this talk I will discuss how some of the ingredients of this proof can be used in the analysis of multiplayer XOR games and to answer an open question of Pisier regarding a variant of Grothendieck's inequality from functional analysis that turns out to be relevant to quantum query algorithms. Based on joint works with Srinivasan Arunachalam, Tom Bannink, Harry Buhrman, Troy Lee, Farrokh Labib, and Carlos Palazuelos

Coutinho, Gabriel
The average mixing matrix of a continuous-time quantum walk
Consider a continuous-time quantum walk in a graph with n vertices, and let M(t) be the mixing matrix of the walk at time t. Take now M' to be the average of M(t) as t goes from 0 to infinity. This matrix satisfies several interesting properties: it is rational, positive semidefinite and doubly stochastic. If the graph has simple eigenvalues, then it cannot be full-ranked, but it can have rank n-1. In this case, any automorphism of the graph has fixed points. In this talk I intend to show how the correct interpretation of M' leads very naturally to this interplay between quantum walks and combinatorics. This talk is based on a joint work with Chris Godsil, Krystal Guo and Hanmeng Zhan.

De las Cuevas, Gemma
Local descriptions of mixed states
Mixed states are positive semidefinite matrices of trace one. Local descriptions thereof face a fundamental challenge: the description that is locally positive is arbitrarily more costly than the most efficient description. This is due to the separation between the rank and positive semidefinite rank. More generally, several decompositions of mixed states correspond, in a particular case, to decompositions of nonnegative matrices such as the nonnegative factorisation, the positive semidefinite factorisation, the completely positive factorisation and the completely positive semidefinite decomposition. I will give several examples of what can be learnt by exploiting this correspondence. Joint work with Tim Netzer.

Fawzi, Hamza
The set of separable states has no finite semidefinite representation except in dimension 3×2
The set of separable states Sep(n,m) plays a fundamental role in quantum information. It is well-known that for (n,m)=(3,2) the set of separable states has a simple description using semidefinite programming: it is given by the set of states that have a positive partial transpose. In this talk we show that for larger values of n and m the set Sep(n,m) has no semidefinite programming description of finite size. As Sep(n,m) is a semialgebraic set this provides a new counterexample to the Helton-Nie conjecture, which was recently disproved by Scheiderer in a breakthrough result. Compared to Scheiderer's approach, our proof is elementary and relies only on basic results about semialgebraic sets and functions.

Kakariadis, Evgenios
Amalgamated free products of operator algebras
The amalgamated free product of C*-algebras has become a standard construction in the subject. It is central in studying group C*-algebras, free probability, quantum information, dynamical systems on C*-algebras, and the Connes Embedding Conjecture via its connection to Kirchberg's conjecture and also to Tsirelson's conjectures. Boca's result on completely positive maps on amalgamated free products has become a useful tool in this endeavor. In this talk we will show a general method of extending unital completely positive maps to amalgamated free products of C*-algebras that covers Boca's Theorem. Several applications on the C*-envelope will be discussed. The talk is based on joint work with Davidson (Proc. R. Soc. Ed. A) and with Davidson and Fuller (Memoirs AMS).

Leung, Debbie
Simple proof of non-closure of quantum correlations using embezzlement
Embezzlement of entanglement is the (impossible) task of producing an entangled state from a product state via a local change of basis, when a suitable *catalytic* entangled state is available. The possibility to approximate this task was first observed by van Dam and Hayden in 2002. In this talk, we show how embezzlement can be used to obtain an explicit nonlocal game that cannot be played optimally with finite amount of entanglement with tight trade-off between winning probability and dimension of the entangled state shared by the players. References: 0201041, 0804.4118, 1802.04926, 1904.02350 (arXiv:quant-ph). Players: van Dam, Hayden, Toner, Watrous, Ji, Vidick, Coladangelo.

Li, Yinan

Quantum asymptotic spectra and quantum Shannon capacities

We introduce and study the asymptotic spectra corresponding to the quantum variations of Shannon capacity that arise in the theory of non-local games and quantum zero-error information theory. We prove that the quantum Shannon capacity of graphs (Mancinska & Roberson, J. Combin. Theory Series B, 2016), the entanglement-assisted Shannon capacity of graphs~(Beigi, Phys. Rev. A, 2010, Duan et al., IEEE Trans. Info. Theory, 2013) and the entanglement-assisted Shannon capacity of non-commutative graphs~(Duan et al. IEEE Trans. Info. Theory, 2013) are equal to the pointwise minimum over their respective asymptotic spectra. We then identify elements in the new asymptotic spectra, and investigate the relations among the three new asymptotic spectra and the asymptotic spectrum of graphs. Besides this we prove that the fractional Haemers bound over the reals is an upper bound on the quantum Shannon capacity, which shows that the quantum Shannon capacity is not equal to the Lovasz theta function. Our dual characterization brings a unified framework and a calculus to the study of asymptotic quantum graph parameters and the asymptotic "quantum relations" among graphs and non-commutative graphs. This leads to various consequences and natural questions. For example, a consequence of our duality is that the aforementioned Shannon capacities are "locally additive if and only if multiplicative''.

Macinska, Laura

Nonlocal games and quantum permutation groups

I will present a connection between quantum information and quantum permutation groups provided by quantum strategies for the so-called graph isomorphism game. I will start by defining the isomorphism game and its quantum strategies as prescribed by two potentially different models: the tensor product model and the commuting model. We will see that the existence of a commuting strategy can naturally be expressed in the language of quantum groups. Specifically, we will see that two connected graphs X and Y are quantum (commuting) isomorphic if and only if there exists x \in V(X) and y \in V(Y) that are in the same orbit of the quantum automorphism group of the disjoint union of X and Y. This connection links quantum groups to the more concrete notion of nonlocal games and physically observable quantum behaviors. Finally, we will see that this connection not only allows us to leverage quantum group theoretic results to learn more about quantum strategies but the methods used to investigate quantum strategies for the isomorphism game can also be applied to get new results about quantum automorphism groups of graphs. This talk is based on arXiv:1712.01820 which is a joint work with Martino Lupini and David E. Roberson.

Naaijkens, Pieter

Classical capacity of channels between von Neumann algebras

Many interesting physical systems with infinitely many degrees of freedom are most conveniently described in an operator algebraic setting. I will give some examples which lead to subfactors, i.e. inclusions of von Neumann algebras with trivial centres. There is a natural quantum channel associated with such a subfactor, and I will outline how the classical capacity of this channel can be found.

Netzer, Tim

Free Spectrahedra, Operator System, and Separable States

Spectrahedra are the feasible sets of semidefinite programming. They have received much attention from an algebraic-geometrical point of view recently. Extending them to matrix-levels reveals many interesting facts about them. This leads to the notion of a free spectrahedron, which is a special case of an abstract operator system, a notion often studied in operator algebra. It turns out that free spectrahedra are in fact those operator systems that admit a concrete realization on a finite-dimensional Hilbert space. A link to theoretical quantum physics is for example given by the fact, that states and separable states form operator systems. We explain how to check whether an abstract operator system is a free spectrahedron, and how this helps understanding classical spectrahedra. We classify completely for which polyhedral structures this is true, and obtain an interesting result about states as a byproduct: every bipartite state of tensor-rank two is separable. We explain how this can help understanding separable states and their decompositions in more general cases.

Paulsen, Vern
Bisynchronous Correlations
Recently the graph isomorphism game has been shown to have better properties than other synchronous games. In particular, as soon as the algebra of a graph isomorphism game is non-zero, then the game has a perfect qc-strategy. The graph isomorphism game turns out to be an example of a bisynchronous game. Perfect strategies for such games satisfy a property that we call bisynchronicity and we prove that any such correlation corresponds to a map that is factorizable in the sense of Haagerup and Musat.

Roberson, David
Quantum isomorphism and counting homomorphisms
In 1967, Lovasz showed that two graphs are isomorphic if and only if they admit the same number of homomorphisms from any graph. More recently in 2018, Dell, Grohe, and Rattan showed that two graphs are not distinguished by the k-dimensional Weisfeiler-Leman algorithm if and only if they admit the same number of homomorphisms from any graph of treewidth at most k. Just this year we have shown that two graphs are quantum isomorphic (in the commuting operator framework) if and only if they admit the same number of homomorphisms from any planar graph. To prove this we first provide a combinatorial characterization of the intertwiners of the quantum automorphism group of a graph based on counting rooted homomorphisms from planar graphs. I will discuss this combinatorial description of intertwiners and the resulting characterization of quantum isomorphism. This is joint work with Laura Mancinska.

Strelchuk, Sergii
Classical and quantum aspects of Permutational Quantum Computing model
I will provide an introduction to the Schur Transform and survey its recent applications in the context of Permutational Quantum Computing (PQC). I will further show that most of its widely-believed quantum features are classically tractable: the outputs of Schur sampling circuits are unconditionally strongly simulatable and weakly simulatable under certain sparsity assumptions. I will also discuss the Kushilevitz-Mansour algorithm which is used to estimate heavy Fourier/Clebsch-Gordan coefficients of the output states of quantum circuits and discuss possible ways of regaining supra-classical power in PQC.

Vacaru, Sergiu
Quantum Geometric Information Flows and (Modified) Gravity Theories
We outline our recent research on classical and quantum geometric information flow theories (respectively, GIFs and QGIFs) when the geometric flow evolution and field equations for modified gravity theories, MGTs, and nonholonomic Einstein systems, NES, are derived from Perelman–Lyapunov type entropic type functionals. There are defined canonical (nonholonomic, i.e. with non-integrable distributions) geometric variables which allow a general decoupling and integration of nonlinear systems of equations describing GIFs and QGIFs and (for self-similar geometric flows) Ricci soliton type configurations. Our approach is more general and different from the ideas and methods used for special classes of solutions with the area–hypersurface entropy, and related holographic and dual gauge–gravity models, involving the concepts and generalizations of the Bekenstein–Hawking entropy and black hole thermodynamics. There are analyzed the most important properties (inequalities) for emergent NES, their geometric flow evolution and QGIF versions of the von Neumann, relative and conditional entropy, mutual information, (modified) entanglement and Rényi entropy. We speculate on applications of GIFs and QGIFs, entanglement and geometry of relativistic and quantum mechanical systems and MGTs and show how to compute the W-entropy and associated thermodynamics for generalized stationary and cosmological solutions in MGTs etc.

Vrana, Peter

Semiring-homomorphic graph parameters

In zero-error communication theory a central role is played by graphs and their non-commutative generalizations, as combinatorial objects associated to classical and quantum channels. Various capacities for zero-error transmission are expressed as parameters of these (non-commutative) graphs and are in general poorly understood. Recently, Zuiddam showed that Strassen's theory of asymptotic spectra, originally developed to study the asymptotic rank of tensors, can be used to give a characterization of the Shannon capacity of graphs as a minimum over the set of graph parameters that are multiplicative under the strong product, additive under the disjoint union and monotone with respect to the cohomomorphism preorder. Subsequently, Li and Zuiddam extended the results to the quantum realm, finding a similar characterization of the zero-error entanglement-assisted classical and quantum capacities. It became therefore a problem of major interest to construct and classify such semiring-homomorphic graph invariants, and it was not long until Fritz provided a unified construction for all the known such invariants: the fractional clique cover number, the Lovász number, the projective rank and the fractional Haemers bound. In my talk I will explain how to extend every semiring-homomorphic monotone to probabilistic graphs and highlight some properties of these extensions. Based on arXiv:1903.01857.

Weaver, Nik

The "quantum" Turan problem for operator systems

Let V be a linear subspace of M_n(C) which contains the identity matrix and is stable under Hermitian transpose. A "quantum k-clique" for V is a rank k orthogonal projection P in M_n(C) for which dim(PVP) = k^2, and a "quantum k-anticlique" is a rank k orthogonal projection for which dim(PVP) = 1. We give upper and lower bounds both for the largest dimension of V which would ensure the existence of a quantum k-anticlique, and for the smallest dimension of V which would ensure the existence of a quantum k-clique.