Analyticity of Riccati equation solutions

Ruth Curtain, University of Groningen, NL.
(Collaboration with Leiba Rodman, College of William and Mary, USA).

Over the fast few years there has been increasing interest in the problem of controlling arrays of identical dynamical systems, for example, very long platoons of vehicles. Some authors use the model

$$\begin{aligned} \dot{x}_r(t) &= \sum_{l=-\infty}^{\infty} A_l x_{r-l}(t) + \sum_{l=-\infty}^{\infty} B_l u_{r-l}(t), \quad -\infty \leq r \leq \infty \\ y_r(t) &= \sum_{l=-\infty}^{\infty} C_l x_{r-l}(t), \quad -\infty \leq r \leq \infty \end{aligned}$$

where A_l, B_l, C_l are matrices of suitable dimensions and $x_l(t)$ is the state, $u_l(t)$ is the control, $y_l(t)$ is the output at position l and time t. For practical implementation it is necessary is to find a localized control law $u_r(t) = \sum_{l=-N}^{N} F_l x_{r-l}(t)$, where N is as small as possible. The linear quadratic design is based on solutions $P_0(z)$ with the property $P_0(z)^* = P_0(z)$ to the Riccati equation for $z \in \mathbb{T}$, the unit circle.

$$A(z)^* P(z) + P(z)A(z) - P(z)B(z)B(z)^* P(Z) + C(z)^* C(z) = 0.$$

Under mild conditions there exists a unique solution $P \in \mathbf{L}_\infty(\mathbb{T}; \mathbb{C}^{n \times n})$. However, this is insufficient to obtain a localized control law. For this we are led to examining the following nonstandard Riccati equation for z in an annulus around $\mathbb{T}$

$$A^{\sim}(z)P(z) + P(z)A(z) - P(z)B(z)B^{\sim}(z)P(z) + C^{\sim}(z)C(z) = 0,$$

where $A(z)$, $B(z)$, $C(z)$ are analytic in the annulus, and $A^{\sim}(z) = A(1/z)^T$. We seek seek solutions such that $P(z) = P^{\sim}(z) := P(1/z)^T$ and it is analytic for all z in an annulus $\mathbb{T}$.

We obtain a complete solution to this problem, including an estimate of the width of the annulus. Thus we arrive at a design for a localized control law.