An Example of Multiobjective Optimization in Mechanics: Scattering on Rough Surfaces and Reverse Magnus effect

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Outline

1. Rough surfaces: Two-dimensional case
2. An application of the theorem: reverse Magnus effect
3. Numerical simulation

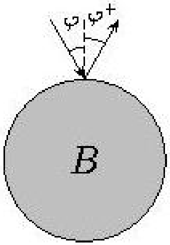
Main tools of research: *billiards, Monge-Kantorovich optimal mass transfer, numerical simulation*

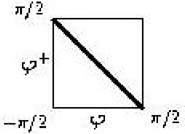
Applications: *Aerodynamics of motion in rarefied media*

Key words: *free molecular flow, shape optimization, Robins-Magnus effect*

1. Rough surfaces: Two-dimensional case

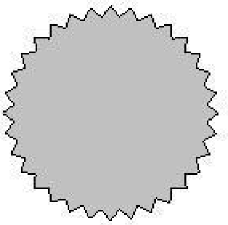
Let $B \subset \mathbb{R}^2$ be a unit circle. Consider the billiard in $\mathbb{R}^2 \setminus B$.
The law of scattering from B is: $\varphi^+ = -\varphi$.



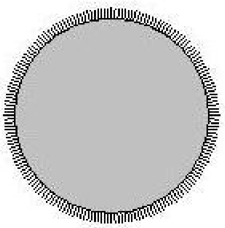


1. Rough surfaces: Two-dimensional case

Now consider a "set" $\mathcal{B}$ obtained by "roughening" the boundary of B.



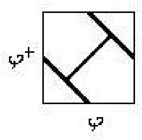
$\mathcal{B}_1$



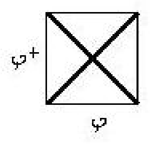
$\mathcal{B}_2$

1. Rough surfaces: Two-dimensional case

The law of reflection from $\mathcal{B}_1$:



The law of reflection from $\mathcal{B}_2$:



A **rough circle** is represented by an expanding family of sets approaching B such that there exists the limiting law of reflection.

Let $\mathcal{M}$ be the set of measures ν on $\square := [-\pi/2,\, \pi/2] \times [-\pi/2,\, \pi/2]$ such that

(a) the marginals are $\pi_1^{\#}\nu = \pi_2^{\#}\nu = \lambda$, *where* $d\lambda(\varphi) = \cos\varphi\, d\varphi$;
(b) ν *is symmetric with respect to the diagonal* $\varphi = \varphi^+$.

Main Theorem

The set of all reflection laws is a weakly dense subset of $\mathcal{M}$.

An application of the theorem: reverse Magnus effect

Determine the force of resistance of a rapidly rotating rough unit circle.

In the real world, there are two opposite effects:

- *Magnus effect (in dense media);*
- *Reverse Magnus effect (in very rare media).*

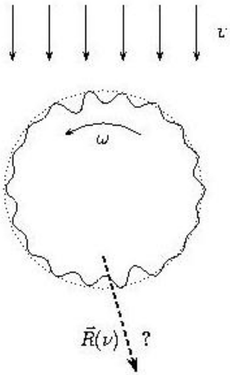
The reverse Magnus effect is due to the two factors:

(a) Lateral friction of the particles [Weidman and Herczynski (2004); Borg, Söderholm and Essén (2003)].

(b) Multiple collisions.

An application of the theorem: reverse Magnus effect

We concentrate on the case (b). The resistance $\vec{R}(\nu)$ depends on the roughness ν. What is the set of all possible values $\vec{R}(\nu)$?



An application of the theorem: reverse Magnus effect

Denote $\lambda = \omega / v$.

Denote resistance and angular momentum of force acting on the circle by $\vec{R}_\lambda(\nu)$ and $R_\lambda^M(B)$, respectively.

One has

$$\vec{R}_\lambda(B) = \iint_\square \vec{c}_\lambda(x,y)\, d\nu(x,y), \qquad R_\lambda^M(B) = \iint_\square c_\lambda^M(x,y)\, d\nu(x,y).$$

An application of the theorem: reverse Magnus effect

(a) $\lambda = 1$:

$$\vec{c}_1(x,y) = 3\sin^2 x \begin{pmatrix} \cos(2x-y)+\cos x \\ \sin(2x-y)+\sin x \end{pmatrix} \cdot \chi(x>0),$$

$$c_1^M(x,y) = 3\sin^2 x\,(\sin x + \sin y)\cdot \chi(x>0).$$

An application of the theorem: reverse Magnus effect

(b) $0 < \lambda < 1$:

$$\vec{c}_\lambda(x,y) = \frac{3(\lambda \sin x + \sin \eta(x))^3}{4 \sin \eta(x)} \cos \frac{x-y}{2} \begin{pmatrix} \cos\left(\eta(x) + \frac{x-y}{2}\right) \\ \sin\left(\eta(x) + \frac{x-y}{2}\right) \end{pmatrix}$$

$$c_\lambda^M(x,y) = \frac{3\left(\lambda \sin x + \sin \eta(x)\right)^3}{8 \sin \eta(x)} \left(\sin x + \sin y\right),$$

where $\eta(x) = \arccos(\lambda \cos x)$.

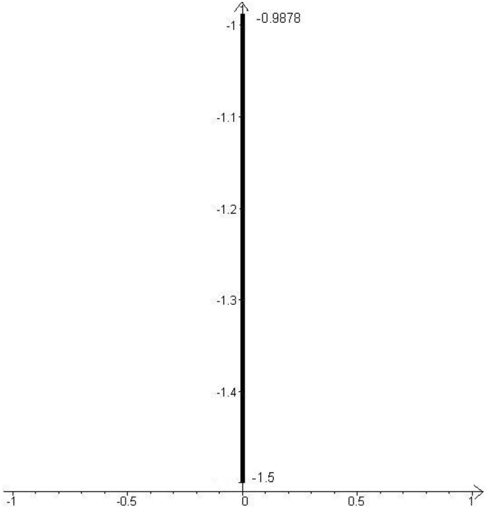
(c) $\lambda > 1$:

$$\vec{c}_\lambda(x,y) = \frac{3\cos\frac{x-y}{2}}{2\sin\eta(x)} \left\{ (\lambda^3 \sin^3 x + 3\lambda \sin x \sin^2 \eta(x)) \cos\eta(x) \begin{bmatrix} \cos\frac{x-y}{2} \\ \sin\frac{x-y}{2} \end{bmatrix} + \right.$$

$$\left. + (3\lambda^2 \sin^2 x \sin\eta(x) + \sin^3 \eta(x)) \sin\eta(x) \begin{bmatrix} -\sin\frac{x-y}{2} \\ \cos\frac{x-y}{2} \end{bmatrix} \right\} \cdot \chi(x > \arccos(1/\lambda)),$$

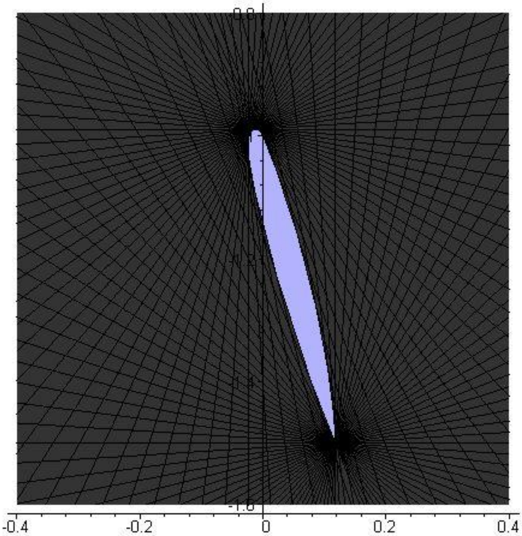
$$c_\lambda^M(x,y) = 3\,(\sin x + \sin y)\, \frac{\lambda^3 \sin^3 x + 3\lambda \sin x \sin^2 \eta(x)}{4\sin\eta(x)} \cdot \chi(x > \arccos(1/\lambda)).$$

Solution in the case $\lambda = 0$

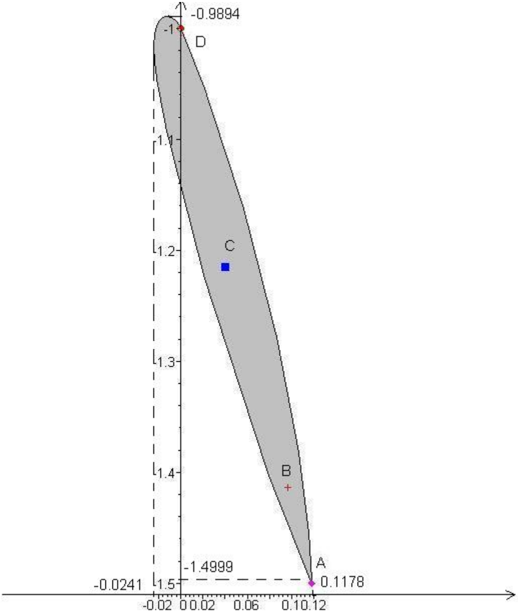


Solution in the case $\lambda = 0.1$

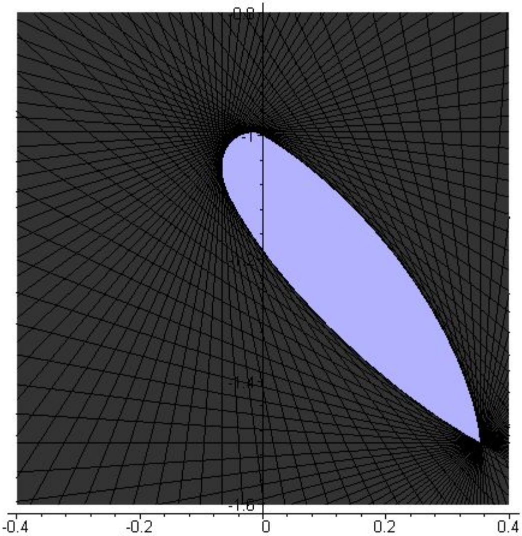
Numerical simulation realized using Xpress Mosel Version 1.6.2 and Xpress Optimizer Version 17.01.00



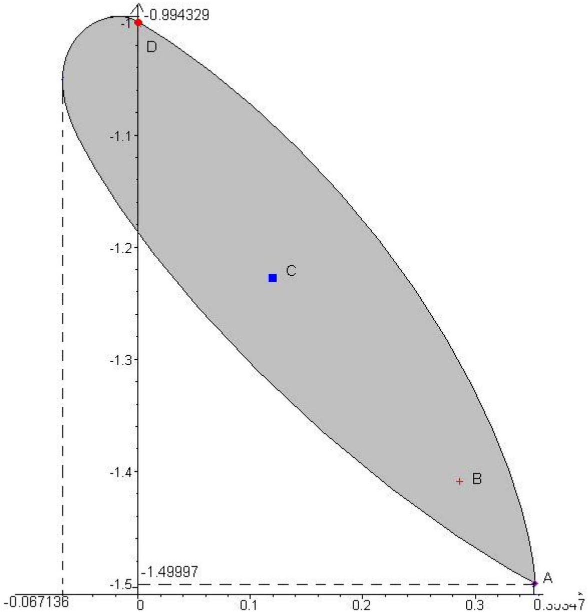
Solution in the case $\lambda = 0.1$



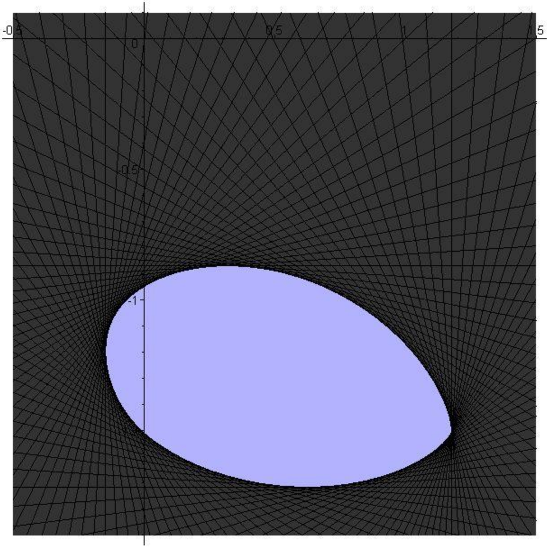
Solution in the case $\lambda = 0.3$



Solution in the case $\lambda = 0.3$



Solution in the case $\lambda = 1$



Solution in the case $\lambda = 1$