

Not

understanding
the physics of
noncommuta-
tive waves and
quantum
symmetries

Shahn Majid

Introduction

Key features
of NCG

Bicrossproduct
model as limit
of QG

Quantum
change of
frame

Not understanding the physics of noncommutative waves and quantum symmetries

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ANY Quantum Gravity \Rightarrow noncommutative geometry \Rightarrow Classical Riemannian Geometry

- NCG provides a framework for first next-to-classical Planck scale effects
- Physics is independent of coordinates or 'choice of generators'
- **Minimal assumptions:** unital $*$ -algebra A over \mathbb{C} , as 'coordinate algebra'
- **Symmetries are formulated as quantum groups** (H, Δ, ϵ, S) where $\Delta : H \rightarrow H \otimes H$, $\epsilon : H \rightarrow \mathbb{C}$ and $S : H \rightarrow H$. Think **like** $A \sim C(M)$, $H \sim U(g)$
- **Differential structure formulated as:**
 - ① Ω^1 an $A - A$ -bimodule $a(\omega b) = (a\omega)b$
 - ② $d : A \rightarrow \Omega^1$, $d(ab) = da \cdot b + a \cdot db$
 - ③ $\Omega^1 = \text{span}_{\mathbb{C}}\{adb\}$
 - ④ (optional connectedness) $\ker d = \mathbb{C} \cdot 1$
 - ⑤ extend to $\Omega = \bigoplus_n \Omega^n$, $d^2 = 0$ as DGA (sometimes canonical)

Key generic features of NCG 'model-indept predictions'

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- 1 NC spacetime generally \leftrightarrow quantum isometry group for consistency. E.g. κ -Poincaré [Lukierski et al early 1992] was taken acting on classical $\mathbb{R}^{1,3}$ until [SM+H. Ruegg, PLB 1994] showed it acts on bicrossproduct model spacetime

$$[x_i, t] = \frac{\imath}{m_p} x_i$$

with canonical isometry qg $H = U(\mathfrak{so}_{1,3}) \bowtie C(\mathcal{H}^{1,3})$ where Lorentz undeformed.

- 2 Noncommutative differential structures generally have **nonassociativity anomaly** \Rightarrow smallest Ω^1 has extra dimensions. Typically, $\theta \in \Omega^1$ with $df = m_p[\theta, f]$.
- 3 NC Fourier transform takes NC spacetime \leftrightarrow curved/**nonAbelian** momentum space. Eg. $\mathcal{H}^{1,3} \Rightarrow$ now to make sense of Feynman rules?
- 4 Accumulation points due to non-linear actions '**special**'.

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NC variational principle needs noncommutative moduli space of functions. We know the quantum action (eg state sum) but hard to expand in particle interpretation.

Two main classes of nc spacetimes that work:

- ① q -deformations eg q -Minkowski space [SM 1990 'braided hermitian matrices', Carow-Watamura, Wess et Al. 1989].
Done but very hard to compute formulae.
- ② Bicrossproduct model with isometry group deformed [SM. 1987 (3D euclidean)] as above.
- ③ (Twisting examples \mathbb{R}_θ^4 etc. and DR model too many conceptual problems – eg no Fourier transform and too 'special')

Warning From the poincare algebra alone **cannot** make a prediction. eg p_0, p_i , say with central element

$$\vec{p}^2 e^{\frac{p_0}{m_p}} - 2m_p^2 \left(\cosh\left(\frac{p_0}{m_p}\right) - 1 \right)$$

this is not a prediction. You could have used some other generators s.t. $\vec{p}^2 - \tilde{p}_0^2!$

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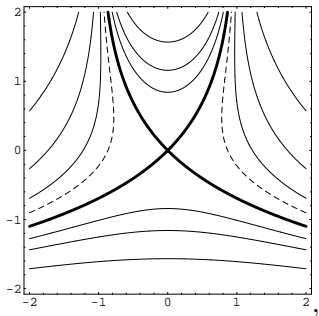
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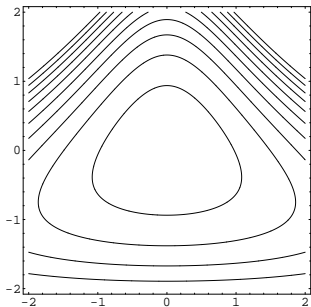
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The 'invariant' point of view is p_0, p_i just coordinates. Physical way: particles are irreps of $U(so_{1,3}) \rtimes C(\mathcal{H}^{1,3}) \leftrightarrow$ orbits of $SO_{1,3}$ in $\mathcal{H}^{1,3}$.. deformed hyperbolae.



Lorentzian case $|\vec{p}| < 1$



Euclidean case [SM'87]

This is the origin of the 'doubly special' feature. Many other examples. Still supposes that p_0 is energy etc – need to relate to experiment. [Amelino-Camelia and S.M. 2000] justified mod disp. relns by 'test' on NC plane waves

$$e^{k \cdot x} e^{i\omega t}$$

Bicrossproduct model as $q \rightarrow 1$ limit of QG

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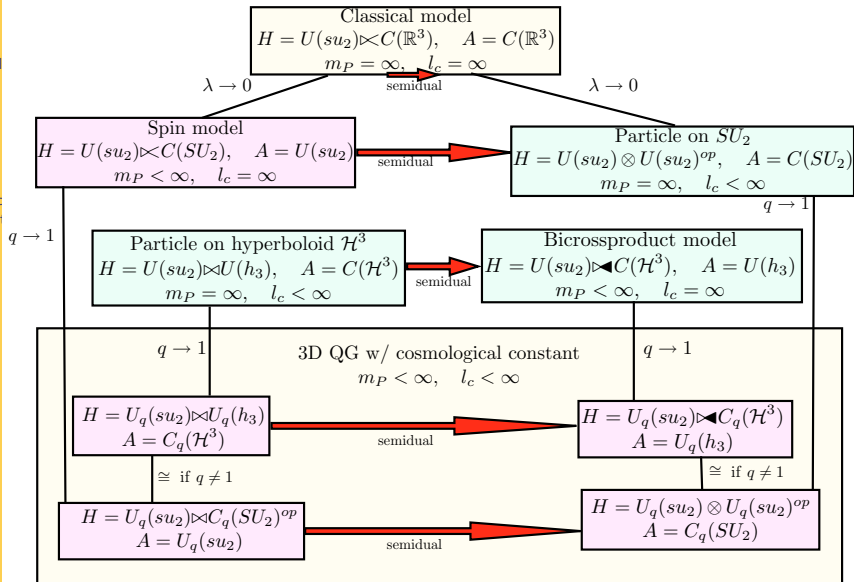
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3D Quantum Gravity with point sources *is* completely understood. Particles controlled by:

$$q = e^{-\frac{1}{m_p l_c}}, \quad H = U_q(su_2) \blacktriangleleft C_q(\mathcal{H}^3) \cong U_q(su_2) \otimes U_q(su_2)$$

$$A = U_q(h_3) \cong C_q(SU_2).$$

The theory on the right is simply a q -deformation of a particle on $C_q(SU_2)$. Irreps are defined by two integers which we can think of as spin and mass (\leftrightarrow eigenvalue of the q -Laplacian Δ_q).

$$C_q(SU_2) \text{ has a matrix of generators } \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \mu = q - q^{-1},$$

$$ba = qab, \quad ca = qac, \quad db = qbd, \quad dc = qcd, \quad bc = cb$$

$$[d, a] = \mu bc, \quad ad - q^{-1}bc = 1$$

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$$\Omega^1(C_q(SU_2)) \text{ basis } \{e_a, e_b, e_c, e_d\}, \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} qa & q^{-1}b \\ qc & q^{-1}d \end{pmatrix} e_a.$$

$$[e_b, \begin{pmatrix} a & b \\ c & d \end{pmatrix}] = \mu \begin{pmatrix} 0 & a \\ 0 & c \end{pmatrix} e_d, \quad [e_c, \begin{pmatrix} a & b \\ c & d \end{pmatrix}] = \mu \begin{pmatrix} b & 0 \\ d & 0 \end{pmatrix} e_a$$

$$[e_d, \begin{pmatrix} a \\ c \end{pmatrix}]_{q^{-1}} = q^{-1} \mu \begin{pmatrix} b \\ d \end{pmatrix} e_b, \quad [e_d, \begin{pmatrix} b \\ d \end{pmatrix}]_q = q^{-1} \mu \begin{pmatrix} a \\ c \end{pmatrix} e_c + q^{-1} \mu^2 \begin{pmatrix} b \\ d \end{pmatrix}$$

where $[x, y]_q \equiv xy - qyx$. Then $d = \frac{q}{\mu}[\theta,]$, $\theta = e_a + e_d$ and define **partial derivatives** by $df = \partial_+ f e_b + \partial_- f e_c + \partial_t f e_t + \partial_0 f \theta$

\Rightarrow

$$\frac{(2)_q}{q\mu} \partial^0 f = q^{-m} (k)_q (n)_q \frac{f}{bc} + \binom{k+n+m}{2}_q \binom{k+n+m}{2}_q + 1 \Big)_q f$$

on $f = c^k b^n d^m$, where $(n)_q \equiv (q^n - q^{-n}) / (q - q^{-1})$. The right hand side is Δ_q .

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Now $C_q(SU_2) \cong U_q(h_3)$ with generators $x_{\pm} = x \pm iy, t$:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} q^{m_p t} & \frac{m_p}{2} \mu x_- \\ \frac{m_p}{2} \mu x_+ & q^{-m_p t} \left(1 + \frac{\mu m_p^2}{4} x_+ x_- \right) \end{pmatrix}$$

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$$q \rightarrow 1 \Rightarrow [x_i, t] = \frac{1}{m_p} x_i, \quad [x_i, x_j] = 0$$

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Bicrossproduct or 'kappa/DSR' model with 4D calculus:

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$$[dx_i, x_j] = \delta_{ij} \frac{2}{m_p} \theta', \quad [dt, x_i] = 0,$$

Quantum change of frame

$$[\theta', x_i] = 0, \quad [\theta', t] = \frac{1}{m_p} \theta', \quad [dt, t] = \frac{1}{m_p} (2\theta' - dt).$$

$$df = \partial^i f dx_i + \partial^t f dt + \partial^0 f \theta' \Rightarrow$$

$$\partial^i (f(x)g(t)) = \frac{\partial f}{\partial x} g, \quad \partial^t (fg) = f m_p (g(t) - g(t - \frac{1}{m_p}))$$

$$\partial^0 (fg) = \frac{1}{m_p} \frac{\partial^2 f}{\partial x^2} g(t + \frac{1}{m_p}) + f m_p \left(g(t + \frac{1}{m_p}) + g(t - \frac{1}{m_p}) - 2g(t) \right)$$

derivative in the extra dimension is the wave operator.

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On nc waves $e^{k \cdot x} e^{i\omega t}$ we have

$$k^2 e^{\frac{\omega}{m_P}} + 2m_P^2 (\cosh(\frac{\omega}{m_P}) - 1)$$

agrees with Δ_q on with $a^m = q^{-tm m_P} = e^{\frac{-tm}{l_c}}$, $m, l_c \rightarrow \infty$ with $\omega = \frac{m}{l_c}$ fixed.

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$$\Rightarrow \text{speed of light} = \left| \frac{\partial \omega}{\partial \vec{k}} \right| = e^{\lambda \omega}$$

In 4D could be tested by LISA and will be by GLAST. γ -ray bursts spread 0.1 – 100 MeV travel cosmological distances L with extra spread arrival times

$$\Delta_T \sim \lambda \Delta_{k^0} \frac{L}{c} \sim 10^{-44} \text{s} \times 100 \text{MeV} \times 10^{10} \text{y} \sim \mathbf{1ms}$$

How would we see other noncommutative waves eg in 'fuzzy space' model $[x_i, x_j] = 2i l_P \epsilon_{ijk} x_k$? We construct coherent states

$$|j, \phi, \psi\rangle = \sum_{k=0}^{2j} 2^{-j} \sqrt{\binom{2j}{k}} (1 + \cos \phi)^{\frac{2j-k}{2}} (1 - \cos \phi)^{\frac{k}{2}} e^{ik\psi} |j, j-k\rangle$$

minimum uncertainty with $|\langle x \rangle| = j l_P$.

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If $H \sim U(\text{poinc})$ then $H^* \sim C(\text{Poinc})$. In bicrossproduct case:

$$H^* = C(SO_{1,3}) \blacktriangleleft U(h_{1,3})$$

with generators Λ_ν^μ, a_μ ,

$$[a_\rho, \Lambda_\nu^\mu] = \beta_{a_\rho}(\Lambda_\nu^\mu), \quad \Delta a_\mu = a_\mu \otimes 1 + a_\nu \otimes \Lambda_\mu^\nu.$$

This 'acts' on bicrossproduct spacetime $x_\mu \in U(h_{1,3})$ by coaction

$$\Delta_R^{\text{poinc}}(x_\mu) = 1 \otimes a_\mu + x_\nu \otimes \Lambda_\mu^\nu.$$

What we would see is:

$$\langle x_\mu \rangle \rightarrow \langle x_\nu \rangle \langle \Lambda_\mu^\nu \rangle + \langle a_\mu \rangle + O(l_P)$$

$$\langle a_\mu a_\nu \rangle = \langle a_\mu \rangle \langle a_\nu \rangle + O(l_P), \quad \langle \Lambda_\mu^\nu a_\rho \rangle = \langle \Lambda_\mu^\nu \rangle \langle a_\rho \rangle + O(l_P)$$

$$\langle \Lambda_\mu^\nu \Lambda_\beta^\alpha \rangle = \langle \Lambda_\mu^\nu \rangle \langle \Lambda_\beta^\alpha \rangle$$

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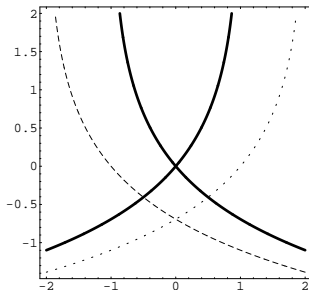
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In bicrossproduct case in 1+1 this was analysed [SM'05]

$$[a_0, \begin{pmatrix} \cosh(\theta) \\ \sinh(\theta) \end{pmatrix}] = \imath l_P s \begin{pmatrix} s \\ c \end{pmatrix}, \quad [a_1, \begin{pmatrix} c \\ s \end{pmatrix}] = \imath l_P (c - 1) \begin{pmatrix} s \\ c \end{pmatrix}$$

$$\Delta(a_0, a_1) = 1 \otimes (a_0, a_1) + (a_0, a_1) \otimes \begin{pmatrix} \cosh(\theta) & \sinh(\theta) \\ \sinh(\theta) & \cosh(\theta) \end{pmatrix}$$



as (p_0, p_i) approaches critical lines (dashed) from above, its action on angle θ blows up (infinite uncertainty). **No simultaneous values of translation and rotations.** Can't move to Planckian center of mass frame without quantum theory of H^* .