From permutation groups to model theory

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Abstracts

Cameron, Peter

Oligomorphic groups and their orbit algebras

A permutation group G on Ω is oligomorphic if it has only finitely many orbits on the set of n-element subsets of its domain, for all natural numbers n. The importance of these groups in model theory, combinatorial enumeration, and other areas suggests that it is interesting to look at the sequences (f_n) counting orbits on n-sets. Examples suggest that such a sequence grows rapidly (with known exceptions) and smoothly. Some results on the first property are known (the best of them due to Dugald Macpherson), but less is known about the second. A tool for investigating these is the orbit algebra, a graded algebra for which the dimension of the nth homogeneous component is f_n. Understanding the structure of this algebra will throw light on the smoothness of growth of the sequence. I will discuss some results on this question, especially results of Maurice Pouzet (giving necessary and sufficient conditions for the orbit algebra to be an integral domain) and more recently Justine Falque and Nicolas Thiéry (showing that the algebra is Cohen–Macauley if the growth is no faster than polynomial).

Chatzidakis, Zoe

Notions of difference closures of difference fields

It is well known that a differential field K of characteristic 0 is contained in a differential field which is differentially closed and has the property that it K-embeds in every differentially closed field containing K. Such a field is called a differential closure of K, and it is unique up to K -isomorphism. The difference closure is what model-theorists call a "prime model". One can ask the same question about difference fields: do they have a difference closure, and is it unique? The immediate answer to both these questions is no, for trivial reasons: in most cases, there are continuum many ways of extending an automorphism of a field to its algebraic closure. Therefore a natural requirement is to impose that the field K be algebraically closed. Similarly, if the subfield of K fixed by the automorphism is not pseudo-finite, then there are continuum many ways of extending it to a pseudo-finite field, so one needs to add the hypothesis that the fixed subfield of K is pseudo-finite. In this talk I will show by an example that even these two conditions do not suffice. There are two (and more) natural strengthenings of the notion of difference closure, and we show that in characteristic 0, these notions do admit unique prime models over any algebraically closed difference field K, provided the subfield of K fixed by the automorphism is large enough. In model-theoretic terms, this corresponds to the existence and uniqueness of a-prime, or $\aleph_1$-prime, models. In characteristic p, no such result can hold.

Cherlin, Gregory

The relational complexity of a finite primitive structure

Kantor, Liebeck, and Macpherson classified the (sufficiently large) finite primitive structures with a bounded number of 5-types. This answered a model theoretic question by Lachlan, who had a classification for structures homogeneous for a finite relational language. One can consider Lachlan's condition as the combination of two quite distinct conditions; the number of variables needed for the relations is the relational complexity p, and in addition one bounds the number of p-types. I've wondered what one can say if one takes the opposite tack from Kantor/Liebeck/Macpherson. Namely, bound the relational complexity without restricting the number of types. In particular, I made a conjecture as to the explicit classification in the binary case (relational complexity 2, and primitive). This was the subject of my last talk at the ICMS, which can be found at: http://sites.math.rutgers.edu/~cherlin/Talks/2014Edinburgh.pdf. One can be more ambitious and ask for estimates on the relational complexity in terms of more familiar invariants of permutation groups. Treating the bounded case is essentially a matter of coming up with weak but non-trivial lower bounds in full generality. In particular, for 'natural' actions we expect sharp estimates: at this point we leave group theory for combinatorics mixed with linear algebra. Recent developments include major progress in the binary case by a battery of permutation group theorists (to date: Dalla Volta, Gill, Hunt, Liebeck, Spiga) with interesting prospects for a full proof, and some new explicit computations for the relational complexity of one of the more natural actions of the symmetric or alternating groups (with Josh Wiscons). At a minimum, I will give some background and say something about the two developments mentioned. There are also a few dozen related questions that suggest themselves and some relevant computational data, some of which may make an appearance.

Chernikov, Artem

Hypergraph regularity and higher dimensional analogs of VC-dimension

Recently a strong form of Szemeredi's regularity lemma for graphs of finite VC-dimension was established by Lovasz and Szegedy. In particular, such graphs can be finitely approximated in a "random-free" way: graphs with bounded VC-dimension admit a regular partition in which the densities between the pieces of the partition are either close to 0 or close to 1. Shelah introduced a generalization of VC-dimension for higher arity relations, studied in model theory under the name of k-dependence (with $k = 1$ corresponding to the finite VC-dimension case). Using measure theoretic methods, we establish a regularity lemma for k-dependent hypergraphs for all k, showing that they admit a similar kind of random-free approximation by relations of arity at most k. Joint work with Henry Towsner.

Droste, Manfred

Normal subgroups of unital automorphism groups

We investigate the group of all those order automorphisms of the reals which are bounded by some power of translation by 1. This is a unital lattice-ordered group which has independent interest due to its recent connections to mv-algebras. It is well-known that the group of all order-automorphisms of the reals has just three proper non-trivial normal subgroups. In contrast, we show that our group has 2^c many normal subgroups, where c denotes the cardinality of the continuum. We give a complete description of this normal subgroup lattice. Joint work with Charles Holland.

Elwes, Richard

The unreasonable effectiveness of logic

There are two reasons to study maths: you are interested in it, or you are interested in something else (physics, computer science, data, etc.) where mathematical tools are useful. Similarly, there are two reasons to study mathematical logic: the sheer fascination of the foundations of our subject, or the usefulness of logical devices for illuminating other mathematical topics. The analogy is not perfect, of course. But in both cases, this apparent divide between the pure and applied branches of the subject is far less clear-cut than it may first seem. Modern physics is full of mathematics which was initially developed when no application seemed conceivable (non-Euclidean geometry is just one example). Likewise, discoveries from the very purest parts of logic have a habit of making themselves extremely useful within mainstream mathematics. This public lecture (which will not presuppose expertise in model theory) will tell the stories of some of these ideas, with a focus on first order logic: a logical system which may initially seem too flimsy to cope with sophisticated mathematical structures, but which over the last few decades has repeatedly yielded stunning insights into core mathematical subjects including algebra, number theory, and graph theory.

Evans, David
Actions of automorphism groups of omega-categorical structures on compact spaces
If G is a topological group, a G-flow X is a non-empty, compact, Hausdorff space on which G acts continuously; it is minimal if all G-orbits are dense. By a theorem of Ellis, there is a (unique) minimal G-flow $M(G)$ which is universal: there is a continuous G-map to every other G-flow. Here, we will be interested in the case where $G = \mathrm{Aut}(K)$ for some structure K, usually omega-categorical. Work of Kechris, Pestov and Todorcevic and others gives conditions on K under which structural Ramsey Theory (due to Nesetril–Rodl and others) can be used to compute $M(G)$. In the first part of the talk I will give a description of the above theory and when it applies (the 'tame case'). In the second part, I will describe joint work with J. Hubicka and J. Nesetril which shows that the omega-categorical structures constructed in the late 1980's by Hrushovski as counterexamples to Lachlan's conjecture and not tame and minimal flows of their automorphism groups have rather different properties to those in the tame case.

Gray, Robert
Cohomology theory of monoids with a single defining relation
The word problem for one-relator groups was solved by Magnus in 1932. Later in 1950 Lyndon published a highly influential paper in which he computed the cohomology of an arbitrary one-relator group. This work of Lyndon implies that all one-relator groups satisfy the homological finiteness property FP infinity, and that torsion free one-relator groups have cohomological dimension at most two. The question of whether one-relator monoids have decidable word problem is an important longstanding open problem. While the question is open in general, it has been solved in a number of special cases e.g. in work of Adjan (1966) and Adjan and Oganessian (1987). In this talk I will present some recent joint work with Benjamin Steinberg (City College of New York) on the cohomology theory of one-relator monoids. I will explain some of the ideas behind our proof that all one-relator monoids are of type FP infinity, and that they have cohomological dimension at most two if their subgroups are torsion-free and infinite cohomological dimension otherwise. These results give a natural monoid analogue of Lyndon's identity theorem for one-relator groups. The fact that one-relator monoids all have type FP infinity is also of interest because of its connection with the question of whether all one-relator monoids admit presentations by finite complete rewriting systems. This in turn relates to the word problem for one-relator monoids. I will explain these connections in my talk, and how topological methods developed in our earlier work were applied to establish these results for one-relator monoids.

Hrushovski, Ehud
On measures and higher amalgamation
Higher amalgamation problems were introduced into model theory by Shelah, for the category of models and elementary maps of a theory T. If T is countable and has few models in an uncountable power, he proved the existence and uniqueness of prime models over independent diagrams of models, indexed by a simplicial complex. For $N = 2$ he also proved amalgamation in the finer category of algebraically closed substructures and partial elementary maps, and further the uniqueness of this 2-amalgamation; 3-amalgamation follows from the latter. This requires the introduction of imaginary sorts. For $N \geq 4$, higher imaginaries are needed. In a modernized, continuous-logic version, the profinite automorphism groups defining the algebraic closure become compact groups; this allowed Kim–Pillay to prove 3-amalgamation for simple theories.

On the other hand, there has been a lot of interest in measures on definable sets, arising from the example of asymptotic classes as well as NIP work. The measure zero ideal is analogous to the forking ideal used in stability, and in fact to some extent is a special case, in that the formula describing the measure of the intersection of two sets is a stable formula. This connects stable 3-amalgamation with a swath of results across different fields, such as the Furstenberg–Zimmer compact factor / weakly mixing dichotomy in ergodic theory and Szemeredi's graph regularity lemma. The latter can be seen as the measure-theoretic version of Shelah's finite equivalence relation theorem. Combinatorics knows higher-dimensional forms of Szemeredi's regularity lemma. This corresponds to an $n-1$-dimensional notion of stationarity that is not familiar to model theorists. A result from Elad Levi's thesis proves it for pseudo-finite fields (thus extending a theorem of Tao's for $N = 2$.) I will present this and discuss the general situation.

Time permitting, I will also describe a recent result with Krupinsky and Pillay, connecting 3-amalgamation and 1st-order amenability (invariant measures).

Liebeck, Martin

Bases for linear groups

Let V be a finite-dimensional vector space, and G a subgroup of $GL(V)$. We call a set B of vectors in V a base for G if its pointwise stabilizer in G is trivial. I shall review some recent results on bounding bases sizes, and connections with the theory of primitive permutation groups and also with algebraic groups.

Macintyre, Angus

Recent work on adele rings and related structures

I consider rings Adeles(K) as K ranges over number fields. K is never interpretable in Adeles(K), but this leaves open how much about K is interpretable. In fact the theory of Adeles(K) conceals much deep information about K, for example its zeta function. I give a general analysis of axioms for Adeles(K), and relate it to old work of Iwasawa and Perlis on the extent to which isomorphism of Adeles(K) and Adeles(L) gives isomorphism of K and L. I show that elementary equivalence and isomorphism coincide for adele rings over number fields. The results and proof methods are related to several ongoing broader collaborations with Paola D'Aquino, Margarita Otero and Jamshid Derakhshan.

Macpherson, Dugald

Jordan permutation groups, treelike structures, and their limits

A transitive permutation group G on a set X is a Jordan group if the pointwise stabiliser in G of some proper non-empty subset of X acts transitively on the complement (with an extra non-degeneracy condition). Combined work of Adeleke and Neumann and also Adeleke and myself yielded in the mid 1990s a structural description of infinite primitive Jordan groups. Key examples arise as groups of automorphisms of treelike structures and also of their rather mysterious 'limits' constructed by Adeleke. I will give an overview of the subject, but focus on certain omega-categorical constructions (so yielding oligomorphic Jordan groups) – joint work with Bhattarcharjee on limits of betweenness relations, and the PhD thesis of Almazaydeh on limits of D-relations.

Malliaris, Maryanthe

Complexity in simple theories

The talk will be about detecting complexity among simple theories.

Mitchell, James

Semigroup topologies on the full transformation monoid

A topology on a group G is called a *group topology* if the functions $G \times G \to G$ defined by $(x;\ y) \mapsto xy$ and $G \to G$ defined by $x \mapsto x^{-1}$ are continuous. For example, if $\mathbb{N}$ is discrete and $\mathbb{N}^{\mathbb{N}}$ has the product topology, then the subspace topology on the symmetric group $\mathrm{Sym}(\mathbb{N})$ is a group topology on $\mathrm{Sym}(\mathbb{N})$; called the topology of *pointwise convergence*. In 1967, Gaughan proved that any Hausdorff group topology on $\mathrm{Sym}(\mathbb{N})$ contains the topology of pointwise convergence. Kechris and Rosendal extended Gaughan's theorem to show that the topology of pointwise convergence is the unique non-trivial separable group topology on $\mathrm{Sym}(\mathbb{N})$.

Motivated by the results of Gaughan, Kechris, and Rosendal, in this talk I will discuss recent work with Z. Mesyan and Y. Péresse, where we consider analogous problems in the context of semigroups.

Müller, Isabel
Stationary independence and the extension property of partial automorphisms
The automorphism group G of a countable first order structure is, equipped with the topology of pointwise convergence, a Polish group. Especially in the case that the structure arises as the limit of a class of finite structures, there is an intimate relation between the combinatorial properties of that class and the topological properties of G. The combinatorial properties we want to discuss in this talk are the extension property of finite partial automorphisms (EPPA) and the amalgamation property with automorphisms (APA) on the one side, which force the existence of ample generics in G, and the existence of a stationary independence relation (SIR) on the other side, which influence the normal subgroup structure of G. We want to give an overview of these notions and connections and pose questions about the relations between EPPA, APA and SIR.

Nesetril, Jaroslav
Modelling limits
The existence of limits of FO-converging sequences of finite structures is an interesting example of interplay of various model-theoretic and combinatorial techniques. We survey this development and particularly the existence of modelling (totaly Borel) limits. This is a joint work with Patrice Ossona de Mendez.

Pillay, Anand
p-adic Nash groups
This is joint work with Ningyuan Yao. We conjecture that any p-adic Nash group G (equivalently group definable in the field Q_p is algebraic (up to finite index and finite kernel), and prove it in the commutative case.

Praeger, Cheryl
Permutation groups with certain finiteness conditions
I will discuss the extent to which various finiteness conditions influence the structure of infinite permutation groups, such as infinite primitive groups. Some of this work is joint with Dugald Macpherson.

Simon, Pierre
Homogeneous structures with few types
We are interested in homogeneous structures which have polynomially many types over finite sets, or equivalently–by a theorem of Dugald Macpherson–at most exponentially many finite substructures up to isomorphism. Model theoretically, this corresponds to the NIP condition. All known examples of such structures are built out of linear orders, circular orders or trees. By making an additional assumption that the structure does not contain trees, we are able to give some form of classification and in particular completely list the rank one case (in a sense to be made precise). As an application, we determine the primitive homogeneous multi-orders along with their reducts.

Siniora, Daoud
A note on automorphisms of meet-trees
A meet-tree is a partial order such that the set of vertices below any vertex is linearly ordered, and for every pair of vertices there is a greatest element smaller than or equal to each of them. I'll report on a work in progress with Itay Kaplan and Tomasz Rzepecki on the behaviour of partial automorphisms of finite meet-tees.

Smith, Simon

The box product and Dugald's early work on distance transitive graphs

One of the most important products in permutation group theory is the wreath product, acting in its product action. The reason for this is that it preserves the fundamental property of primitivity. Primitive permutation groups are indecomposable in some sense, and for finite groups they are the basic building blocks from which all permutation groups are comprised. A new product for permutation groups, called the box product, was recently discovered. It is fundamentally different to the wreath product in product action. Nevertheless, it preserves primitivity under surprisingly similar conditions. The box product was used to easily solve a well-known open problem from topological group theory, and it has an important role to play in the structure theory of infinite permutation groups. The inspiration for the box product came, in part, from Dugald's 1982 work on infinite distance transitive graphs of finite valency. In this talk I will give an introduction to the box product, and I will describe how Dugald's work on distance transitive graphs led to its discovery.

Solecki, Slawomir

Projective Fraïssé *limits and compact spaces*

Fraïssé theory is a method in classical Model Theory of producing canonical limits of certain families of finite structures. For example, the random graph is the Fraïssé limit of the family of all finite graphs. It turns out that this method can be dualized, with the dualization producing projective Fraïssé limits, and applied to the study of compact metrizable spaces. I will describe recent results, due to several people, on connections between projective Fraïssé limits and the structure of some canonical compact spaces and their homeomorphism groups (the pseudoarc, the Menger curve, the Lelek fan, simplexes with the goal of developing a projective Fraïssé homology theory).

Steinhorn, Charlie

An application of o-minimality in mathematical economics

This talk deals with preference and utility theory in the context of o-minimal expansions $\mathcal{R}$ of the ordered field of real numbers. We give a description of all preferences that can be defined in such a structure $\mathcal{R}$ and when such preferences admit a utility function.

Tent, Katrin

Ampleness in strongly minimal structures

The notion of ampleness captures essential properties of projective spaces over fields. It is natural to ask whether any sufficiently ample strongly minimal set arises from an algebraically closed field. In this talk I will explain the question and present recent results on ample strongly minimal structures.

Thomas, Simon

Characters of inductive limits of finite alternating groups

If $G \ncong \mathrm{Alt}(\mathbb{N})$ is an inductive limit of finite alternating groups, then the indecomposable characters of G are precisely the associated characters of the ergodic invariant random subgroups of G

Truss, John

Ehrenfeucht–Fraïssé games on linear orders and coloured linear orders

Ehrenfeucht–Fraïssé games provide an effective method of comparing structures. In the n-move game on (relational) structures A and B, players I and II play alternately, starting with I. On each of his moves I chooses a point of A or B (the choice doesn't need to be from the same structure each time), and II responds by choosing a point in the other structure 'as similar as possible' to I's move. After n moves, points $a_1, \dots, a_n$ of A and $b_1, \dots, b_n$ of B have been chosen, and II wins if the map taking a_i to b_i for each i is an isomorphism of induced substructures. If II has a winning strategy, then A and B are said to be n-equivalent. This is the same as saying that A and B satisfy the same sentences of quantifier-depth at most n. Thus n-equivalence may be regarded as a refinement of elementary equivalence. We study n-equivalence for some classes of linear orders, mainly finite, ordinals, and some scattered ones, also in some cases with colours allowed. This is joint work with Feresiano Mwesigye

Vargas-Garcia, Edith

Reconstructing the topology on monoids and polymorphism clones of some relational structures

Transformation Monoids and *Clones* on a set A carry a natural topology, induced by the topology of point-wise convergence. The endomorphism monoids $\mathrm{End}(\mathcal{A})$ and polymorphism clones $\mathrm{Pol}\mathcal{A}$ of a relational structure $\mathcal{A}$ are viewed abstractly as topological monoids and topological clones, respectively. Their topology is the natural one. In this talk we show how to reconstruct the topology on the monoid of endomorphisms and the polymorphism clone of some relational structures, among others are: Reducts of the rationals $\mathbb{Q}$. Presenting a small joint work with Mike Behrisch & John K. Truss.