BERGMAN AND BESOV SPACES ON TUBE DOMAINS OVER SYMMETRIC CONES

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We will essentially consider the forward light cone in $\mathbb{R}^n$, with $n \geq 3$:

$$\Omega := \left\{ y \in \mathbb{R}^n \mid y_1^2 - y_2^2 - \ldots - y_n^2 > 0, \;\; y_1 > 0 \right\}$$

and the associated tube domain $\mathcal{D} := \mathbb{C}^n + i\Omega$. While Hardy spaces have been the object of many counter-examples, the theory of Bergman spaces presents a variety of new phenomena compared to the classical setting of the upper half-plane. The exact range of p for which the Bergman projection is bounded in L^p is not known: there is a gap between known necessary conditions on one side, sufficient ones on the other side.

We will see how the boundedness of the Bergman projection is linked to other properties, such as duality for Bergman spaces and Hardy inequality for the wave operator

$$\Box := \partial_{z_1}^2 - \partial_{z_2}^2 - \ldots - \partial_{z_n}^2$$

in place of the derivative in the classical setting.

The dual space of the Bergman space A^1 is called the Bloch space, and defined as the space of holomorphic functions f such that $\Delta^m(y)|\Box^m f(x+iy)|$ is bounded for m large enough. We will give new necessary conditions on m, but critical values are not known.

These results have been obtained jointly with D. Békollé, G. Garrigós, F. Ricci and B. Sehba [1].

A last problem will be mentioned. In the Lie ball, which is the bounded realization of this tube domain, one does not know the best value q for which all functions in the Bloch class are in L^q. New partial results have been obtained jointly with G. Garrigós and C. Nana.

References

[1] Békollé, D.; Bonami, A.; Garrigós, G.; Ricci, F.; Sehba, B., Analytic Besov spaces and Hardy-type inequalities in tube domains over symmetric cones, to appear in Crelle's Journ.