Investment Timing, Incomplete markets and Corporate Control

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Introduction

• Standard real options approach to investment timing equates the opportunity to invest in a project with an American call option on the project, with the investment timing decision being analogous to the exercise of the option. (Dixit and Pindyck(1994))

• The standard real options models of investment timing assume:
(i) complete markets or the existence of assets such that the risk of the investment project is spanned
(ii) the owner of the option makes the exercise decision

- In reality: (i) payoffs are not spanned by other assets, so incomplete markets
(ii) well-diversified shareholders delegate investment decisions to risk averse managers, and managers are subject to corporate control

- We aim to model the impact of incompleteness and corporate control on a risk averse manager's investment timing

Corporate Control

• In our model, the manager being subject to corporate control or control challenges means that if he deviates from firm value maximization, he may be dismissed

• Dismissal can arise from internal or external mechanisms. Eg. board of directors or takeover market

Agency Conflicts

- Agency theory evaluates the conflict of interest between principals (shareholders) and agents (managers)
- Here the risk averse manager makes the investment decision on behalf of shareholders
- There are many other agency conflicts between shareholders and managers. Eg. perks and entrenchment, quiet life, short-termism, overconfidence
- Distinction between these and incomplete markets is that our manager still has concave utility and maximizes expected utility

Model Assumptions

• There is a single irreversible investment decision which can be made over an infinite horizon

• The project pays a one-off amount V_τ at time τ for a cost K. Exercise or investing at time τ yields $(V_\tau - K)^+$.

• Take zero interest rates

• V is not a traded asset

• Investment project payoff V follows geometric Brownian motion

$$dV/V = \nu dt + \eta dW = \eta(\xi dt + dW)$$

• The market asset P is tradeable and follows

$$dP/P = \mu dt + \sigma dB = \sigma(\lambda dt + dB)$$

where $dBdW = \rho dt$. Write $dW = \rho dB + \rho^\perp dZ$ for a Brownian motion Z independent of B.

• Standard real options models (Dixit and Pindyck (1994), McDonald and Siegel (1986)) assume that $\rho = 1$ and thus the investment project is perfectly spanned by the traded market asset.

• We consider $|\rho| < 1$ for:
(1) Shareholders: well-diversified and do not require compensation for idiosyncratic risk
(2) Risk averse manager: not well-diversified and does require compensation for idiosyncratic risk

Shareholder's Investment Timing

- Shareholders solve

$$\mathcal{S}(v) = \sup_{\tau} \mathbb{E}\left[\mathcal{D}^0_\tau (V_\tau - K)^+ | V_0 = v\right]$$

where $\mathcal{D}^0_t$ denotes the state price density which assigns zero market-price-of-risk to the independent Brownian motion Z. We have $B^0_t = B_t + \lambda t$ and $Z^0_t = Z_t$ are independent Brownian motions, giving P and V follow

$$\frac{dP}{P} = \sigma dB^0 \qquad \frac{dV}{V} = \eta[\rho dB^0 + \sqrt{1-\rho^2} dZ - (\lambda\rho - \xi)dt].$$

- Standard arguments give $\mathcal{S}(v)$ solves

$$0 = \frac{1}{2}\eta^2 v^2 \frac{\partial^2 \mathcal{S}}{\partial v^2} + \eta(\xi - \lambda\rho) v \frac{\partial \mathcal{S}}{\partial v}$$

subject to boundary, value-matching and smooth pasting conditions:

$$\mathcal{S}(0) = 0;\ \ \mathcal{S}(\bar{V}^s) = \bar{V}^s - K;\ \ \left.\frac{\partial \mathcal{S}}{\partial v}\right|_{\bar{V}^s} = I_{\{\bar{V}^s > K\}}$$

Proposition 1 *Denote by* $\beta = 1 - \frac{2(\xi - \lambda\rho)}{\eta}$ *the non-zero root of the quadratic*

$$\phi(\phi - 1)\eta^2/2 + \eta\phi(\xi - \lambda\rho) = 0.$$

Suppose $\beta > 1$. *(i) Investment/exercise takes place at the first passage time* $\tau_{\bar{v}} = \inf\{t : V_t \geq \bar{v}\}$ *for some constant* $\bar{v}$. *Then the shareholder's value of the investment option under any* $\bar{v}$ *is given by*

$$\mathcal{S}_{\bar{v}}(v) = \begin{cases} (\bar{v} - K)(\frac{v}{\bar{v}})^{\beta}; & v < \bar{v} \\ v - K; & v \geq \bar{v} \end{cases}$$

(ii) Optimizing over thresholds $\bar{v}$ *gives* $\bar{v}^* = \bar{V}^s = \frac{\beta}{\beta - 1}K$.
The shareholder's value of the investment option under the optimal threshold is

$$\mathcal{S}(v) \equiv \mathcal{S}_{\bar{V}^s}(v) = \begin{cases} (\bar{V}^s - K)(\frac{v}{\bar{V}^s})^{\beta}; & v < \bar{V}^s \\ v - K; & v \geq \bar{V}^s \end{cases}$$

The Manager's Investment Timing Problem Under Corporate Control

- The manager is risk averse and cannot hedge against the idiosyncratic risk factor Z.
- Let X denote manager's wealth from investing in the market asset P and risk-free bond. Dynamics are

$$dX = \theta_t dP/P$$

where θ_t is the cash amount invested in P at time t.

- The manager will choose an investment time τ and a hedge θ in the market to maximize his expected utility
- At the investment time, his position consists of two components: $(V_\tau - K)^+$ and accrued wealth X_τ.

Implicit is assumption that manager's private wealth is contingent on the value of the investment option/value of firm. Reasonable since manager's typically receive stock and option compensation

Control Challenges

Let the probability the manager faces a control challenge at time τ be given by

$$p(V_\tau) = 1 - e^{-\Phi(V_\tau)}$$

where

$$\Phi(v) = c[\mathcal{S}(v) - (v - K)]/K$$

We have

$$\Phi(\bar{V}^s) = 0 \quad \Phi(v) \geq 0$$

- c represents the strength or level of threat of the challenge

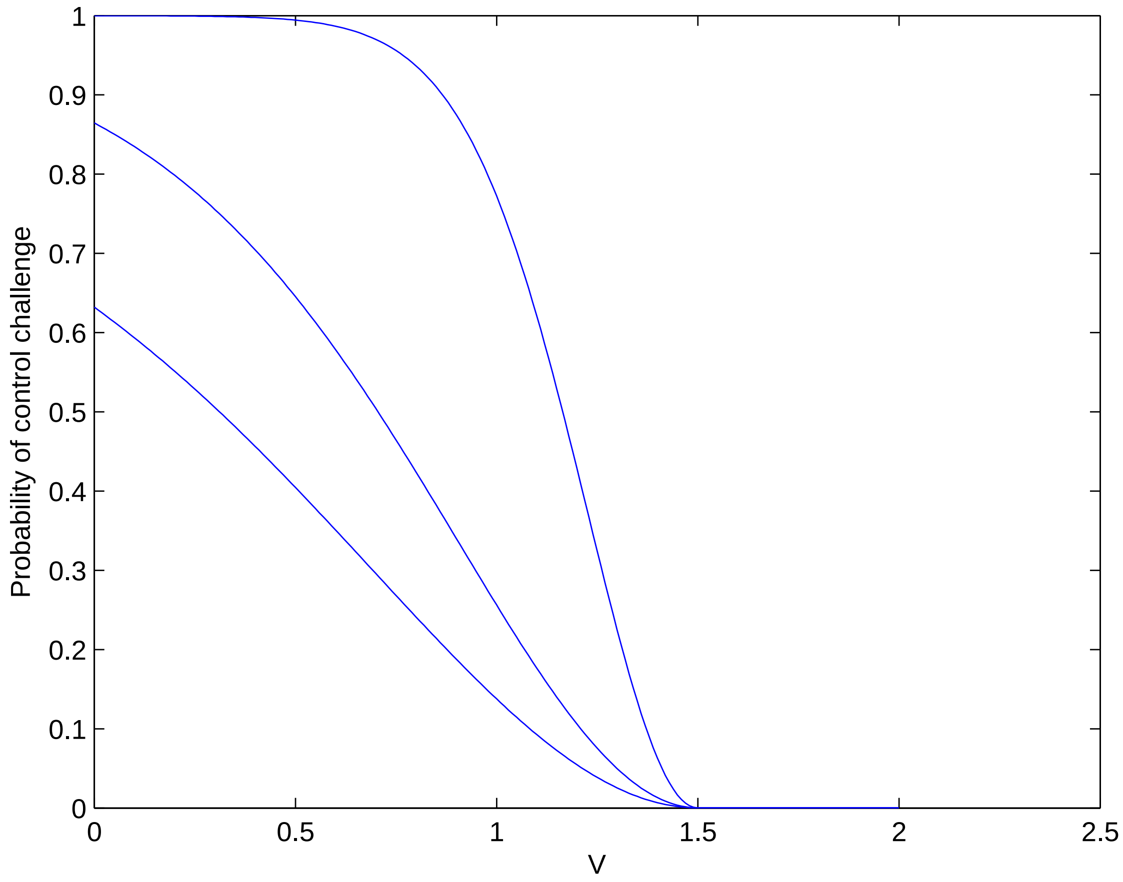

Figure 1: The probability of a control challenge $p(V_{\mathcal{T}})$. The three lines from lowest to highest take $c = 1, 2, 10$. Parameters are $K = 1$, $\beta = 3$ giving the shareholder's threshold $\bar{V}^S = 1.5$.

The Manager's Investment Timing Problem Under Corporate Control

- The manager solves

$$\sup_{t \le \tau} \sup_{(\theta_s)_{t \le s < \tau}} \mathbb{E}_t \left[e^{-\zeta\tau} U \left(X_\tau + (V_\tau - K)^+ I_\tau \right) \Big| X_t = x, V_t = v \right]$$

where ζ is a discount factor, $U(x) = -\frac{1}{\gamma} e^{-\gamma x}$ and

$$I_\tau = \begin{cases} 1; & \text{if the manager is not terminated} \\ 0; & \text{if the manager is terminated} \end{cases}$$

Horizon-Unbiased Utilities

• If we choose $\zeta = -\frac{1}{2}\lambda^2$ and define $\tilde{U}(t,x) = e^{\frac{1}{2}\lambda^2 t}U(x)$, then $\tilde{U}(t, X_t)$ is a supermartingale under any strategy and a martingale under the optimal strategy. In fact, for any (bounded) τ,

$$\tilde{U}(t,x) = \sup_{(\theta_u);t\leq u\leq\tau} \mathbb{E}[\tilde{U}(\tau, X_\tau)|X_t = x]$$

so that in the absence of the investment option, the manager has no preference over choice of horizon τ

• We call this property *horizon-unbiased*, see Henderson (2004) and Henderson and Hobson (SPA, 2007)

• Avoids *artificial* incentives to exercise/wait based on the set-up of the portfolio choice problem

• The manager faces a standard Merton portfolio problem after τ, so there is an opportunity cost associated with delaying investment The factor $e^{\frac{1}{2}\lambda^2\tau}$ represents this opportunity cost

• If the problem had a finite horizon T, $\tilde{U}(t,x)$ is just the implied/indirect utility function or value function:

$$\tilde{U}(t,x) = \sup_{(\theta_u);t\leq u\leq T} \mathbb{E}[U(X_T)|X_t = x]$$

Define

$$H(x,v) = \sup_{\tau \geq t} \sup_{\theta_u, t \leq u \leq \tau} \mathbb{E}\left[-\frac{1}{\gamma} e^{\frac{1}{2}\lambda^2(\tau - t)} e^{-\gamma(X_\tau + (V_\tau - K)^+ I_\tau)} | X_t = x, V_t = v \right]$$

By time-homogeneity, we deduce the manager invests at

$$\tau = \inf\{t : V_t \geq \bar{V}^c\}$$

where $\bar{V}^c$ is a constant threshold we need to find.

Proposition 2 *In the continuation region, H solves the following non-linear HJB equation*

$$0 = \frac{1}{2}\lambda^2 H + \xi\eta v H_v + \frac{1}{2}\eta^2 v^2 H_{vv} - \frac{1}{2}\frac{(\lambda H_x + \rho\eta v H_{xv})^2}{H_{xx}}$$

subject to boundary, value-matching and smooth-pasting conditions:

$$\begin{aligned}
H(x,0) &= -\frac{1}{\gamma}e^{-\gamma x} \\
H(x,\bar{V}^c) &= -\frac{1}{\gamma}e^{-\gamma x}[1 + e^{-\Phi(\bar{V}^c)}(e^{-\gamma(\bar{V}^c - K)^+} - 1)] \\
H_v(x,\bar{V}^c) &= \frac{1}{\gamma}e^{-\gamma x}e^{-\Phi((\bar{V}^c)}\Big\{\gamma e^{-\gamma(\bar{V}^c - K)^+} \\
&+ \Phi'(\bar{V}^c)(e^{-\gamma(\bar{V}^c - K)} - 1)\Big\}
\end{aligned}$$

Proposition 3 *Suppose $\beta > 1$. Define $\kappa(v) = 1 - e^{-\gamma(v-K)}$ and $D(v) = 1 - e^{-\Phi(v)}\kappa(v)$. The manager's constant investment threshold $\bar{V}^c$ solves*

$$\begin{aligned} & \frac{\beta}{\bar{V}^c}\left[D(\bar{V}^c)^{\rho^2} - D(\bar{V}^c)\right] \\ = \quad & (1-\rho^2)e^{-\Phi(\bar{V}^c)}\left(\gamma e^{-\gamma(\bar{V}^c - K)} - \kappa(\bar{V}^c)\Phi'(\bar{V}^c)\right) \end{aligned}$$

and the value function $H(x, v)$ is given by

$$H(x, v) = -\frac{1}{\gamma}e^{-\gamma x}\left[1 + \left\{D(\bar{V}^c)^{1-\rho^2} - 1\right\}\left(\frac{v}{\bar{V}^c}\right)^{\beta}\right]^{1/(1-\rho^2)}$$

The Manager's Timing Problem without Control

• If the manager is not subject to control but faces incomplete markets, then setting $c = 0$ gives

Proposition 4 (Henderson(2004)) *The manager facing incomplete markets invests at the first passage time of V to the constant threshold $\bar{V}^0$,*

$$\tau = \inf\{t : V_t \geq \bar{V}^0\}$$

where $\bar{V}^0$ solves

$$\bar{V}^0 - K = \frac{1}{\gamma(1-\rho^2)} \ln\left[1 + \frac{\gamma(1-\rho^2)\bar{V}^0}{\beta}\right]$$

Model Implications: Investment Timing - Incomplete Market

Proposition 5 *(i)* $\lim_{\gamma \to 0} \bar{V}^0 = \bar{V}^s$; $\lim_{\gamma \to \infty} \bar{V}^0 = K$
(ii) $\bar{V}^0$ *decreasing in* γ, *so for* $\gamma > 0$, *we have* $K < \bar{V}^0 < \bar{V}^s$

- Waiting to invest involves fluctuations in the possible payout. Investing locks-in the value ("lump-sum"). Risk averse manager dislikes uncertainty and hence prefers to act earlier than shareholders.
- If extremely risk averse, manager carries this behavior to its extreme and invests at the npv threshold.

Model Implications: Investment Timing - Shareholders vs Managers subject to Control

Proposition 6 *When correlation $\rho = 0$, the thresholds are ordered as*

$$\bar{V}^0 \leq \bar{V}^c \leq \bar{V}^s$$

Proposition 7 *For small γ and $-1 < \rho < 1$ the thresholds are ordered as*

$$\bar{V}^0 \leq \bar{V}^c \leq \bar{V}^s$$

- These imply the manager who also faces control challenges moderates his investment timing to be closer to the shareholder's choice.

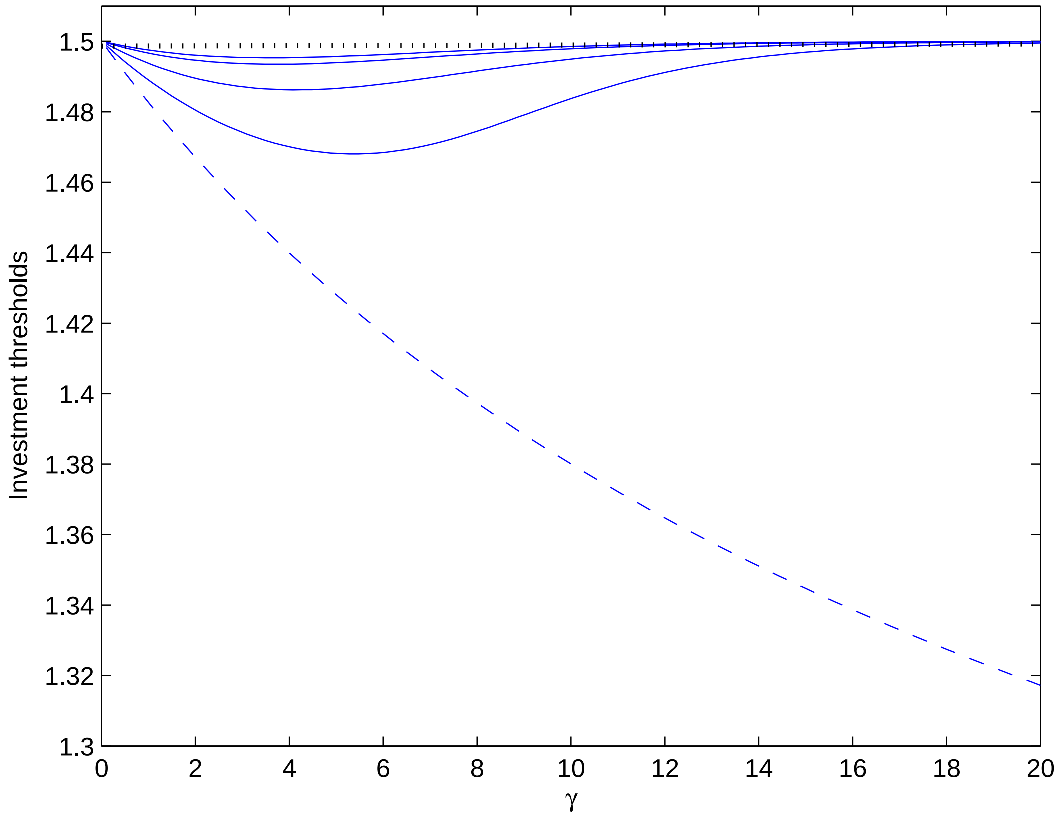

Figure 2: *Investment thresholds as a function of risk aversion. Parameters are* $\rho = 0.95$, $K = 1$, $\beta = 3$, *giving shareholder's threshold* $\bar{V}^s = 1.5$. *The dashed curve is* $\bar{V}^0$ *and the solid lines are* $\bar{V}^c$ *for values of* $c = 1, 3, 7, 10$ *(from lowest to highest).*

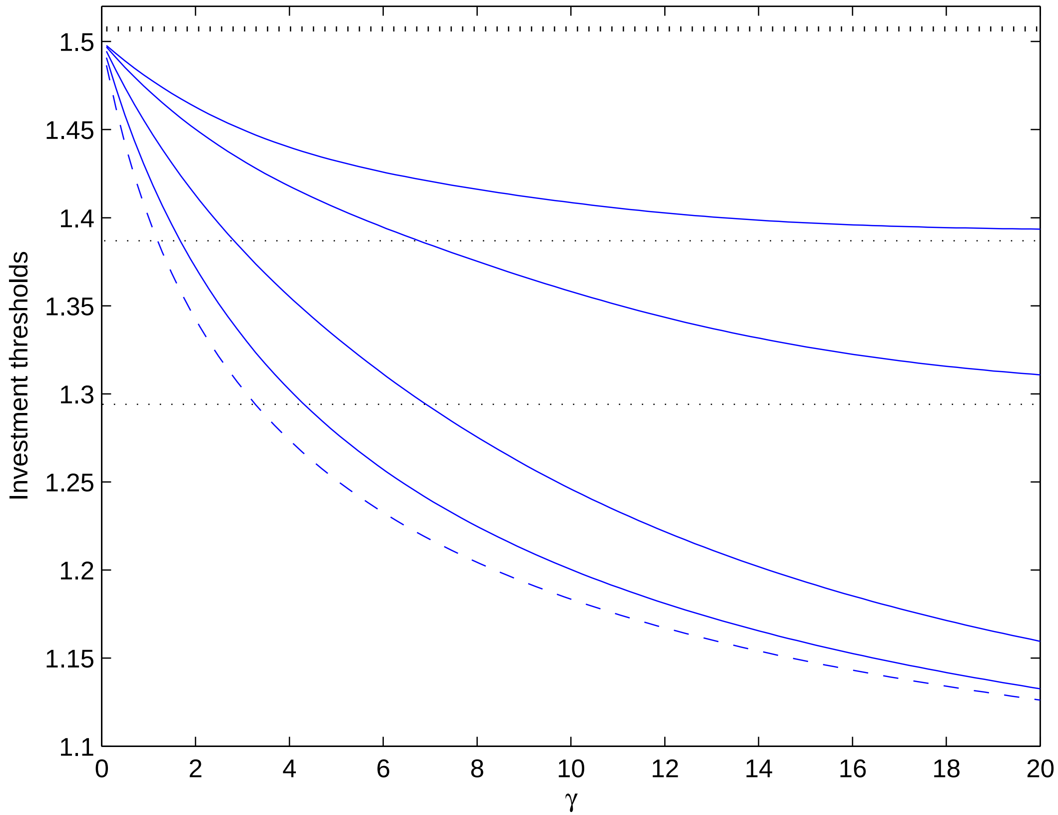

Figure 3: *Investment thresholds as a function of risk aversion. Parameters are* $\rho = 0.5$, $K = 1$, $\beta = 3$, *giving shareholder's threshold* $\bar{V}^s = 1.5$. *The dashed curve is* $\bar{V}^0$ *and the solid lines are* $\bar{V}^c$ *for values of* $c = 1, 3, 7, 10$ *(from lowest to highest). Also shown are the limiting values for* $c = 7, 10$.

Model Implications: Investment Timing

• The graphs show that $\bar{V}^c$ has a different shape depending on level of correlation ρ and strength of control challenge c

• Depends on relative importance of two influences -

(1) risk aversion towards idiosyncratic risk

(2) risk of control challenge by shareholders

• It can be shown that $|\rho| = \frac{1}{\sqrt{2}}$ is the switching point between these two situations

Characterization of critical c and limiting threshold

Proposition 8 *Suppose* $\rho = 0$. *Let* $c^* = \frac{\beta}{1-(\frac{\beta-1}{\beta})^{\beta-1}}$.
If $c \leq c^*$ *then as* $\gamma \to \infty$, $\bar{V}^c \to K$.
If $c > c^*$ *then as* $\gamma \to \infty$, $\bar{V}^c \to \tilde{v}(c) \in (K, \bar{V}^s)$ *where* $\tilde{v}(c)$ *solves*

$$\frac{\beta}{\tilde{v}(c)} - \frac{c}{K}\left[1 - \frac{\tilde{v}(c)}{\bar{V}^s}\right] = 0$$

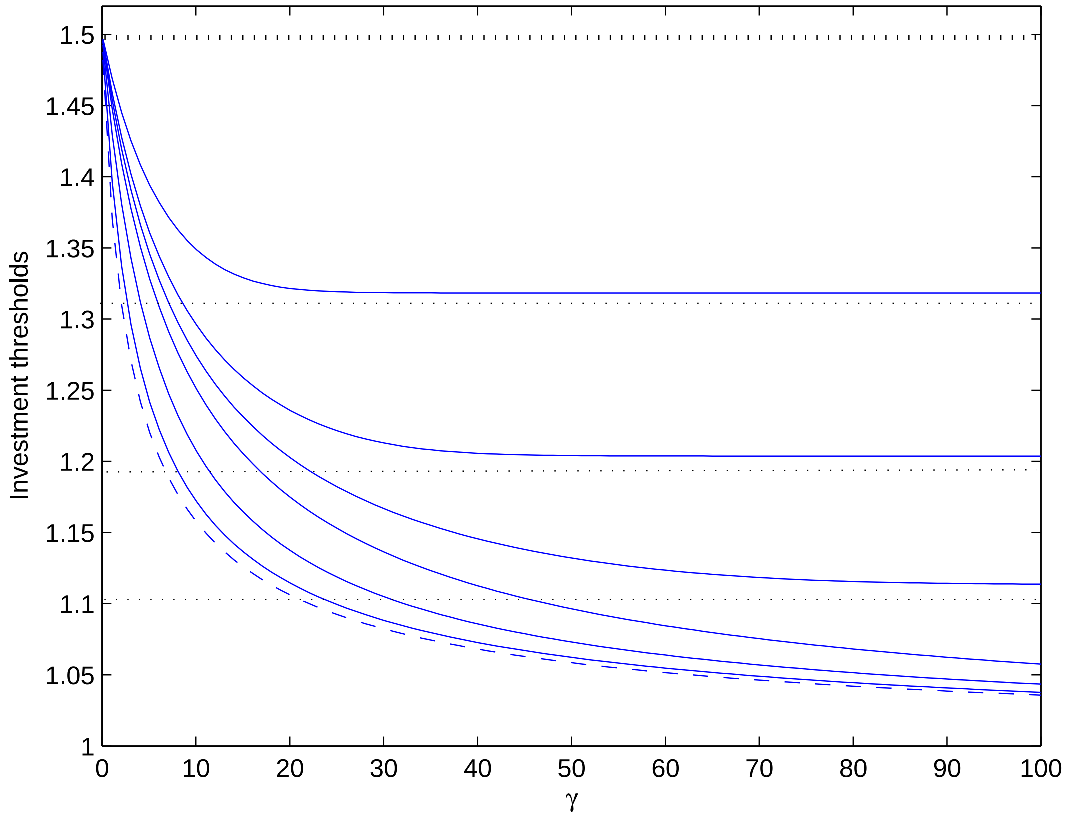

Figure 4: *Investment thresholds when $\rho = 0$, as a function of risk aversion. Parameters are $K = 1$, $\beta = 3$ so $\bar{V}^s = 1.5$. The dashed curve is $\bar{V}^0$ and the solid lines are $\bar{V}^c$ for $c = 1, 3, 5, 6, 7, 10$ (from lowest to highest). The critical value of c for these parameters is $c^* = 5.4$. Also shown (by horizontal dotted lines) are the limiting values for $c = 6, 7, 10$.*

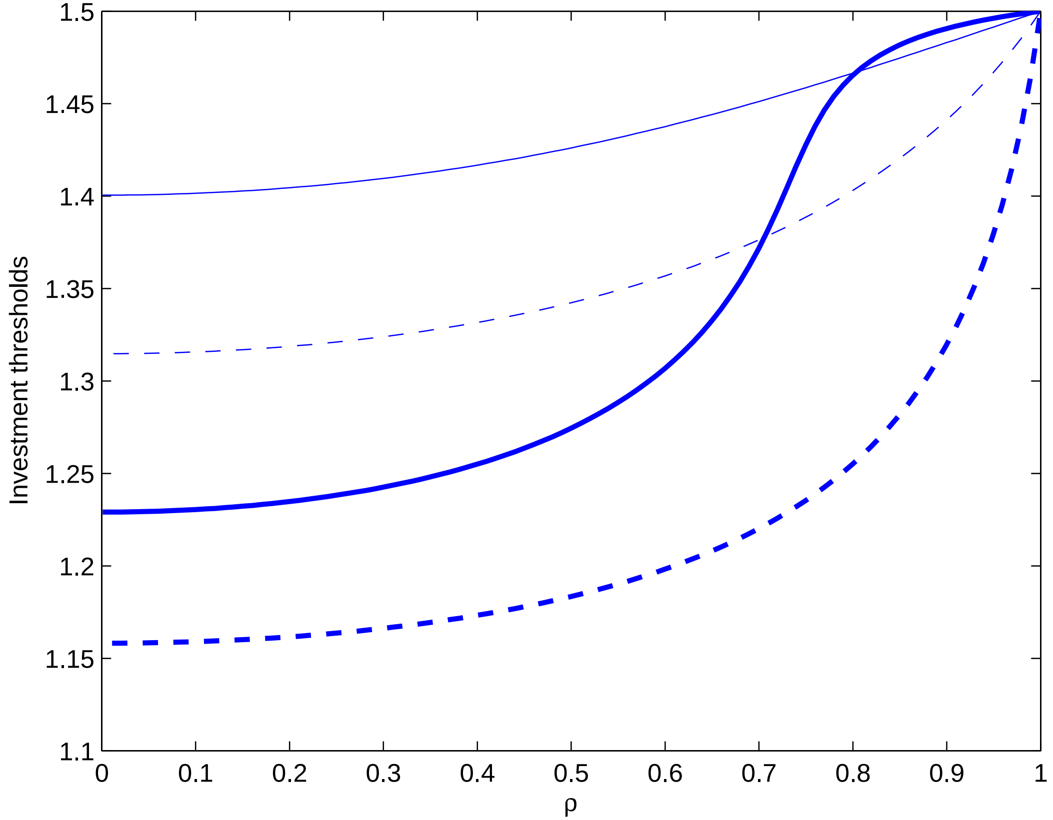

Figure 5: *Investment thresholds as a function of correlation. The dashed lines represent* $\bar{V}^0$*. The solid lines are thresholds* $\bar{V}^c$*, with the strength of control being* $c = 4$*. The pair of bold lines are for risk aversion level* $\gamma = 10$*. The non-bold lines take* $\gamma = 2$*. Other parameters are* $K = 1$*,* $\beta = 3$ *giving* $\bar{V}^s = 1.5$*.*

- Two regimes based on correlation
- We conclude corporate control is *ineffective* in the low correlation regime but *effective* in the high correlation regime

Comparison with Literature:
Hugonnier and Morellec (JEDC, 2007)

• HM compute a single threshold at which a manager subject to both incompleteness and control will invest
• They only find one regime : their manager's threshold is always monotonically decreasing with risk aversion, approaching the npv threshold, K
• $\rightarrow$ control is *always* relatively ineffective

Two reasons for different conclusions:
(i) their discounting assumptions
(ii) their manager's payoff does not depend on V

(i) When their discount factor $q = 0$, it corresponds to our discount factor $\zeta = -\frac{1}{2}\lambda^2$. HM assume $q > 0$. In the special case of *no control risk* their model gives

$$\sup_{\tau} \mathbb{E} e^{-(q/\Phi)\tau}$$

where $\Phi > 0$, and so $\tau^* = 0$. ie. their manager is *not indifferent to investment timing* in this case although he should be...Indifference would require $q = 0$.
In their model with control risk, if we took $q = 0$, it leads to the choice $\bar{V}^s$.
(ii) Our model does not degenerate in the case $q = 0$ because the payoff depends on V_τ

Model Predictions

- When there is a well functioning market for corporate control, should observe differences depending on degree of incompleteness

If "fairly complete":

* manager's invest early creating agency costs - but as risk aversion increases, manager's invest close to the shareholder's threshold and agency costs minimal

* Paddock, Siegel, Smith (1988) find empirical support for standard real options investment model on offshore oil reserve data

If "fairly incomplete":

* manager's invest much earlier than shareholder's prefer creating large agency costs - as risk aversion increases, manager invests closer to the npv threshold

* for large risk aversion, support for npv model

Model Predictions

• In situations without a well functioning market for corporate control or internal control mechanisms, we ought to see:
* managers deviating more from firm value maximization due to incomplete markets
* manager's investing much earlier than shareholder's prefer (large agency costs)
* differences in investment timing increasing with risk aversion

When? La Porta et al (1999) find nearly 50% of firms in a dataset from 27 countries are controlled by large shareholders many of whom are managers → difficult to discipline managers

Conclusions

• We consider the impact of incompleteness and corporate control on manager's investment timing decisions - importantly, we distinguish between the two effects

• We find two correlation regimes - control is effective in the high correlation regime and ineffective in the low correlation regime