Instabilities of higher dimensional rotating black holes

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Introduction

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Higher-dimensional black holes

d-dimensional Schwarzschild:

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{d-2}^2, \qquad f = 1 - \left(\frac{r_+}{r}\right)^{d-3}$$

Rotating generalization discovered by Myers and Perry (1986)

- Arbitrary d
- Horizon topology S^{d-2}
- Uniquely specified by mass M and $[(d-1)/2]$ angular momenta J_i

The other known family of higher-dimensional black holes are the *black rings* (Emparan & HSR 01, Pomeransky & Senkov 06)

- $d = 5$
- Horizon topology $S^1 \times S^2$
- 3-parameters determine mass M and angular momenta J_1, J_2

Natural questions:

- What else is there?
- Which black holes are classically stable?

Stability of higher-dimensional black holes

Consider a black *p-brane*:

$$ds^2 = -fdt^2 + f^{-1}dr^2 + r^2 d\Omega^2 + dz_i dz_i, \qquad f = 1 - \frac{r_+}{r}$$

Gregory & Laflamme (1993) showed that linearized gravitational perturbations with sufficiently long wavelength in z-directions grow exponentially with time.

A black ring with large angular momentum J_1 has S^1 radius much greater than S^2 radius: locally looks like a boosted black string ($p = 1$), suggests instability (Emparan & HSR 01).

Ultraspinning instability

A *singly-spinning* Myers-Perry black hole (i.e. one with $J_1 \neq 0$, other J_i vanishing) has superficial resemblance to Kerr. *But* Kerr has upper bound on J for given M. This is not true for MP with $d \geq 6$. Such black holes can have arbitrarily large J for given M: these are called *ultraspinning* black holes.

In limit of large J, ultraspinning MP black hole resembles black *brane*, suggests instability (Emparan & Myers 03)

These arguments are heuristic. Demonstrating existence of an instability requires a study of linearized gravitational perturbations. Hard! But, for Myers-Perry, progress has been made in past year.

Linearized gravitational perturbations: known results

- Schwarzschild solution is stable for all $d > 4$ (Ishibashi & Kodama 03)
- Singly spinning MP: time-independent mode appears at critical value of J (for given M). Checked for $6 \leq d \leq 11$. Black hole expected to be unstable for larger J. (Dias *et al* 09,10)
- Most symmetrical MP black hole has equal angular momenta ($J_i = J$) with odd d. Solution is cohomogeneity-1. There is an upper bound on J. Saturating this bound gives extreme black hole, as for Kerr. For $d = 9$, solutions close to extremality exhibit linearized perturbations growing exponentially with time. (Dias *et al* 10).

Nonlinear results

Shibata and Yoshino (09,10) have solved numerically the nonlinear Einstein equation to follow the evolution of initial data corresponding to a perturbed singly spinning MP solution with $d = 5, 6, 7, 8$. They found an instability for large J. There is a qualitative difference from the linearized results just discussed.

Let $k + \Omega_H m$ denote the Killing vector tangent to the horizon generators. The linearized analyses performed so far restrict to perturbations preserving symmetry generated by rotational Killing vector field m. The solutions of Shibata and Yoshino break this symmetry. Their instability involves emission of gravitational waves, and then settling down to a MP solution with reduced J.

The Shibata-Yoshino instability appears at *lower* J than the m-invariant instabilities found in linearized analyses.

This talk.

The rest of this talk will concern linearized gravitational perturbations of higher-dimensional rotating black holes.

So far, work on this subject has been limited to particular Myers-Perry black holes which exhibit *symmetry enhancement*. A general MP solution has $\mathbb{R} \times U(1)^N$ isometry group, $N = [(d-1)/2]$. Singly spinning MP solutions, and equal angular momenta, odd d, solutions have much larger isometry groups. This makes the study of linearized perturbations tractable. How can we make progress with the general case?

In 4d, the tractability of the study of linearized gravitational perturbations of Kerr derives from the remarkable decoupling property discovered by Teukolsky (1972). We shall explore the extent to which this extends to $d > 4$ dimensions.

Review of 4d results: gauge invariance

As discovered by Teukolsky, and emphasized by Stewart and Walker (1974), decoupling is closely related to the existence of gauge-invariant local variables describing metric perturbations.

Consider linearized perturbations of a 4d spacetime. We can separate any scalar X into a background value $X^{(0)}$ and a perturbation $X^{(1)}$. There is gauge freedom corresponding to infinitesimal diffeomorphisms, under which $X^{(1)} \to X^{(1)} + \xi \cdot \partial X^{(0)}$. Hence $X^{(1)}$ is gauge-invariant iff $X^{(0)} = \mathrm{const.}$

Teukolsky's approach employs the Newman-Penrose formalism, based on a null tetrad $\{\ell, n, m, \bar{m}\}$. Let Ψ_A, $A = 0, 1, 2, 3, 4$ denote the NP scalars encoding the Weyl tensor. Now we have gauge freedom corresponding to infinitesimal tetrad rotations. However:

$\Psi_0^{(1)}$ *is invariant under infinitesimal coordinate transformations and infinitesimal tetrad rotations iff* $\Psi_0^{(0)} = \Psi_1^{(0)} = 0$, *i.e., iff the background spacetime is algebraically special.*

Other quantities, e.g. $\Psi_1^{(1)}$, are *not* gauge invariant.

Summary: for linearized metric perturbations of an algebraically special spacetime, there exists a local quantity, linear in the metric perturbation, that is invariant under infinitesimal diffeomorphisms and infinitesimal changes of basis.

Review of 4d results: decoupling

Gauge-invariant quantity $\Psi_0^{(1)}$ is a complex scalar: 2 physical degrees of freedom, just as for gravitational field. Plausible that it encodes all information about metric perturbation. Furthermore, one can *decouple* $\Psi_0^{(1)}$ from the other $\Psi_A^{(1)}$:

$\Psi_0^{(1)}$ satisfies a second order, linear, homogeneous, hyperbolic, partial differential equation: the Teukolsky equation.

This fact renders tractable the study of Kerr perturbations.

Does any of this extend to $d > 4$ dimensions?

Higher dimensions: null bases

Start by figuring out the $d > 4$ analogue of Ψ_0.

Introduce a null basis $\{\ell, n, m_i\}$, $i = 2, \ldots d-1$, where ℓ, n are null, m_i are spacelike, and $\ell \cdot m_i = n \cdot m_i = 0$, $m_i \cdot m_j = \delta_{ij}$, $\ell \cdot n = 1$.

In 4d, $\Psi_0 \equiv C_{abcd}\ell^a m^b \ell^c m^d$. The $d > 4$ analogue is therefore

$$\Omega_{ij} = C_{abcd}\ell^a m_i^b \ell^c m_j^d$$

This is a $(d-2) \times (d-2)$ traceless, symmetric, matrix: same number of degrees of freedom as gravitational field.

Gauge invariance

Consider linearized perturbations, decompose quantities into background (e.g. $\Omega_{ij}^{(0)}$) and perturbation ($\Omega_{ij}^{(1)}$). Follow same argument as in 4d:

$\Omega_{ij}^{(1)}$ *is invariant under infinitesimal coordinate transformations and infinitesimal basis transformations iff*

$$\Omega_{ij}^{(0)} = \Psi_{ijk}^{(0)} = 0 \qquad (*)$$

Here, $\Psi_{ijk} \equiv C_{abcd}\ell^a m_i^b m_j^c m_k^d$ is the $d > 4$ analogue of Ψ_1.

Classification of the Weyl tensor in d dimensions

The Petrov classification was extended to d dimensions by Coley *et al* (2004).

ℓ is a *Weyl aligned null direction* (WAND) iff $\Omega_{ij} = 0$.

ℓ is a *multiple WAND* iff $\Omega_{ij} = \Psi_{ijk} = 0$.

For $d = 4$, these are equivalent to the notions of principal null direction and repeated principal null direction.

We shall say that a spacetime is *algebraically special* if it admits a multiple WAND.

The Myers-Perry solutions are algebraically special. They are type D, i.e., they admit two distinct multiple WANDs. Black rings are not algebraically special.

Summary so far

- $\Omega_{ij} = C_{abcd}\ell^a m_i^b \ell^c m_j^d$ is the d-dimensional analogue of Ψ_0
- $\Omega_{ij}^{(1)}$ has the same number of degrees of freedom as the gravitational field.
- $\Omega_{ij}^{(1)}$ is invariant under infinitesimal coordinate transformations and infinitesimal basis transformations iff the background spacetime is algebraically special.
- The Myers-Perry solutions are algebraically special.

Does $\Omega_{ij}^{(1)}$ satisfy a decoupled equation?

Decoupling in higher dimensions

Superficially, it appears that $\Omega^{(1)}_{ij}$ will couple to the perturbations in other Weyl components, e.g., $\Psi^{(1)}_{ijk}$. Under what circumstances can one decouple $\Omega^{(1)}_{ij}$?

We analyzed this following the approach of Stewart and Walker (1974). They used the Geroch-Held-Penrose (1973) formalism. This was extended to d dimensions recently (Durkee *et al* 10).

Sketch of method:

- Use Bianchi identity to obtain an equation in which the only Weyl components on which second derivatives act is $\Omega_{ij}^{(1)}$.
- Use Bianchi identity to eliminate first derivatives of as many other Weyl components as possible.
- First derivatives of other Weyl components remain. Impose conditions on *coefficients* of these terms to eliminate them. These coefficients involve first derivatives of ℓ.
- Demand that coefficients of non-derivative terms involving Weyl components other than $\Omega_{ij}^{(1)}$ also vanish.

Result of decoupling analysis

$d = 4$: $\Omega^{(1)}_{ij} \sim \Psi^{(1)}_0$ satisfies a decoupled equation (in vacuum) iff ℓ is geodesic and shearfree. Guaranteed by Goldberg-Sachs theorem!

$d > 4$: $\Omega^{(1)}_{ij}$ decouples iff ℓ is geodesic with vanishing expansion, rotation and shear.

The condition that ℓ be geodesic is not restrictive: any algebraically special spacetime admits a geodesic multiple WAND (Durkee & HSR 09).
A spacetime admitting a null geodesic congruence with vanishing expansion, rotation and shear is called a *Kundt spacetime*. Any such spacetime is algebraically special.
Unfortunately, black hole spacetimes are not Kundt spacetimes.

Near-horizon geometries: motivation

The near-horizon geometry of an extreme black hole *is* a Kundt spacetime: can study gravitational perturbations using our decoupled equation.

Why is this interesting?

We will suggest that, under certain circumstances, an instability of a near-horizon geometry implies instability of the full extreme black hole.

Instability of an extreme black hole suggests that near-extreme black holes also will be unstable.

Near-horizon geometry: definition

Consider a stationary black hole with a degenerate Killing horizon. Introduce Gaussian null coordinates near the horizon (Isenberg & Moncrief 85):

$$ds^2 = -r^2 F(r,x)dv^2 + 2dvdr + 2rh_i(r,x)dvdx^i + \gamma_{ij}(r,x)dx^i dx^j$$

$\partial/\partial v$ is Killing vector field, horizon is at $r = 0$. Near-horizon limit defined by $v \to v/\epsilon$, $r \to \epsilon r$ and $\epsilon \to 0$ (HSR 02), giving

$$ds^2 = -r^2 F(x)dv^2 + 2dvdr + 2rh_i(x)dvdx^\mu + \gamma_{ij}(x)dx^i dx^j$$

$\partial/\partial r$ tangent to null geodesics with vanishing expansion, rotation and shear: Kundt spacetime.

Symmetry enhancement

$$ds^2 = -r^2 F(x) dv^2 + 2dvdr + 2rh_i(x)dvdx^\mu + \gamma_{ij}(x)dx^i dx^j$$

2d non-abelian symmetry group generated by $v \to v + \alpha$ and $v \to v/\lambda$, $r \to \lambda r$.

The near horizon geometry of any *known* extreme black hole solution has more symmetry. With n commuting rotational Killing vector fields, it takes form (Kunduri *et al* 07,08)

$$\begin{aligned} ds^2 &= L(y)^2 \left(-R^2 dT^2 + \frac{dR^2}{R^2} \right) + g_{MN}(y) dy^M dy^N \\ &+ g_{IJ}(y) \left(d\phi^I - k^I R dT \right) \left(d\phi^J - k^J R dT \right) \end{aligned}$$

Isometry group $SL(2,\mathbb{R}) \times U(1)^n$.

$$
\begin{aligned}
ds^2 &= L(y)^2 \left(-R^2 dT^2 + \frac{dR^2}{R^2} \right) + g_{MN}(y) dy^M dy^N \\
&+ g_{IJ}(y) \left(d\phi^I - k^I R dT \right) \left(d\phi^J - k^J R dT \right)
\end{aligned}
$$

$-dT \pm dR/R^2$ both are tangent to null geodesics with vanishing expansion, rotation and shear: this spacetime is "doubly Kundt".

Perturbations of near-horizon geometries

Proceed phenomenologically: consider perturbations of certain near-horizon geometries and see how results correlate with stability properties of full black hole.

Start with near-horizon extreme Kerr (NHEK) geometry (Bardeen & Horowitz 99). Let **m** denote Killing vector field that generates axisymmetry of Kerr.

NHEK can be viewed as S^2 bundle over AdS_2. Linearized gravitational perturbations governed by Teukolsky eq. Expand in harmonics on S^2: labelled by integers (l, m), $|m| \leq l$, $l \geq 2$. Eq reduces to eq of a massive charged scalar in AdS_2 with a homogeneous electric field.

Breitenlöhner-Freedman bound

In AdS_d (with radius ℓ), a free scalar field is stable iff its mass μ respects the BF (1982) bound:

$$\mu^2 \geq -\frac{(d-1)^2}{4\ell^2}$$

We have scalar with mass μ and charge q in AdS_2 with homogeneous electric field. μ and q are complex but $\mu^2 - q^2$ is real. Behaviour of solutions suggests that "effective BF bound" is

$$\mu^2 - q^2 \geq -\frac{1}{4\ell^2}$$

If this is violated then we shall say that the near-horizon geometry is unstable.

NHEK stability (Amsel *et al*, Dias *et al*, 09)

Recall: NHEK modes labelled by (l, m). m is eigenvalue of perturbation w.r.t. generator of axisymmetry $\mathbf{m}$, i.e., $\mathcal{L}_{\mathbf{m}} h = imh$.

Modes with $m = 0$, i.e., axisymmetric modes, respect effective BF bound.

Modes with $|m| \sim l$ violate effective BF bound: unstable.

But Kerr is (believed to be) stable!

Our proposal: instability of near-horizon geometry implies instability of full black hole only for modes preserving certain symmetry. In the case of Kerr, the symmetry is axisymmetry.

Near-horizon Myers-Perry stability (Durkee & HSR)

Myers-Perry with equal angular momenta in odd d is cohomogeneity-1.

Near-horizon geometry is homogeneous, and can be viewed as a S^{d-2} bundle over AdS_2.

Study gravitational perturbations using decoupled eq: expand in harmonics on S^{d-2} to reduce to eq of massive charged scalar in AdS_2 with homogeneous electric field.

Write Killing field tangent to horizon generators as $\mathbf{k} + \Omega_H \mathbf{m}$.

Certain modes that violate symmetry generated by $\mathbf{m}$ violate effective BF bound for any d: analogous to non-axisymmetric unstable modes in NHEK. We claim that this tells us nothing about stability of full black hole.

Modes that respect symmetry generated by $\mathbf{m}$ respect BF bound for $d = 5$.

A few modes that respect symmetry generated by $\mathbf{m}$ violate BF bound for $d \geq 7$. We predict that these correspond to instability of full black hole.

Comparison with known results

Previous studies of perturbations of cohomogeneity-1 Myers-Perry:

$d = 5$: Murata & Soda (2008) found no evidence of any instability.

$d > 5$: Dias *et al* (2010) found instability for $d = 9$, conjectured to exist also for $d > 9$. Instability respects symmetry generated by $\mathbf{m}$.

Appears to be a discrepancy between these results and prediction from near-horizon geometry.

J. Santos has repeated the analysis of Dias *et al* for black holes *very* close to extremality. He finds instability for $d = 7, 9, 11, 13, 15$. In all cases, the modes that we predict to be unstable do indeed turn out to be unstable.

Why does this work?

Consider a scalar field in a stationary rotating black hole background. Impose symmetry conditions on the scalar to reduce its eq of motion to the form

$$-\frac{\partial^2 \Phi}{\partial t^2} = \mathcal{A}\Phi$$

where $\mathcal{A}$ involves only spatial derivatives. Introduce scalar product $(,)$ so that $\mathcal{A}$ self-adjoint. Let λ_0 denote lowest eigenvalue of $\mathcal{A}$. Then, if $\lambda_0 < 0$, Φ is unstable, growing as $\exp(\sqrt{-\lambda_0}t)$.

λ_0 can be estimated using the Rayleigh-Ritz formula:

$$\lambda_0 \leq \frac{(\psi, \mathcal{A}\psi)}{(\psi, \psi)}$$

where ψ is any trial function.

For an extreme black hole, if the scalar field violates the BF bound in the near-horizon geometry, then can find a trial function ψ such that RHS above is negative hence full black hole is unstable.

Key step of argument was to impose symmetry condition on scalar field to eliminate first time derivatives. In 4d, this is just axisymmetry, for cohomogeneity-1 it is invariance under symmetry generated by **m**.

Gravitational perturbations

How do we extend this argument to gravitational perturbations? Need to impose symmetry conditions on perturbation to bring linearized Einstein eq to form

$$-\frac{\partial^2 \Phi_\alpha}{\partial t^2} = \mathcal{A}_\alpha{}^\beta \Phi_\beta$$

where Φ_α encodes perturbation. Then need to find scalar product to make $\mathcal{A}$ self-adjoint. Can this be done?

Kerr: yes, for axisymmetric perturbations (Friedman & Schutz, Chandrasekhar).

Cohomogeneity-1 or single spinning MP: probably, for **m**-invariant modes. Why else would unstable modes have purely imaginary ω?

Summary

- Many higher-dimensional black holes are unstable if they rotate sufficiently rapidly.
- There exist a gauge-invariant quantity for describing perturbations of algebraically special spacetimes, e.g., Myers-Perry black holes.
- This quantity satisfies a decoupled equation only in a Kundt background.
- This decoupled equation can be used to study gravitational perturbations of near-horizon geometries of extreme black holes: much easier than studying full black hole!
- It appears that, if certain symmetries are respected, then instability of a near-horizon geometry implies instability of full black hole.