Setting the scene: geometry in ecological dynamics

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Rothwald, Austria

an ecologist’s introduction to …

spatial structure and spatial statistics

dynamics of spatial structure

growth in neighbourhoods

spatial heterogeneity and evolution

spatial structure and spatial statistics

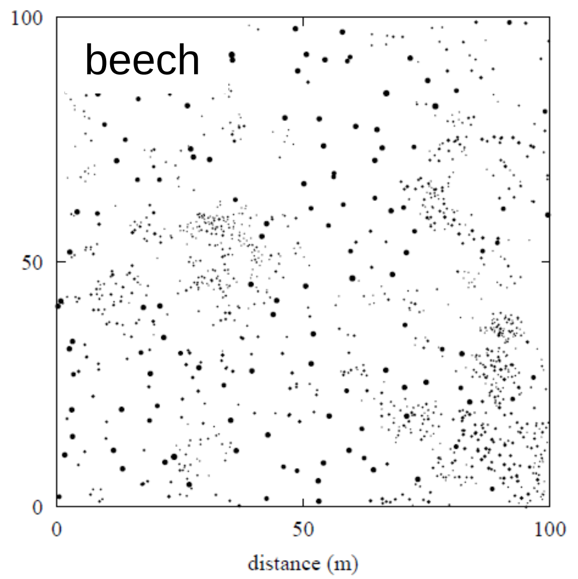
spatial structure is ubiquitous (almost)

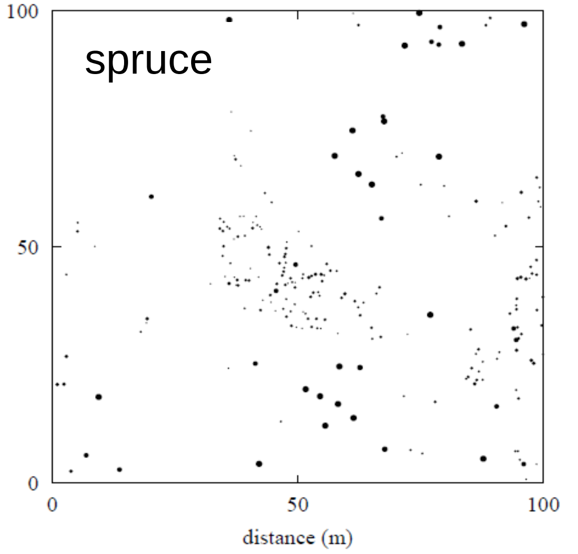
beech and spruce at Rothwald (Austria)

dipterocarps at SInharaja (Sri Lanka)

beech and spruce at Rothwald

spatial patterns





marked point pattern
from Georg Gratzer

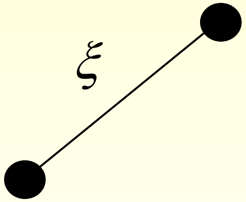
beech and spruce at Rothwald

measures of spatial structure

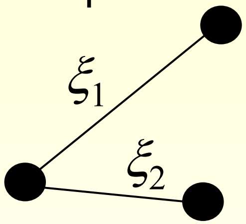
individual level

individuals

pairs



triplets



population level

density

N

pair density

$\tilde{C}(\xi)$

triplet density

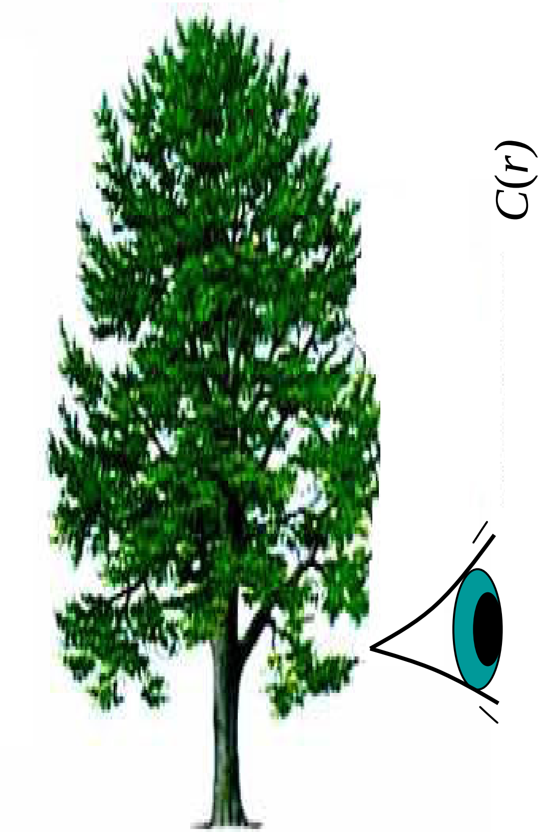
pair correlation function

$$C(\xi) = \tilde{C}(\xi)/N^2$$

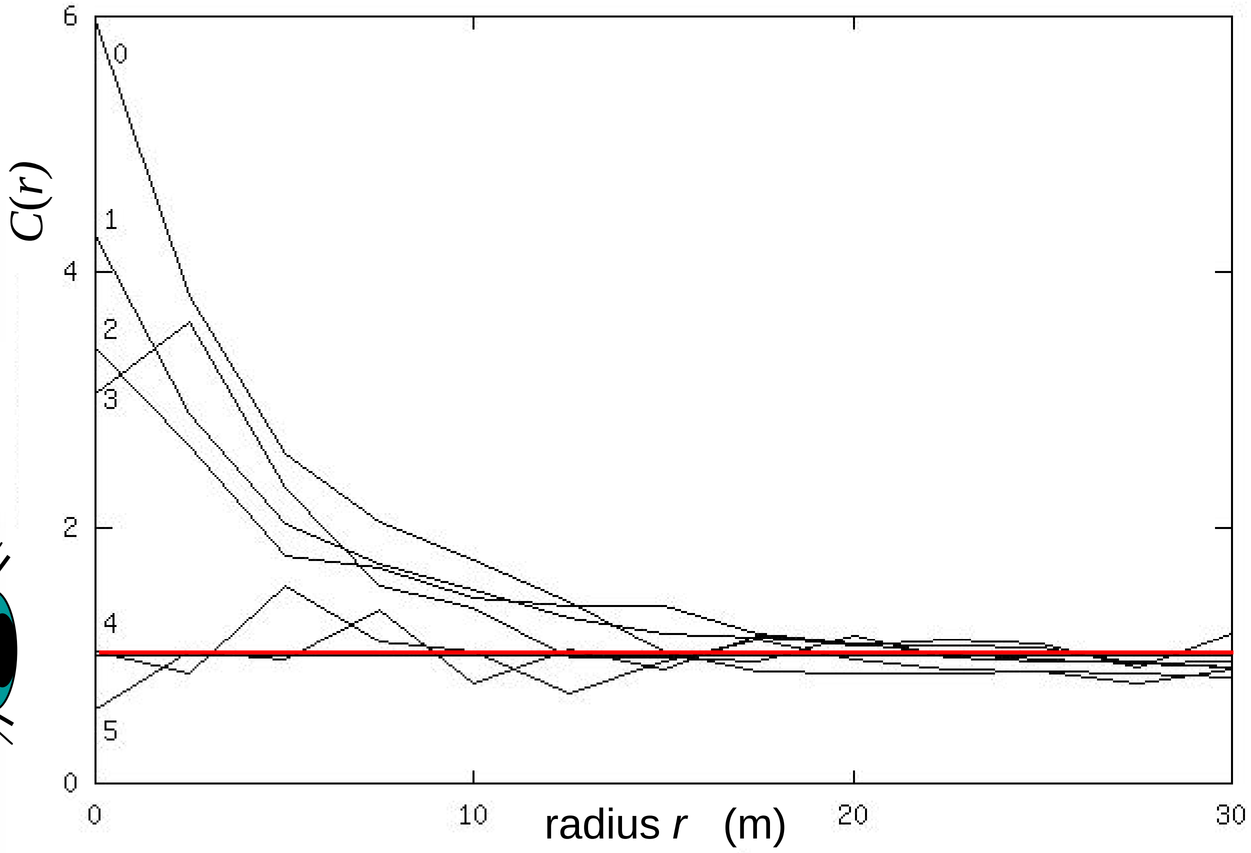
beech and spruce at Rothwald

pair correlation functions

‘plant’s-eye’ view

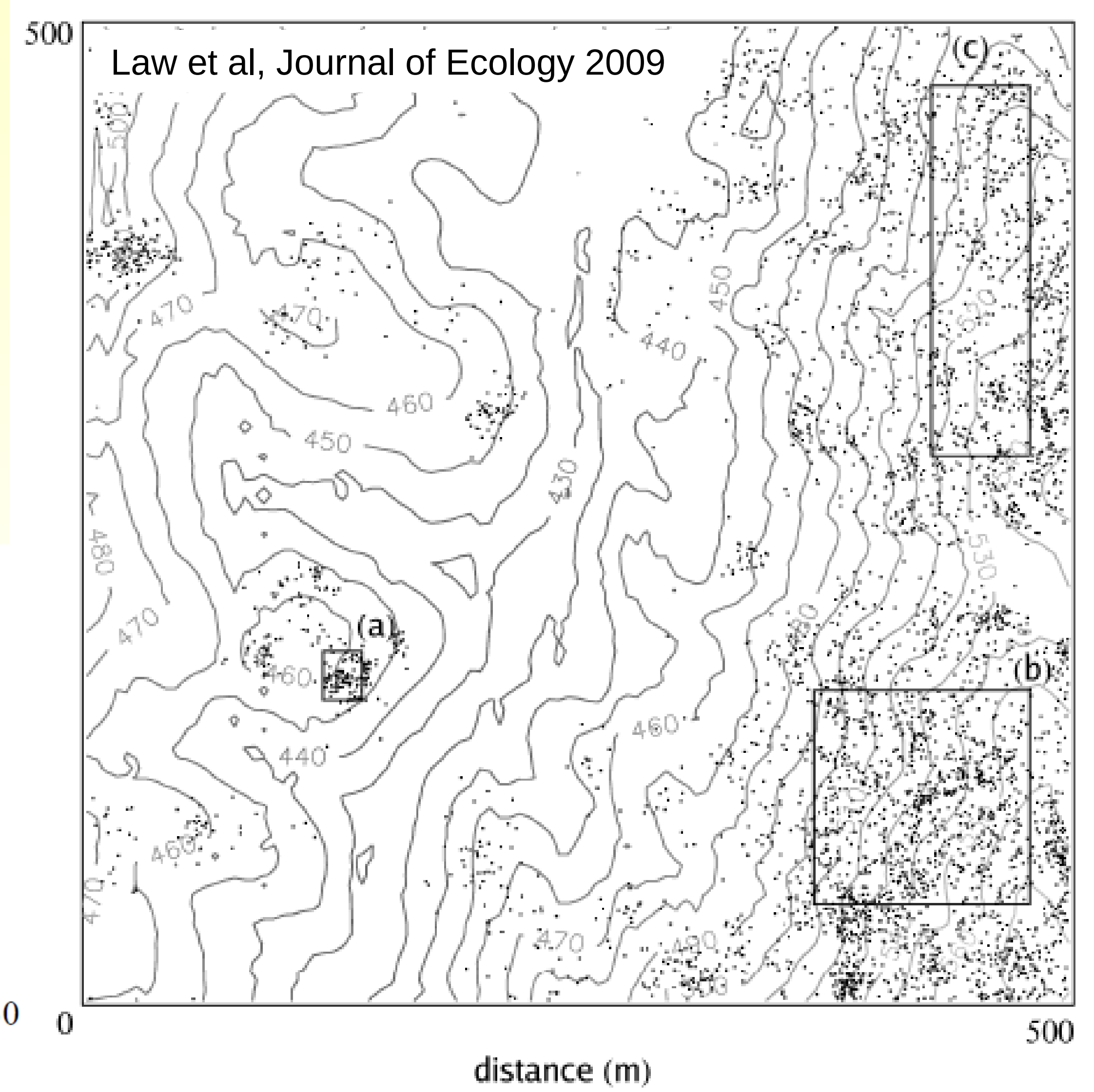


beech trees of dbh classes: 0 (1-2 cm), 1 (2-4 cm), …

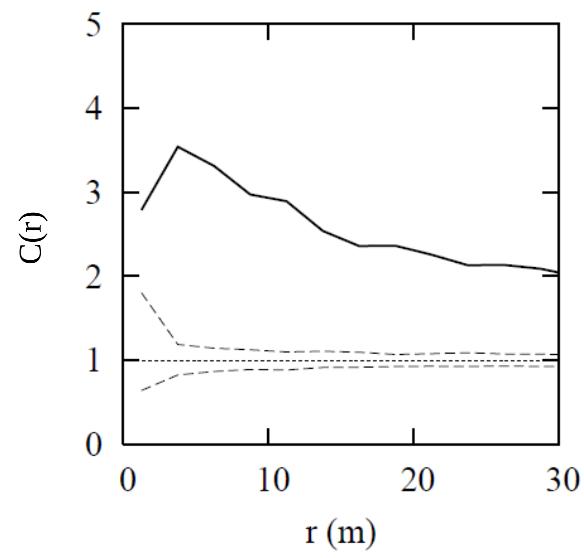


dipterocarps at Sinharaja

Shorea affinis



Law et al, Journal of Ecology 2009



dynamics of spatial structure

spatial structure is important for dynamics

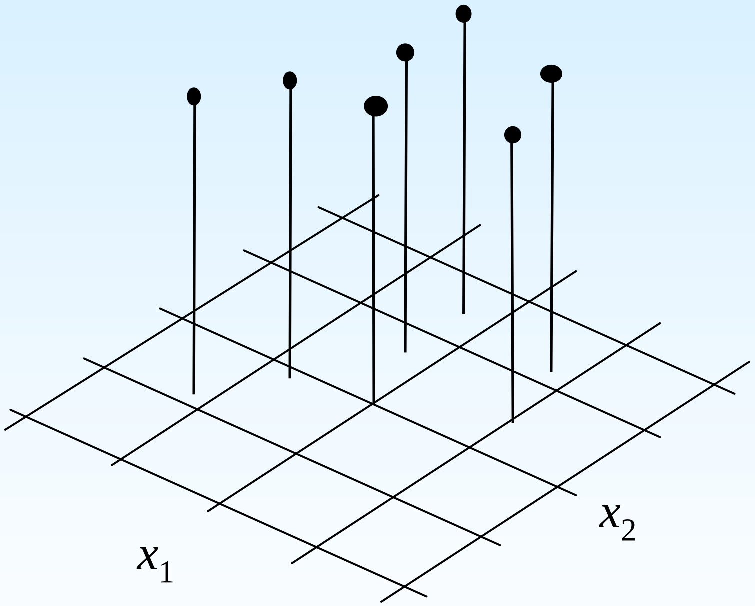
stochastic birth-death process

stochastic realisations

deterministic model

two-species competition

stochastic process: spatial pattern of individuals



continuous 2D space: $x = (x_1, x_2)$

plant at location x_l: $x_l = (x_{1l}, x_{2l})$

denoted by: $\delta(x_l - x)$

spatial pattern: $p(x) = \sum_l \delta(x_l - x)$

pattern of type i: $p_i(x)$

changing over time: $p_i(x,t)$

stochastic process: rules for birth and death rates

non-spatial

spatial

births: $B(x, x', p) \;=\; b \;\; . \; m(x'-x)$

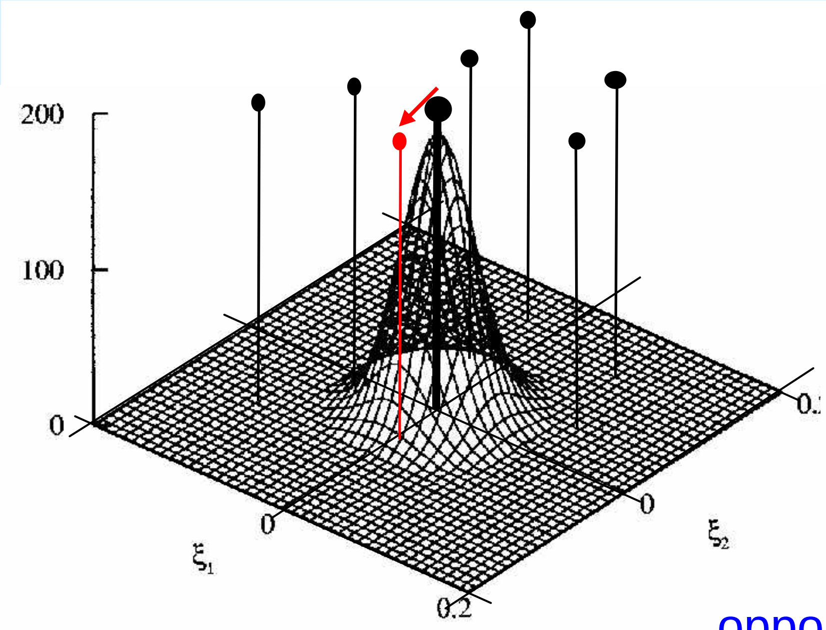
deaths: $D(x, p) \;=\; d \;\; + d' \int w(x'-x) \;\left[p(x',t)-\delta_x(x')\right]dx'$

(no growth of individuals)

stochastic process: kernels

births

dispersal: $m(x'-x)$



local dispersal of offspring

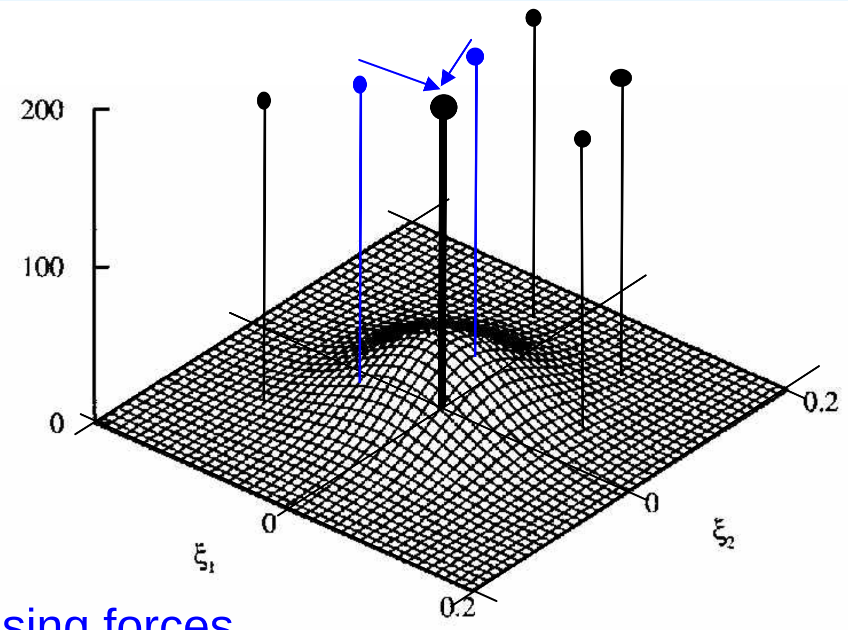
⇩

local clustering

opposing forces on spatial pattern

deaths

competition: $w(x'-x)$

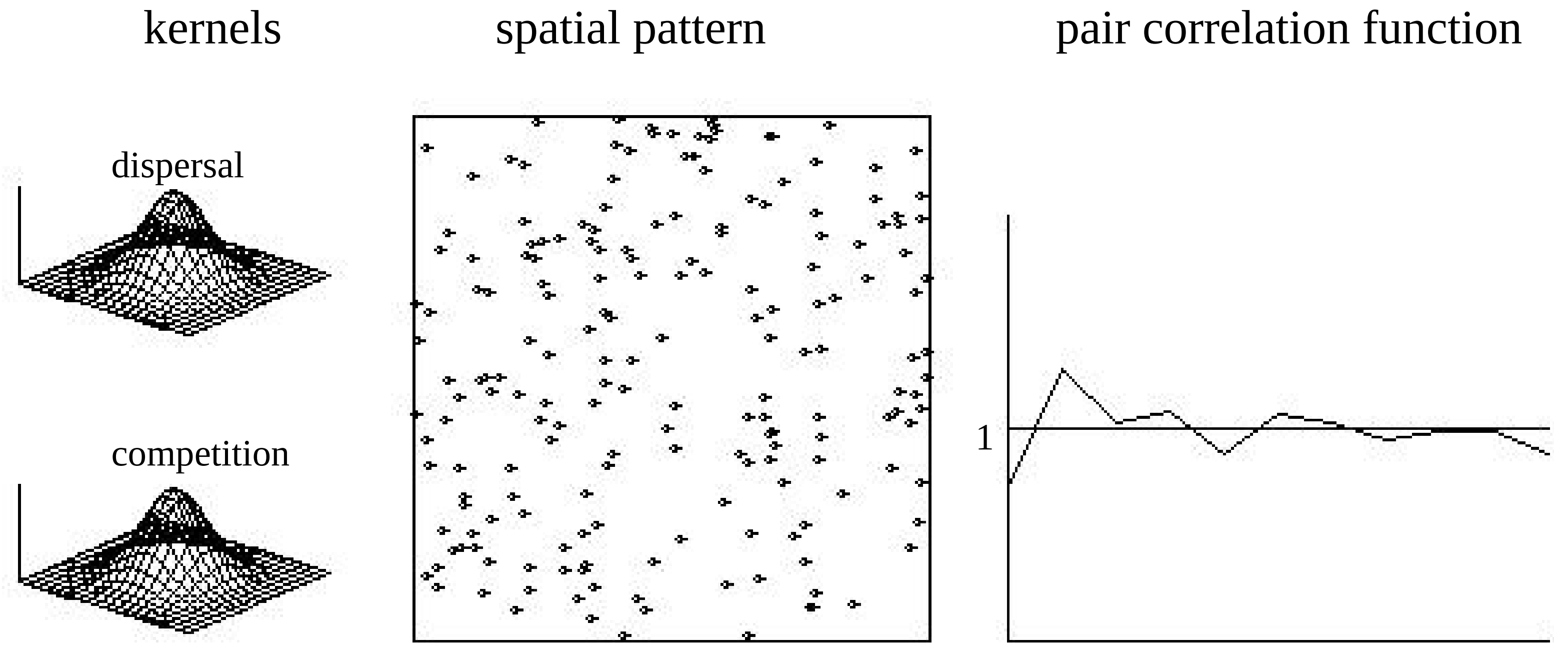


local competition

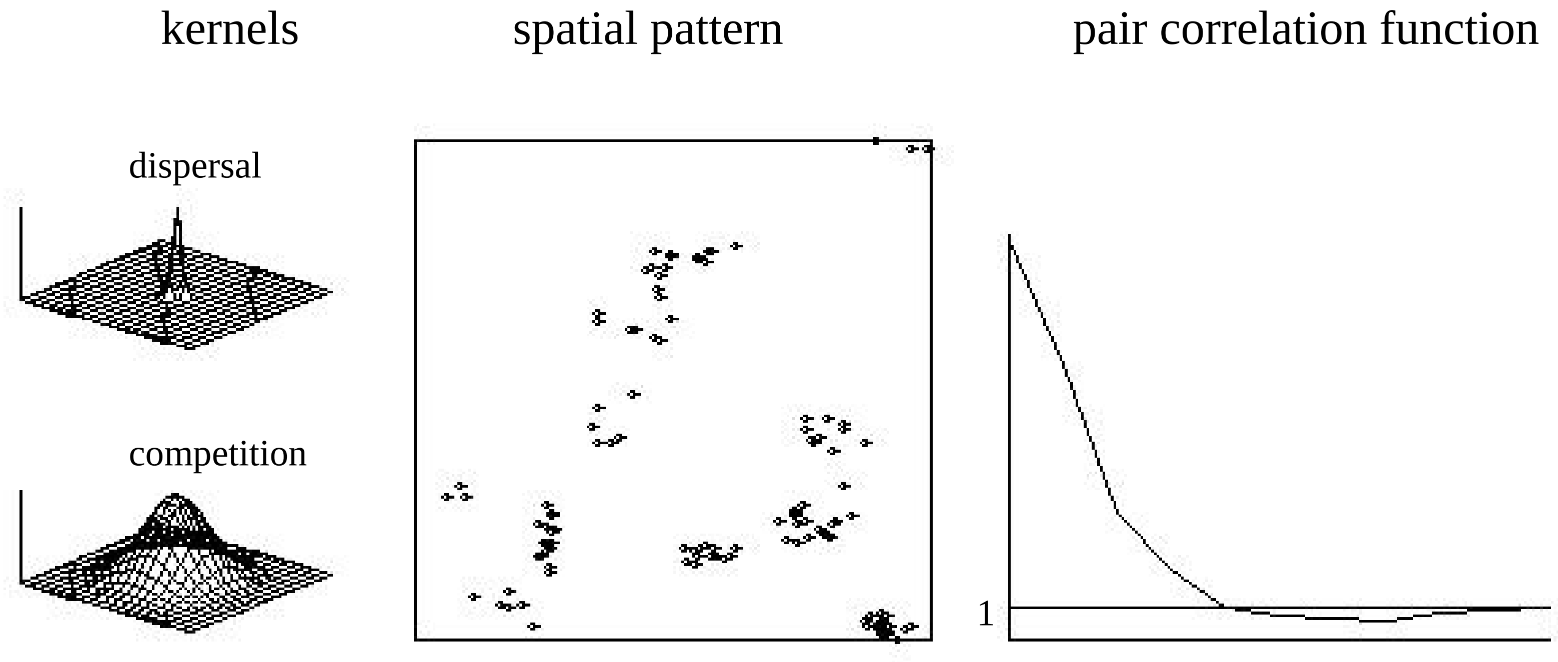
⇩

local inhibition

stochastic realisation: random spatial pattern



stochastic realisation: aggregated spatial pattern



kernels

spatial pattern

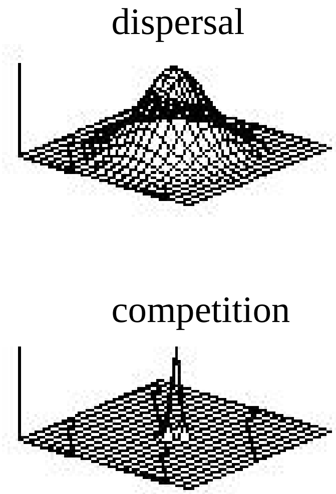
pair correlation function

dispersal

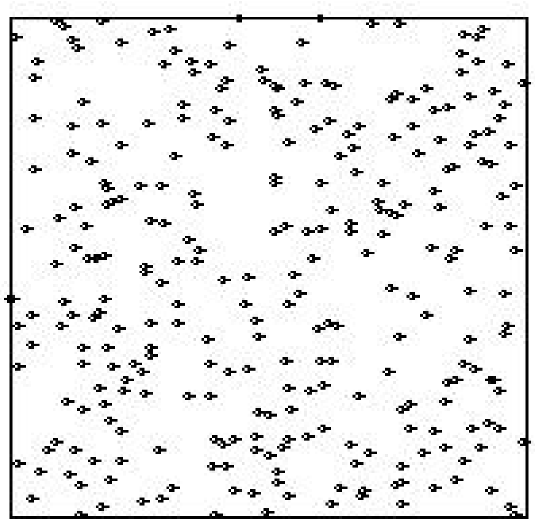
competition

stochastic realisation: regular spatial pattern

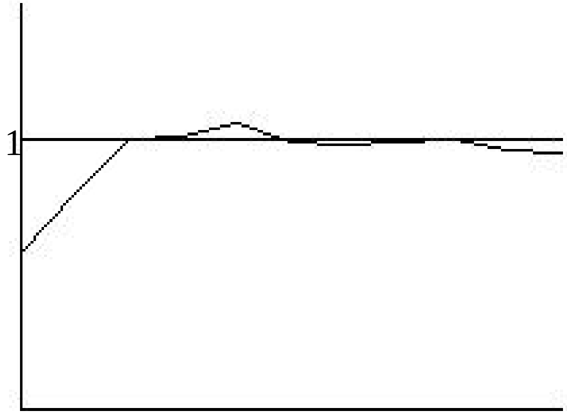
kernels



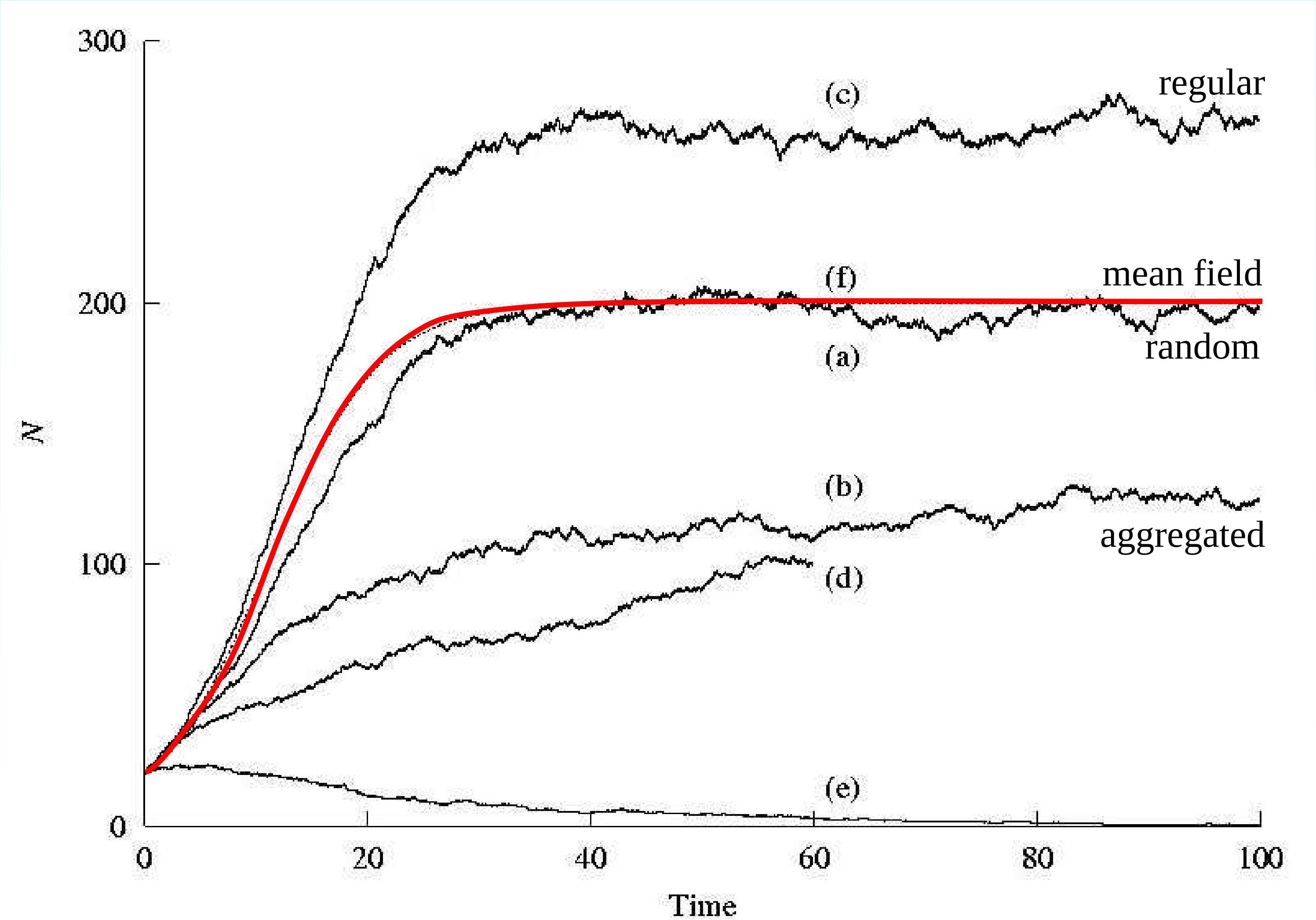
spatial pattern



pair correlation function



stochastic realisations over time



Law, Murrell and Dieckmann, Ecology 2003

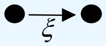
deterministic model: general scheme

Dynamical system of spatial moments (derived from the master equation)

$$\frac{d}{dt}N = F_1(N,\tilde{C})$$

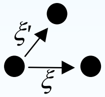
first moment N

$$\frac{d}{dt}\tilde{C}(\xi) = F_2(N,\tilde{C},T)$$



second moment $\tilde{C}$

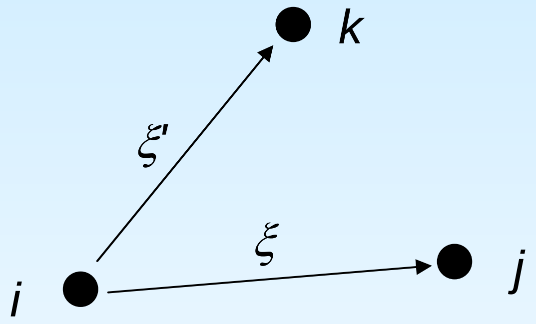
$$\frac{d}{dt}T(\xi,\xi') = F_3(N,\tilde{C},T,\ldots)$$



third moment T

Each equation coupled to the next one in the hierarchy

deterministic model: closure of hierarchy



$$T_{ijk}(\xi,\xi',p) = \frac{1}{A}\int p_i(x)\; p_j(x+\xi)\; p_k(x+\xi')\; dx$$

Truncate hierarchy at second order

Replace *T* with some function of *N* and *C*

deterministic model: logistic equations

Dynamics of first moment

$$\dot{N} = \underbrace{b\,N - d\,N}_{\text{intrinsic births and deaths}} - \underbrace{d' \int w(\xi')\,\tilde{C}(\xi')\,d\xi'}_{\text{neighbour-dependent deaths}}$$

Non-spatial logistic model:

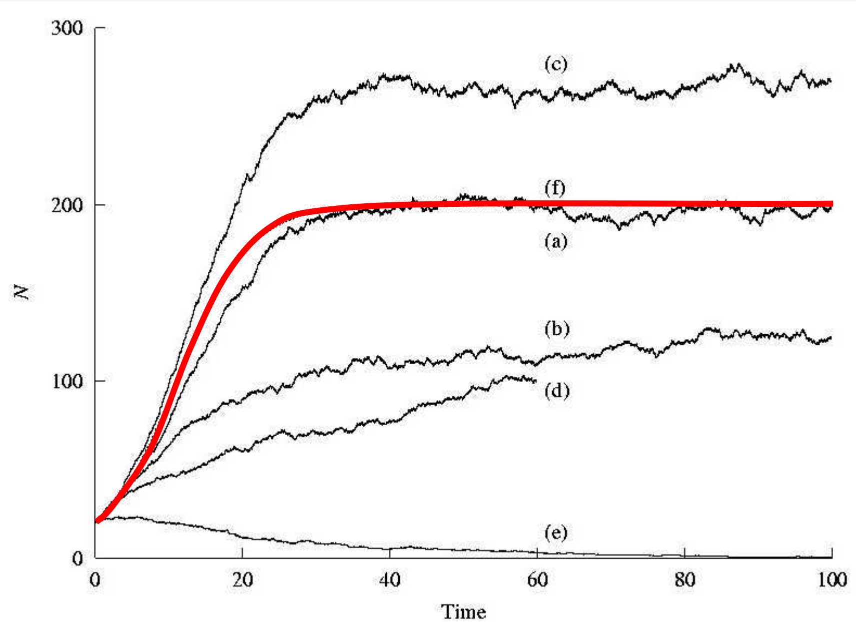
$$\dot{N} = b\,N - d\,N - d'\,N^2$$

Dynamics of $\tilde{C}(\xi)$ more complicated

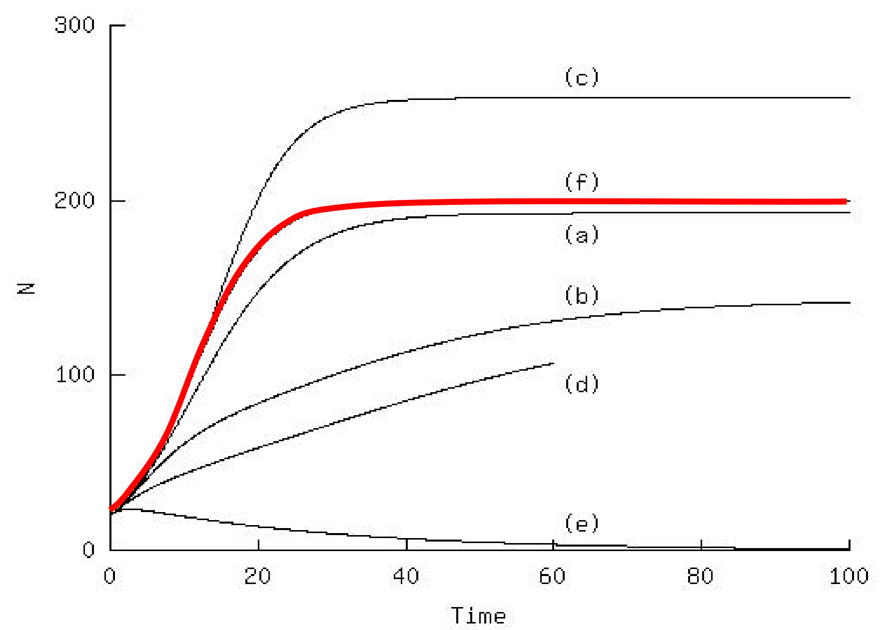
Law, Murrell and Dieckmann, Ecology 2003

deterministic model: numerical integration

stochastic realisations



numerical integrations



Law, Murrell and Dieckmann, Ecology 2003

two-species competition: moment dynamics

$$\boxed{\frac{dN_1}{dt} = (b_1 - d_1)N_1 - d'_{11}I_{11}} - d'_{12}I_{12}$$

$$\frac{dN_2}{dt} = (b_2 - d_2)N_2 - d'_{22}I_{22} - d'_{21}I_{21}$$

where $I_{ij} = \int w_{ij}(\xi)\, C_{ij}(\xi)\, d\xi$

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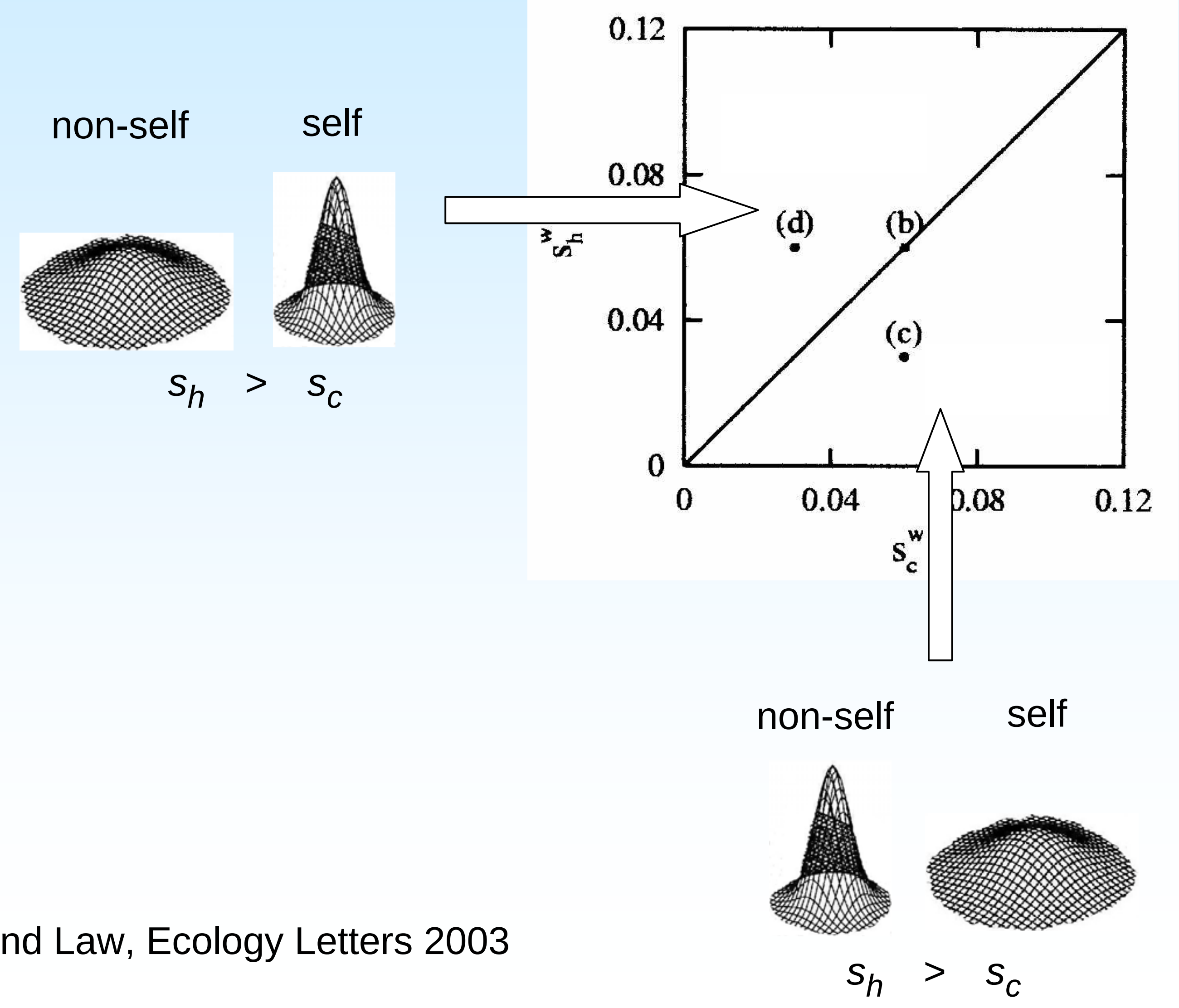
with more complicated expression for rates of change of:

$$C_{11}(\xi), C_{22}(\xi), C_{21}(\xi) = C_{12}(\xi)$$

Murrell and Law, Ecology Letters 2003

two-species competition: competition kernels



non-self self

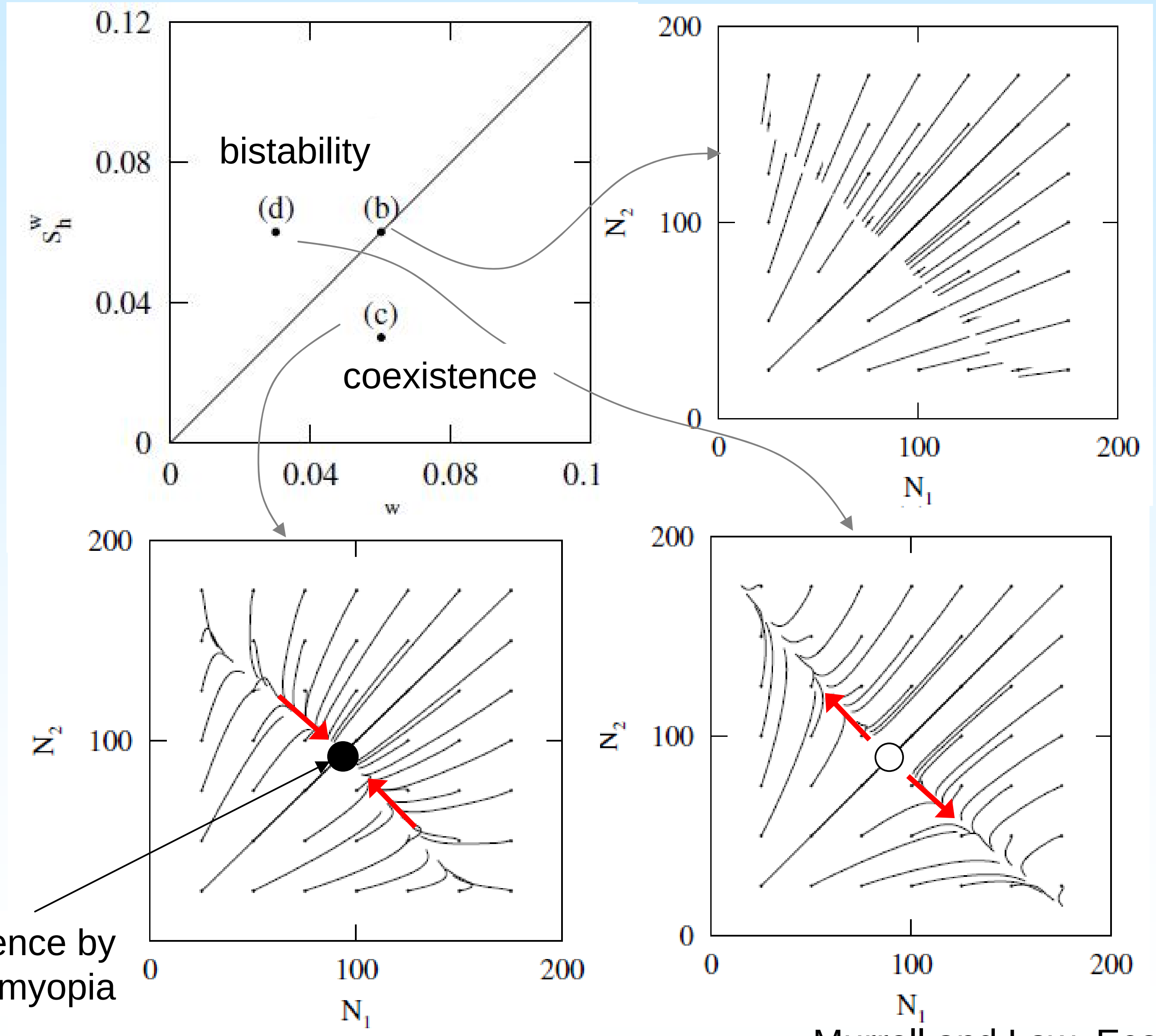
$s_h > s_c$

non-self self

$s_h > s_c$

Murrell and Law, Ecology Letters 2003

two-species competition: outcomes



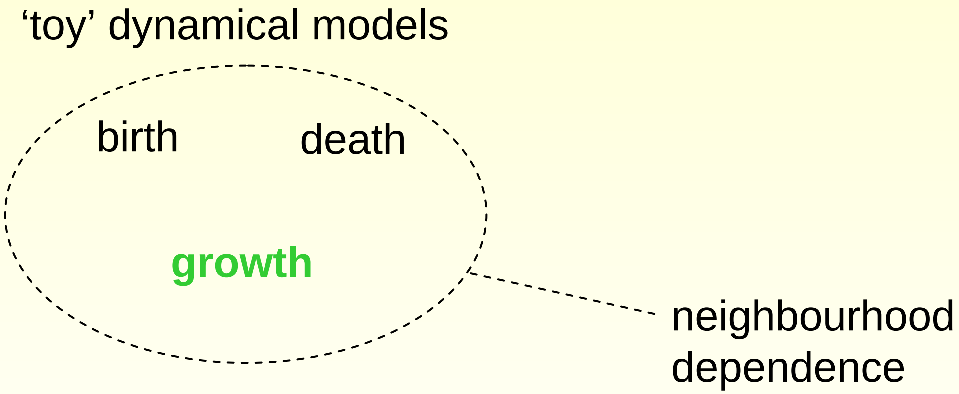
bistability

coexistence

coexistence by heteromyopia

Murrell and Law, Ecology Letters 2003

growth in neighbourhoods



‘toy’ dynamical models

birth

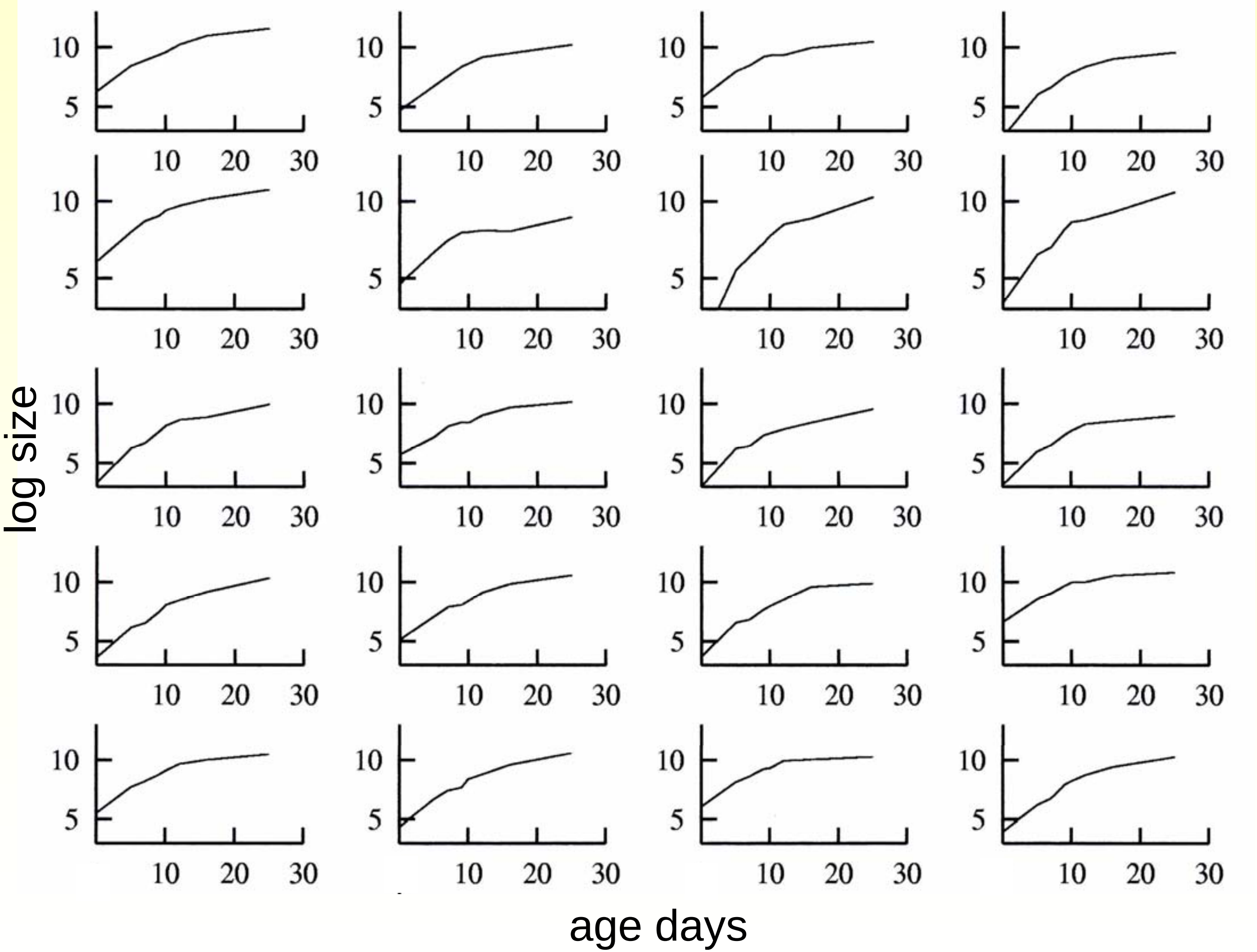
death

growth

neighbourhood
dependence

also needed in plant ecology

growth depends on neighbourhood



Schneider, Law and Illian 2006

dynamics of plant growth

System of n Gompertz growth equations coupled through neighbourhood kernel functions:

$$\frac{dy_1}{dt} = \theta_0 - \theta_1 \, y_1 - \theta_2 \sum_{j \neq 1} F(y_1, y_j, d_{1j}) \qquad \text{(plant 1)}$$

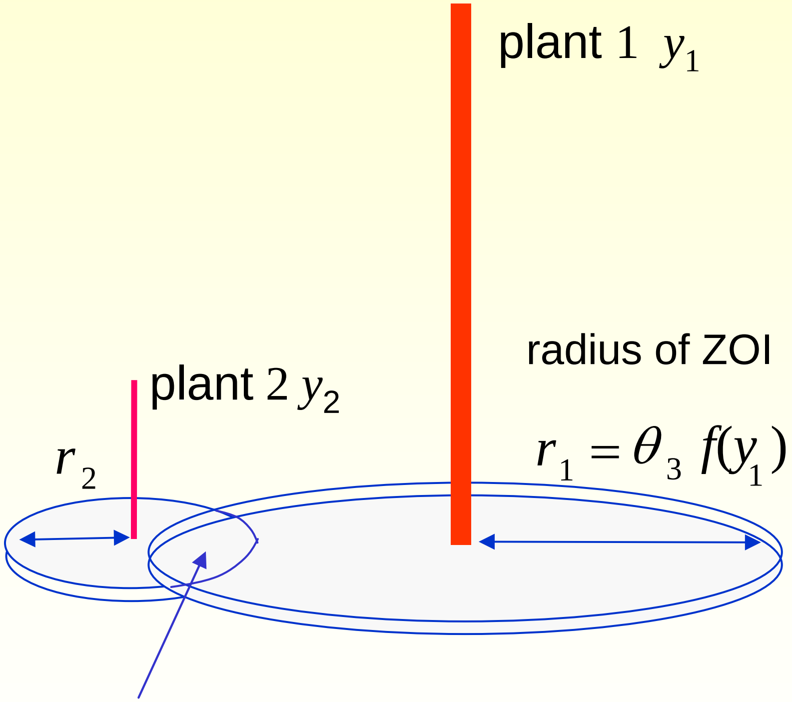
$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots$$

$$\frac{dy_n}{dt} = \underbrace{\theta_0 - \theta_1 \, y_n}_{\text{Gompertz growth}} - \underbrace{\theta_2 \sum_{j \neq n} F(y_n, y_j, d_{nj})}_{\text{coupling through nbrhd competition}} \qquad \text{(plant } n\text{)}$$

little known about F

Schneider, Law and Illian 2006

ZOI kernel



plant 1 y_1

radius of ZOI

$$r_1 = \theta_3 f(y_1)$$

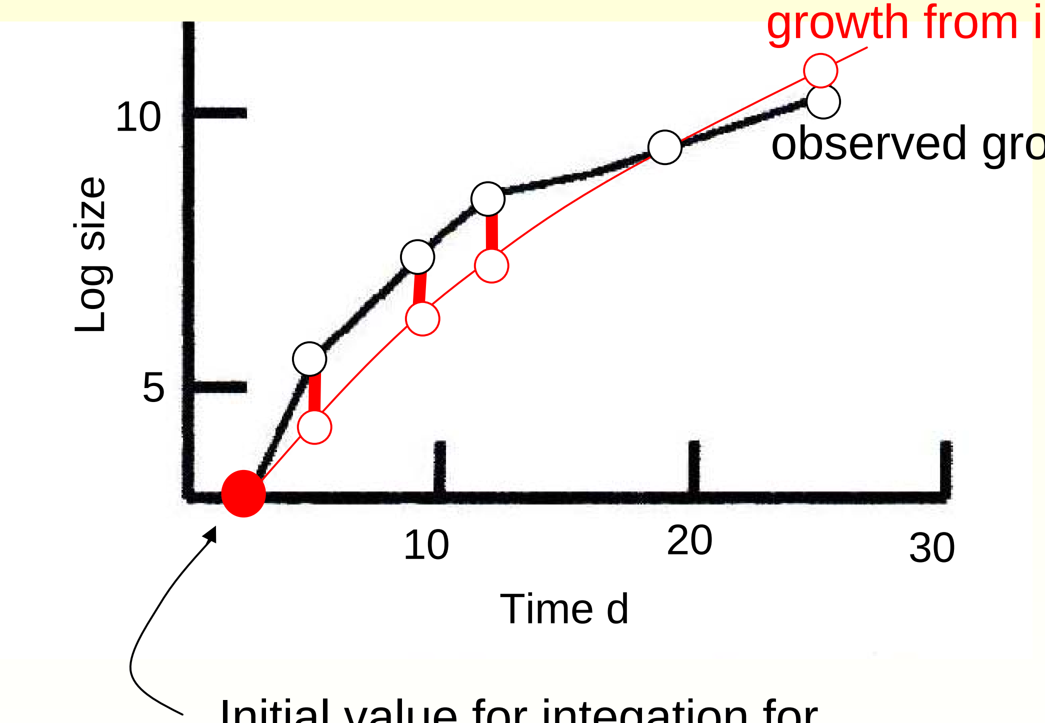
plant 2 y_2

r_2

growth reduced by
amount of overlap

fit growth model to data

For plant *i*:



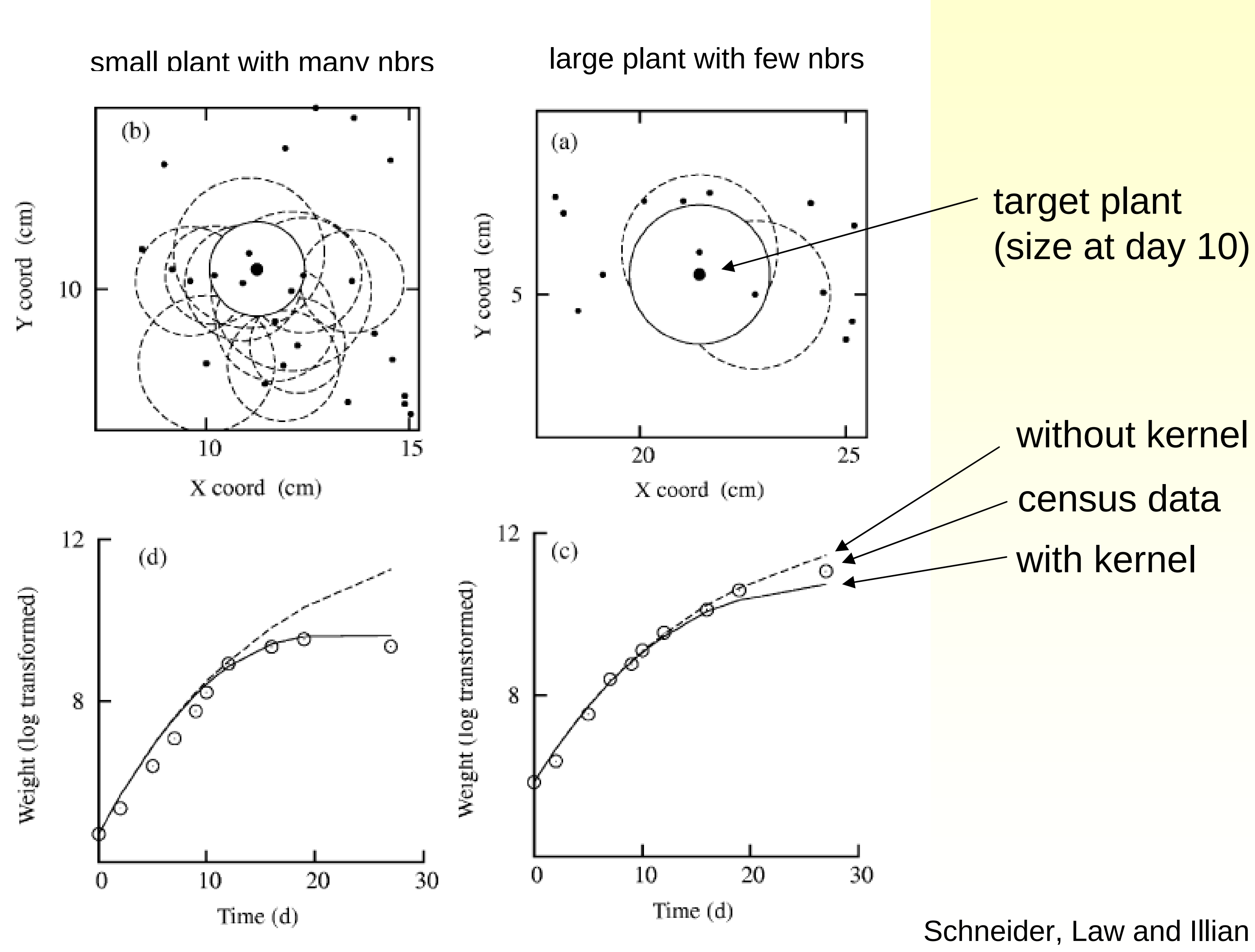
growth from integrating model with θ

observed growth of plant

Initial value for integation for plant *i*

Schneider, Law and Illian 2006

estimated growth curves



small plant with many nbrs

large plant with few nbrs

target plant
(size at day 10)

without kernel

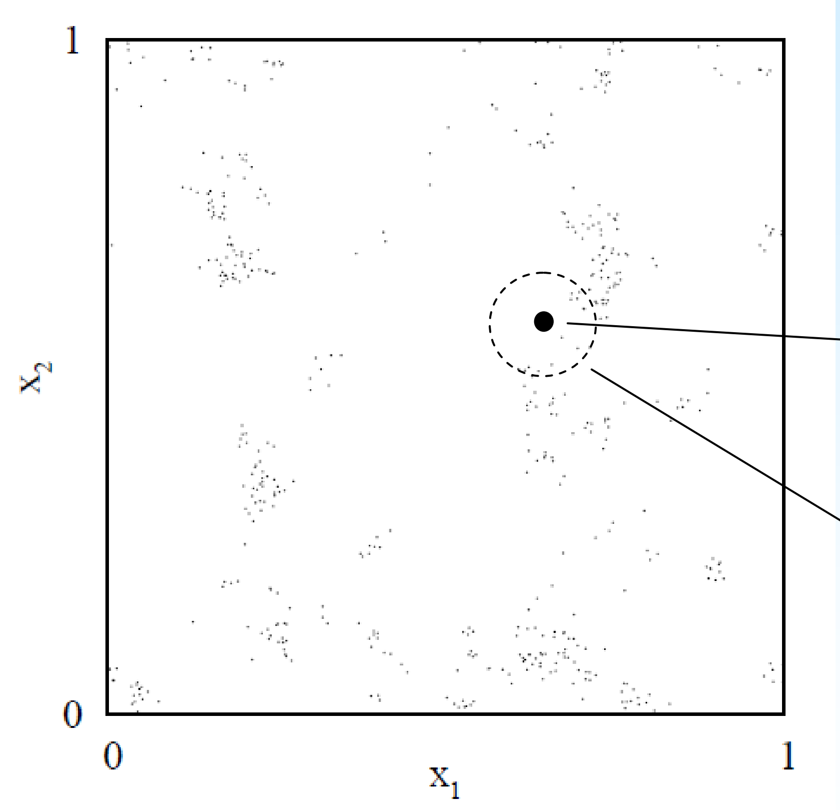
census data

with kernel

Schneider, Law and Illian 2006

heterogeneous environments and evolution

heterogeneous environment: rules for living



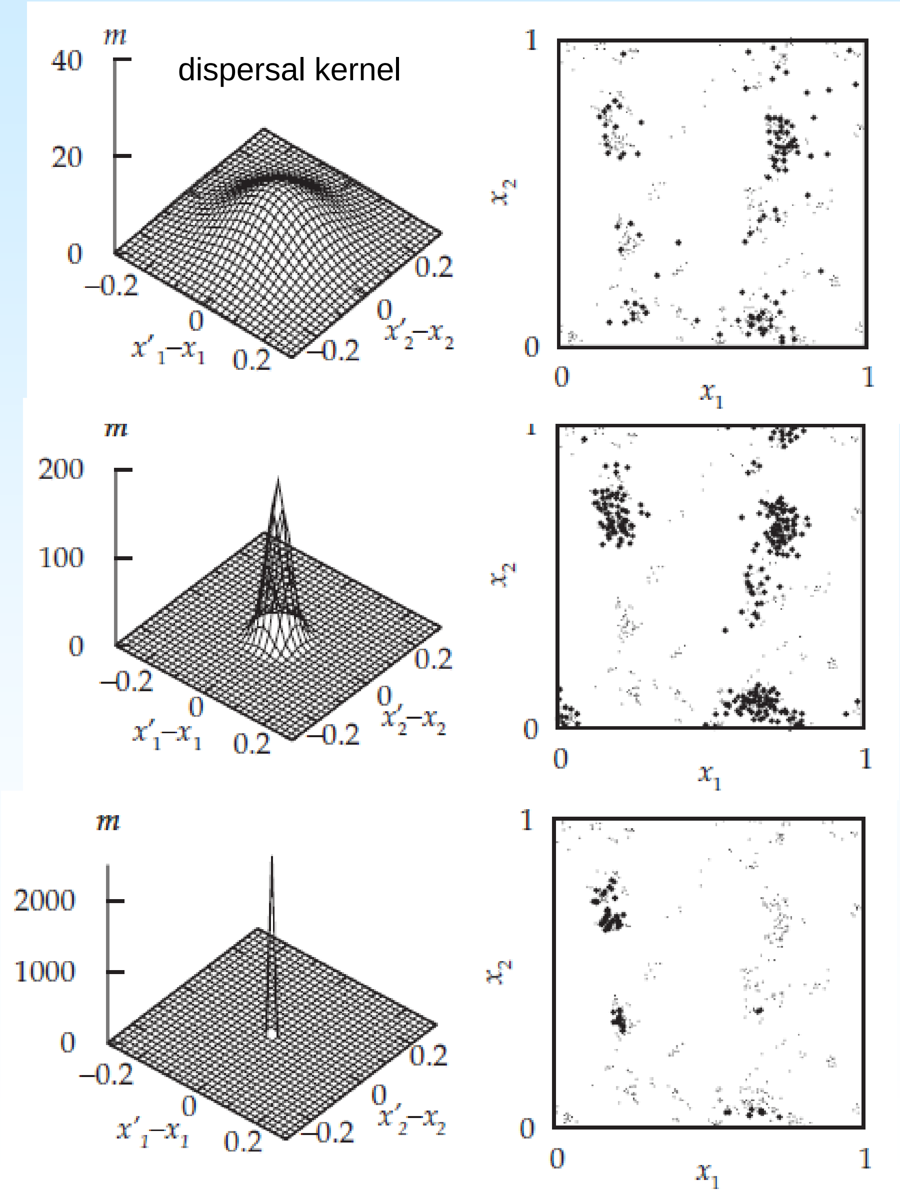
points here are high-quality locations in the environment (fixed in space)

organisms scattered across the space

death rate falls with the number of high-quality locations in a region around the organism

Cousens, Dytham and Law 2008. Dispersal in Plants. CUP

heterogeneous environment: realisations



long distance dispersal:

offspring often fall into low quality regions

medium dispersal:

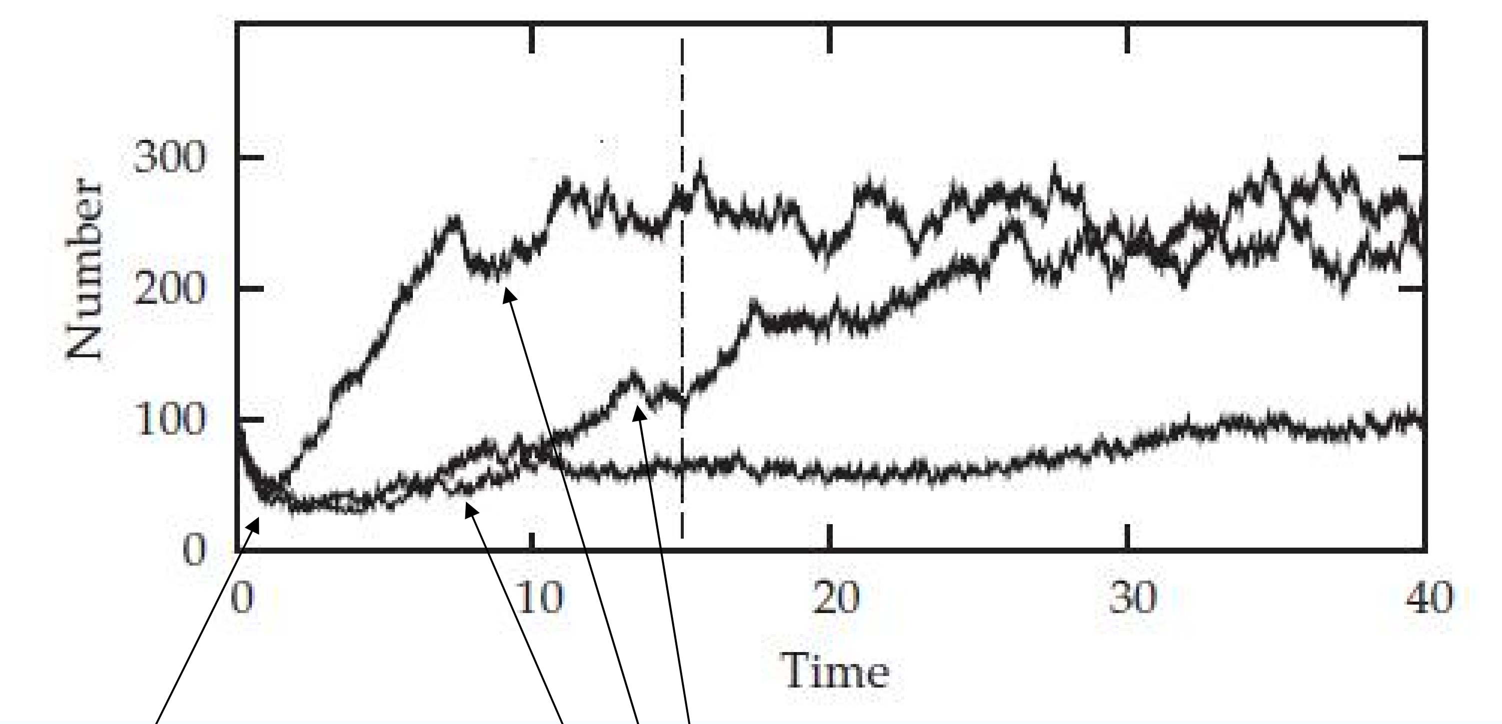
offspring spread across high-quality regions

short distance dispersal:

offspring don’t escape from competition with parents

Cousens, Dytham and Law 2008.
Dispersal in Plants. CUP

heterogeneous environment: realisations



initial decline – 100 individuals located uniformly at random across space

long dispersal: intermediate popn growth

medium dispersal: fast growth

short dispersal: slow growth

Cousens, Dytham and Law 2008. Dispersal in Plants. CUP

heterogeneous environment: evolution

evolution: change in gene frequency

e.g. genes affecting dispersal kernel

homogeneous environment:

mutant with greater dispersal wins:

determinants of geometry evolving

heterogeneous environment:

mutant with dispersal matching the spatial scale of environmental heterogeneity wins

many other possibilities