Uncertainty Quantification, May 24th–May 28th 2010
Workshop report

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1. Relation to original proposal

The scope and aims of the meeting did not deviate from the original proposal. However, the format of the workshop did change. The first one and a half days of the meeting went as originally planned. That is, Monday and Tuesday morning were reserved for three instructional courses on numerical methods for PDEs with random data, high-dimensional sampling methods, and sensitivity analysis, delivered by Max Gunzburger, Ian Sloan and Andrea Saltelli, respectively. Originally we intended to fill the rest of the week with invited talks of 45 minutes in length. However, it became clear to us that many participants wished to present their work and indeed would have interesting things to say. To accommodate this, we invited all participants who wished to, to give a short talk (15 minutes in length) and a poster. This also meant that all participants were treated equally. That is, established researchers were not given preference over younger researchers. In the end, there were 50 short contributed talks, interspersed with three poster sessions of two hours in length. On Thursday afternoon, we additionally staged a Knowledge Transfer event, comprising four half-hour presentations from industrialists followed by a panel discussion.

2. Short description of the meeting

In deterministic mathematical modelling, complete knowledge of input parameters is assumed. This leads to simplified, tractable computations and produces simulations of outputs that correspond to specific choices of inputs. However, most physical, biological, social, economic and financial processes involve some degree of uncertainty. Uncertainty quantification (UQ) is the task of determining statistical information about the outputs of a process of interest, given only statistical (i.e. incomplete) information about the inputs. It encompasses many tasks that are crucial to both public and private enterprise. This includes, crucially, risk assessments that are typically used to inform policy makers e.g. on sensitive issues such as the safety of a nuclear waste repository.

It has long been recognised that mathematical models need to account for uncertainty but progress has been hampered by a lack of mathematical analysis and computing resources. The science of UQ has been in its infancy in many application areas until relatively recently but is now rapidly developing. The workshop brought together numerical analysts, probabilists, computer scientists and industrialists (amongst whom there is traditionally very little communication) to disseminate important advances currently being made in sparse high-dimensional sampling, high-dimensional quadrature, approximation theory, model-order reduction, sensitivity analysis, statistics and scientific computing.

3. Comprehensive workshop report

The meeting took place at the premises of the Royal Society of Edinburgh on May 18th–May 24th 2010. It was attended by sixty-six people, including nine women and thirteen young researchers. There were twenty-four participants from the UK, twenty-eight from the USA, eleven from Europe (France 3, Germany 4, Italy 2, Norway 1, Switzerland 1), and one participant from each of Saudi Arabia, Australia and Korea. Funding for the workshop was secured from the ICMS (in total £25,000, £1,000 of which was from the LMS), the US National Science Foundation (NSF, $44,100) and the US Air Force Office for Scientific Research (AFOSR, $18,185). Attendance at the workshop was by invitation only

and participants were selected by the organisers for the quality of their research pertaining to diverse UQ issues in mathematical modelling. The disciplines of numerical analysis, probability, statistics, computer science and linear algebra were all represented.

The meeting began with three short courses to bring all participants up to speed with current state of the art methodologies. These were much appreciated by all and received very positive feedback. On Monday morning, Max Gunzburger delivered a three-hour instructional course on the basic numerical techniques for approximating solutions to partial differential equations (PDEs) with random data, paying particular attention to elliptic PDEs. He surveyed Monte Carlo, stochastic Galerkin and stochastic collocation methods, including a discussion on basic methodologies for discretising second-order random fields via Karhunen-Loève expansions and Polynomial Chaos expansions. Monday afternoon saw Ian Sloan deliver instructional lectures on integration and sampling techniques in high dimensions. Particular attention was paid to sparse grid methods and Quasi-Monte Carlo approximation. The talks included outline proofs of many of the important results. On Tuesday morning Andrea Saltelli talked about sensitivity analysis and dimension reduction. He presented techniques for exploring high dimensional parameter spaces and identifying the most important parameters in a problem. He also emphasised that there is still a large gap between the theory and methodologies that have been developed and practice, and he stressed the importance of sensitivity analysis in situations where the results of modelling inform important decisions.

Following the instructional courses, on Tuesday, Wednesday, Thursday and Friday morning, there were fifty contributed short talks. Each speaker was also asked to prepare a poster (there were three two-hours sessions spread throughout the week). The speakers' brief was was to use their talk to convince other participants to come and discuss their research in more depth at the poster sessions. The talks covered a wide range of topics and were of a uniformly high standard. In particular many of the early stage researchers gave excellent presentations. The poster sessions were very successful and were the main focus of interactions between the participants. The feedback on this format was, on the whole, very positive. The following is a typical comment from a participant.

> *Having been slightly apprehensive prior to the meeting about the effectiveness of this setup, in retrospect I have to say it worked very well.*

On Thursday afternoon, we staged a Knowledge Transfer (KT) session. Representatives from The British Geological Survey, Rolls Royce, Serco Technical Services and Statoil gave half-hour presentations, outlining the importance of quantifying uncertainty in their respective industries, and the associated mathematical challenges that remain to be met. It was very clear from the presentations that UQ is of major concern to the industries represented. The talk from the British Geological Survey was particularly topical in view of the recent eruption at Eyjafjallajoekull in Iceland and its impact on air travel. The presentations were followed by a panel discussion and questions from the audience. Following the KT session, representatives from two of the meeting's funding bodies (the NSF and the AFSOR) gave presentations on current funding opportunities for research centered on UQ. Unfortunately for UK participants, it was not possible for a representative from the EPSRC to attend this session.

On Friday afternoon we concluded with a wrap-up session where we asked participants to comment on future important directions in UQ. The following main issues emerged from the discussion:

- The curse of dimensionality remains a major problem. Model reduction is a promising future direction for research in this context.
- There is still a gap between the real large-scale problems arising from applications and what can be achieved with current techniques.

- There is a continuing need for the different communities to interact, in particular the statistics and numerical analysis/scientific computation communities. There is also a pressing need for researchers developing new methods to engage with researchers in application areas.
- Inverse problems and use of data will play an increasingly important role in UQ.
- Extending the methods and analysis to large dynamical systems, and nonlinear problems exhibiting bifurcation behaviour is likely to be a fruitful line of research.

All in all, we feel that the meeting was a success. The intended objectives of the workshop were: 1) to review developments in the rapidly developing field of UQ; 2) to bring together internationally leading experts working in the relevant fields of mathematics and enable an effective dialogue between them; 3) to expose scientists in industry to the most recent developments in the field and mathematical scientists to the important problems facing industry; 4) to promote communication between the various relevant mathematical disciplines; 5) to encourage young researchers to work in the field; 6) to strengthen the UK's presence in this fundamentally important field.

Based on the formal feedback and on informal discussion with participants it is clear that objectives 1-4 were achieved. Here are three typical comments from participants:

> *This is one of the best workshops I have attended. I learned a huge amount, from both the tutorials and the research presentations.*
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> *Since there was a significant tutorial component (and not just latest research), I believe the academic value was high. My PhD students were in attendance and the tutorial sessions were invaluable for them.*
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> *The workshop was not just a presentation of methods and procedures but was an opportunity to discuss available methods and set future research guidelines.*

We are confident that objective 5 was also achieved, having received very positive comments from a number of the early stage researchers present. This workshop was one of the first events wholly dedicated to the topic of UQ and that in itself will have contributed to objective 6. We are hopeful that the UK will continue to have an influential presence in this research area.

Finally, the local organisation of the workshop by ICMS staff was superb! Edinburgh is clearly a very popular venue for scientific meetings and virtually the only criticism we received was that we didn't give participants an afternoon off for sight-seeing.