

Wireless Random-Access Networks: Fairness, Performance & Stability

Sem Borst

Bell Labs Alcatel-Lucent & Eindhoven University of Technology

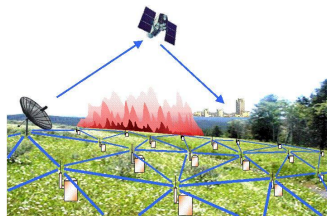
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Joint work with Johan van Leeuwen, Alexandre Proutiere,
Patrick Thiran, Peter van de Ven

Wireless networks



Wireless mesh network



Wireless sensor network

Emerging large-scale wireless networks

- huge numbers of devices, gadgets, sensors (*Internet of Things*)
- lack **centralized** control and infrastructure
- nodes operate autonomously, and share medium in **distributed** fashion

Radical paradigm shift:

devices do not just **use** the network, they **are** the network

Randomized medium-access algorithm

Randomized algorithms provide popular mechanism for distributed medium-access control

- simple operation, low implementation complexity
- however, dynamic interaction among nodes yields highly complex behavior on macroscopic level

Carrier-Sense Multiple-Access Collision Avoidance (CSMA/CA):

Each node attempts to access medium after random back-off time

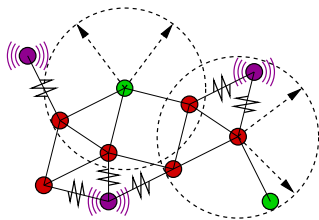
- back-off timer frozen while medium is sensed busy
- or back-off timer is resampled in case medium is sensed busy when it expires

Model description

Edges in interference graph indicate which pairs of nodes interfere:

neighbors are prevented from simultaneous activity

[Boorstyn *et al.* (1980), Wang & Kar (2005), Durvy & Thiran (2006)]



Each node activates at exponential rate when none of its neighbors are currently active

Or equivalently, potential activation epochs occur as Poisson process

Transmission durations are exponentially distributed

Notation

- ν_i : exponential activation rate of node i
- μ_i : exponential rate of transmission period of node i
- interference graph $G = (V, E)$
- $\mathcal{S} := \{x \in \{0, 1\}^N : x_i + x_j \leq 1 \forall i, j \in E\}$: set of all feasible activity states (collection of independent sets in interference graph G)
- $X(t) = (X_1(t), \dots, X_N(t)) \in \mathcal{S}$: activity states at time t

Stationary distribution

$X(t)$ is reversible Markov process with stationary distribution

$$\pi(x) \propto \prod_{i=1}^N \sigma_i^{x_i}, \quad x \in \mathcal{S},$$

with $\sigma_i := \nu_i / \mu_i$ potential activity factor ('offered' load)

Throughput ('carried' load) of node i is

$$\theta_i = \sum_{x \in \mathcal{S}} \pi(x) I_{\{x_i=1\}}$$

Insensitivity

So far we assumed back-off periods and transmission durations to be exponentially distributed

Connection with loss networks readily implies that stationary distribution is in fact insensitive to distribution of transmission durations

Partial balance properties show that stationary distribution is insensitive to distribution of back-off periods as well, irrespective of whether or not back-off process is frozen during activity of neighbors

In case back-off process is not frozen, insensitivity also follows from similarity with Engset network [Bonald (2006)]

Outline

- Long-term spatial fairness - Markov random-field approach
- Short-term fairness - rare-event analysis
- Stability & throughput performance

High activation rate

Suppose all nodes have common activation rate ν , and unit-mean transmission period, then

$$\pi(x) \propto \nu^{\sum_{i=1}^N x_i}, \quad x \in S$$

Maximum number of simultaneously active nodes is

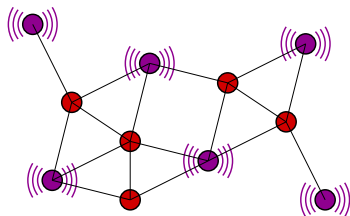
$$M := \max_{x \in S} \sum_{i=1}^N x_i$$

Define $S^* := \{x \in S : \sum_{i=1}^N x_i = M\}$ as set of all states with maximum number of active nodes (maximum independent sets in interference graph)

States $x \in S^*$ become asymptotically dominant as $\nu \rightarrow \infty$:

$$\lim_{\nu \rightarrow \infty} \pi(x) = \begin{cases} 1/|S^*| & x \in S^* \\ 0 & x \notin S^* \end{cases}$$

High activation rate (cont'd)

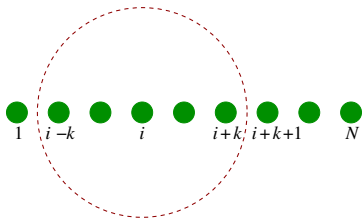


Aggregate system throughput approaches maximum achievable value M as $\nu \rightarrow \infty$, so asymptotically efficient

However, nodes that do not belong to maximum independent set suffer **complete starvation** as $\nu \rightarrow \infty$, so **throughput distribution highly unfair!!**

Spatial unfairness

Consider linear topology with k -hop interference model, i.e., all nodes within k -hop range are interferers



In case $N = 2k + 1$, limiting throughput $\theta_i^* = \lim_{\nu \rightarrow \infty} \theta_i(\nu)$ is given by

$$\theta_i^* = \theta_{2k+2-i}^* = \frac{k+1-i}{k(k+1)/2}$$

In particular $\theta_1^* = \theta_{2k+1}^* = \frac{2}{k+1}$ while $\theta_{k+1}^* = 0!!$

Spatial fairness

Can we achieve fairness, i.e., balance throughputs $\theta_1 = \theta_2 = \dots = \theta_N$ by selecting suitable node-dependent activation rates $\nu_1, \nu_2, \dots, \nu_N$?

Yes, we can: setting

$$\begin{aligned}\nu_1 &= \nu_N = \alpha \\ \nu_2 &= \nu_{N-1} = \alpha(1 + \alpha) \\ &\dots \\ \nu_k &= \nu_{N-k+1} = \alpha(1 + \alpha)^{k-1} \\ \nu_{k+1} &= \nu_{k+2} = \dots = \nu_{N-k} = \alpha(1 + \alpha)^k\end{aligned}$$

yields uniform throughputs $\theta_i \equiv \theta = \frac{\alpha}{1+(k+1)\alpha}$

Markov random-field approach

Stationary distribution

$$\pi(x) \propto \prod_{i=1}^N \nu_i^{x_i}, \quad x \in S \quad (1)$$

Stationary distribution may be equivalently expressed in terms of Markov random field [Spitzer, Kelly]

$$\begin{aligned} \pi(x) &= \pi(x_l, \dots, x_{l+k-1}) \prod_{m=1}^{l-1} P_m(x_m | x_{m+1}, \dots, x_{m+k}) \\ &\times \prod_{n=l+k}^N P_n(x_n | x_{n-1}, \dots, x_{n-k}), \quad x \in \{0, 1\}^N \end{aligned} \quad (2)$$

Markov random-field approach (cont'd)

For compactness, denote $\pi_i := \mathbf{P}\{X_i = 1\}$,

$\psi_i := \mathbf{P}\{X_{i-k}, \dots, X_{i-1} = 0\}$, and $a_i := P_i(0|0, \dots, 0)$, then

$$\pi_i = (1 - a_i)\psi_i \quad (3)$$

for all $i = k + 1, \dots, N$

Form of (1) (or local balance property) yields

$$\mathbf{P}\{X_i = 1, X_j = 0, j \in \mathcal{N}_i\} = \nu_i \mathbf{P}\{X_i = 0, X_j = 0, j \in \mathcal{N}_i\},$$

with \mathcal{N}_i set of neighbors of node i , for all $i = 1, \dots, N$

Form of (2) yields $\mathbf{P}\{X_i = 0, X_j = 0, j \in \mathcal{N}_i\} = \psi_{k+1} \prod_{j=k+1}^{i+k} a_j$

for $i = 1, \dots, k$, while

$$\frac{\mathbf{P}\{X_i = 1, X_j = 0, j \in \mathcal{N}_i\}}{1 - a_i} = \frac{\mathbf{P}\{X_i = 0, X_j = 0, j \in \mathcal{N}_i\}}{a_i \prod_{j=i+1}^{\min\{i+k, N\}} a_j}$$

for all $i = k + 1, \dots, N$

Markov random-field approach (cont'd)

Also, note that $\psi_i + \sum_{j=i-k}^{i-1} \pi_j = 1$ for all $i = k + 1, \dots, N$,
and in particular $\psi_{k+1} + \sum_{j=1}^k \pi_j = 1$

Combining above two sets of equations, we obtain

$$\pi_i = \nu_i \left(1 - \sum_{j=1}^k \pi_j\right) \prod_{j=k+1}^{i+k} a_j, \quad (4)$$

for $i = 1, \dots, k$, while

$$1 - a_i = \nu_i a_i \prod_{j=i+1}^{\min\{i+k, N\}} a_j \quad (5)$$

for all $i = k + 1, \dots, N$

Markov random-field approach (cont'd)

A solution to (5) is provided by $a_i = a$ and $\nu_i = (1 - a)a^{-h_i}$, or equivalently $\nu_i = \alpha(1 + \alpha)^{h_i - 1}$, with $\alpha = 1/a - 1 > 0$ and

$$h_i := \min\{i + k, N\} - i + 1 = \begin{cases} k + 1 & i = k + 1, \dots, N - k \\ N - i + 1 & i = N - k + 1, \dots, N \end{cases}$$

Taking $\nu_i = (1 - a)a^{-i}$, $i = 1, \dots, k$, we obtain from (4)

$$\pi_1 = \dots = \pi_k = \pi = (1 - a)(1 - k\pi),$$

i.e., $\pi_1 = \dots = \pi_k = \pi = \frac{1-a}{1+k(1-a)} = \frac{\alpha}{1+(k+1)\alpha}$, and (3) then yields

$$\pi_i = \frac{1-a}{1+k(1-a)} = \frac{\alpha}{1+(k+1)\alpha} \text{ for all } i = k + 1, \dots, N$$

Short-term fairness

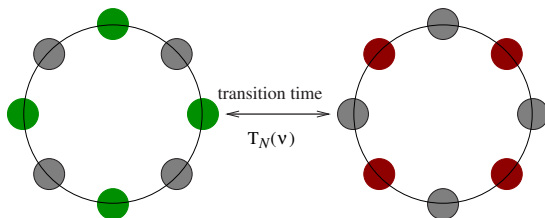
In some situations, **long-term spatial fairness** is guaranteed for uniform activation rates ν , e.g., by virtue of symmetry

Maximum fair throughput will be achieved for high activation rate

However, high activation rate may cause **severe short-term unfairness**

Symmetric ring network

Consider ring network of $2N$ nodes with 1-hop interference range



When all N green nodes are active, it will take long time for any of N red nodes to gain access to the medium

Once one of green nodes has turned off, adjacent green node must turn off as well before first one turns on again, in order for red node to gain access

Probability of latter event is of the order $1/\nu$,
and transition time $T_N(\nu)$ is of the order ν

Some notation

Let $S \subseteq \{0, 1\}^{2N}$ be set of all feasible activity states

For given activity state $x \in S$, let

$$y(x) := \sum_{i=1}^N x_{2i-1} \text{ and } z(x) := \sum_{i=1}^N x_{2i}$$

be number of active green and red nodes, respectively

For any pair of activity states $u, x \in S$, denote

$$T_{x \rightarrow u} := \text{time to move from state } x \text{ to state } u$$

Also, for any subset of activity states $U \subseteq S$, denote

$$T_{x \rightarrow U} := \min_{u \in U} T_{x \rightarrow u}$$

Lower bound

Let $U_i := \{x \in S : y(x) = i\}$ be set of activity states with exactly i green nodes active

In order to move from dominant all-green state x_g to dominant all-red state x_r , process must pass through some state $x \in U_i$, $i = 1, \dots, N - 1$

Thus $T_N(\nu) \geq_{st} T_{x_g \rightarrow U_i}$ for all $i = 1, \dots, N - 1$

Define

$$M_N(i) := i + \max_{x \in U_i} z(x)$$

as maximum total number of nodes that can be active given that exactly i green nodes are active,

$$M_N := \min_{i=1, \dots, N-1} M_N(i), \quad i^* := \arg \min_{i=1, \dots, N-1} M_N(i)$$

Maximum total number of nodes that can remain continuously active when moving from all-green state x_g to all-red state x_r is $M_N - 1$

Rare-event analysis

Using results for reversible Markov chains with exponentially small transition probabilities [Olivieri & Scoppola (1995)], we can establish two asymptotic results as $\nu \rightarrow \infty$:

- $\frac{T_{x_g \rightarrow U_{i^*}}}{\mathbf{E}\{T_{x_g \rightarrow U_{i^*}}\}} \rightarrow \exp(1)$
- $\log \mathbf{E}\{T_{x_g \rightarrow U_{i^*}}\} \sim \log H(\nu)$

with

$$H(\nu) = \frac{\pi(x_g)}{\max_{x \in U_{i^*}} \pi(x)} = \frac{\nu^N}{\nu^{M_N}} = \nu^{N-M_N}$$

representing ratio between stationary probabilities of **all-green state** x_g and some critical (max-min) intermediate state on path to **all-red state** x_r

For ring network, $M_N = N - 1$, and thus

$$\log \mathbf{E}\{T_{x_g \rightarrow U_{i^*}}\} \sim \log(\nu) \quad \text{as } \nu \rightarrow \infty$$

Upper bound

Also, $T_N(\nu) \leq T_{x_g \rightarrow U_i} + \max_{x \in U_i} T_{x \rightarrow x_r}$ for all $i = 1, \dots, N - 1$

It may be shown that $\max_{x \in U_i^*} T_{x \rightarrow x_r}$ is bounded from above by random variable that is independent of ν , and thus

$$T_N(\nu) \sim T_{x_g \rightarrow U_i^*} \quad \text{as } \nu \rightarrow \infty$$

Asymptotic results for $T_{x_g \rightarrow U_i^*}$ yield

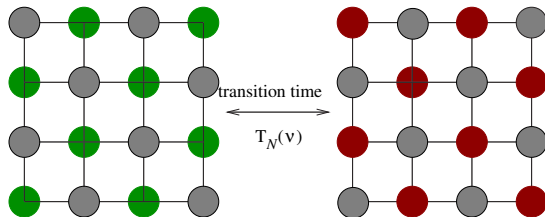
- $\frac{T_N(\nu)}{\mathbf{E}\{T_N(\nu)\}} \rightarrow \exp(1)$
- $\log \mathbf{E}\{T_N(\nu)\} \sim \log(\nu)$

Using more detailed arguments, one can in fact show [Durvy (2007)] that

$$\mathbf{E}\{T_N(\nu)\} \sim \nu \quad \text{as } \nu \rightarrow \infty$$

Symmetric grid network

Consider $2N \times 2N$ grid network with 1-hop interference range



When all $2N^2$ green nodes are active, it will take long time for any of red nodes to gain access to the medium

Rare-event analysis

Let maximum total number of nodes that can remain continuously active when moving from **all-green state** x_g to **all-red state** x_r again be $M_N - 1$

We can establish two asymptotic results as $\nu \rightarrow \infty$:

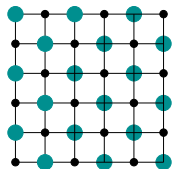
- $$\frac{T_{x_g \rightarrow U_{j^*}}}{\mathbf{E}\{T_{x_g \rightarrow U_{j^*}}\}} \rightarrow \exp(1)$$
- $$\log \mathbf{E}\{T_{x_g \rightarrow U_{j^*}}\} \sim \log H(\nu) \rightarrow 1$$

with

$$H(\nu) = \frac{\pi(x_g)}{\max_{x \in U_{j^*}} \pi(x)} = \frac{\nu^{2N^2}}{\nu^{M_N}} = \nu^{2N^2 - M_N}$$

representing ratio between stationary probabilities of **all-green state** x_g and some critical (max-min) intermediate state on path to **all-red state** x_r

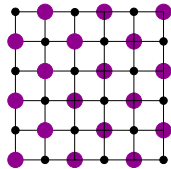
Rare-event analysis (cont'd)



Blue dominant state



Intermediate
state



Purple dominant state

Conjecture ($N \geq 2$):

$$2N^2 - M_N = \begin{cases} N & \text{free boundary} \\ 2N & \text{wrap-around boundary} \end{cases}$$

Queueing dynamics

So far we assumed saturated buffer conditions, where nodes always have packets pending for transmission

In reality, however, buffer contents fluctuate as packets are randomly generated and transmitted over time, giving rise to **queueing dynamics**

In particular, buffers may empty from time to time, and nodes may refrain from competition for medium during these periods

Stability conditions

Packets generated at node i according to renewal process with rate λ_i

Denote by $\rho_i := \lambda_i/\mu_i$ traffic intensity at node i

Recall that θ_i is throughput of node i in saturated buffer conditions

Proposition

If $\rho_i > \theta_i$ for all $i = 1, \dots, N$, then all nodes are unstable

Reverse statement:

If $\rho_i < \theta_i$ for all $i = 1, \dots, N$, then all nodes are stable

Does **not** hold!

Stability conditions (cont'd)

Consider 4-node ring topology with 1-hop interference range

Assume $\nu_i \equiv \nu$ and $\mu_i \equiv 1$ for all $i = 1, 2, 3, 4$

Then

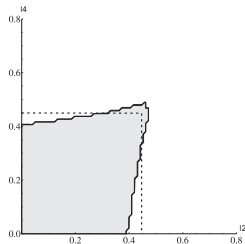
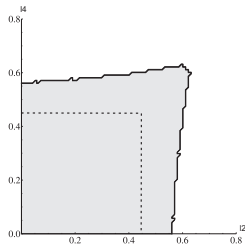
$$\theta_i \equiv \theta = \frac{\nu + \nu^2}{1 + 4\nu + 2\nu^2}$$

Suppose $\lambda_1 = \lambda_2 = \lambda_3 \equiv \lambda < \theta$, while $\lambda_4 = 0$

Stability conditions (cont'd)

Then node 2 can be shown to be unstable for λ close to θ

Intuitive explanation: node 2 loses cooperation from node 4 in joint competition for medium with nodes 1 and 3



Stability regions in terms of λ_2 , λ_4 for fixed $\lambda_1 = \lambda_3 = \lambda_{13}$ and $\nu = 10$;
 $\lambda_{13} = 0.3$ (left); $\lambda_{13} = 0.45$ (right)

Complete interference graphs

Assume that nodes are ordered such that

$$\frac{\lambda_1}{\nu_1} \leq \frac{\lambda_2}{\nu_2} \leq \dots \leq \frac{\lambda_N}{\nu_N}$$

Recall that $\sigma_i = \nu_i/\mu_i$ is potential activity factor of node i , and denote

$$\tau_i = \frac{\sigma_i}{1 + \sum_{j=i}^N \sigma_j} \left(1 - \sum_{j=1}^{i-1} \rho_j\right),$$

representing fraction of time that node i is active, assuming that nodes $1, \dots, i-1$ are all stable, while nodes i, \dots, N are all saturated

Also, define $i_{\max} = \max\{i \in \{1, \dots, N\} : \rho_i < \tau_i\}$, with $i_{\max} = 0$ when $\rho_i > \tau_i$ for all $i = 1, \dots, N$, and assume $\rho_{i_{\max}+1} > \tau_{i_{\max}+1}$ in case $i_{\max} < N$

Proposition

Nodes $1, \dots, i_{\max}$ are stable, while nodes $i_{\max} + 1, \dots, N$ are unstable

Star topology

Consider star topology: leaf nodes $1, \dots, N - 1$ all interfere with root node N , but not with each other

Stability conditions:

$$\rho_i < (1 - \rho_N) \frac{\sigma_i}{1 + \sigma_i}, \quad i = 1, \dots, N - 1,$$

$$\rho_N < \tau_N$$

with τ_N representing fraction of time that root node N is active, assuming that all leaf nodes $1, \dots, N - 1$ are stable

There does not seem to be any closed-form expression available for τ_N in general when $N \geq 3$

General interference graphs

Any network that is not complete graph (or collection of complete graphs) contains 3-node star network as induced subgraph

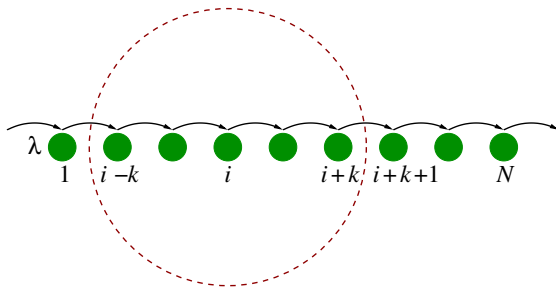
Strong dichotomy:

- simple necessary and sufficient stability conditions for complete interference graphs
- explicit characterization of stability region difficult for all other graphs

Note: conditions for existence of stabilizing activation rates 'easy'
[Jiang & Walrand (2008), Liu *et al.* (2008), Rajagopalan *et al.* (2009)]

Multi-hop scenario

Now consider multi-hop scenario with queues and packet transfers



- Packets generated at node 1 as Poisson process of rate λ
- Packets are forwarded in multi-hop fashion, i.e., node i transfers packets to node $i+1$
- Packets leave network after transmission on link N

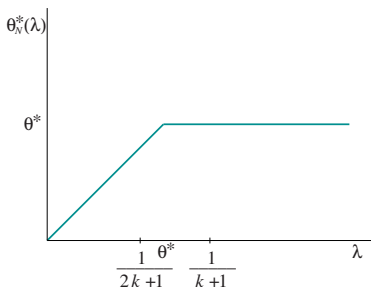
Otherwise similar dynamics as before

Multi-hop scenario (cont'd)

Denote by $\theta_i^*(\lambda) = \lim_{\nu \rightarrow \infty} \theta_i(\lambda, \nu)$ limiting throughput of node i as function of Poisson packet arrival rate λ

Define saturation throughput $\theta_N^* = \lim_{\lambda \rightarrow \infty} \theta_N^*(\lambda)$

One might expect that $\theta_N^*(\lambda) = \min\{\lambda, \theta^*\}$ with $\theta^* \in \left(\frac{1}{2k+1}, \frac{1}{k+1}\right)$

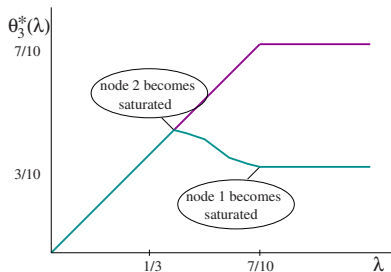


Multi-hop scenario (cont'd)

Let us focus on network with $N = 3$, $k = 1$, and determine saturation throughput $\theta_3^* = \lim_{\lambda \rightarrow \infty} \theta_3^*(\lambda)$

Queueing analysis yields $\theta_3^* = 3/10$

Note that $\theta_3^*(\lambda) = \lambda$ for all $\lambda < 1/3$!



Multi-hop scenario (cont'd)

Suppose we use fair rather than uniform activation rates:

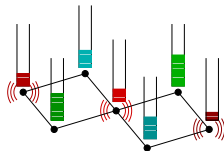
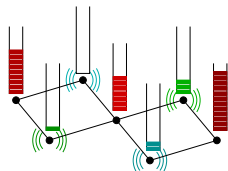
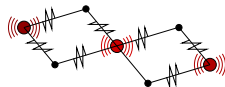
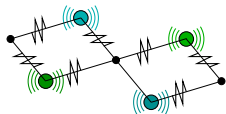
$$\nu_1 = \nu_3 = \alpha, \nu_2 = \alpha(1 + \alpha)$$

Simulation experiments suggest that end-to-end system throughput

$$\theta_3(\lambda) = \lambda \text{ for all } \lambda < \theta(\alpha) = \frac{\alpha + \alpha^2}{1 + 3\alpha + 2\alpha^2} \uparrow \frac{1}{2} \text{ as } \alpha \rightarrow \infty$$

Buffer content: quasi-stationarity??

Long starvation periods of individual nodes cause huge queue build-ups



Dynamic interaction between meta-stable activity state and buffer content yields **coupled oscillations** for large load ρ