Noether analysis for theories with canonincal spacetime noncommutativity

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· G. Amelino-Camelia, F. Briscese, G. G., A. Marcianò, P. Martinetti, F. Mercati , Phys. Rev. D 78, 025005 (2008)

· G. Amelino-Camelia, G. G., A. Marcianò, P. Martinetti, F. Mercati, D. Pranzetti, R.A. Tacchi, Prog. Theor. Phys. Supplement No.171 (2007) pp. 65-78

Canonical noncommutative spacetime

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}$$

with $\theta_{\mu\nu}$ observer-independent and coordinate-independent

- Much recent work on twisted Hopf symmetries of observer-independent-θ-NCST, but no Noether analysis leading to the identification of conserved charges
- Some authors had even suggested that it was not possible to obtain conserved charges

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- I will show that conserved charges can be obtained following steps which are similar to the ones that resulted successful in κ-Minkowski noncommutative spacetime
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Necessity of the definition of a new algebra of symmetries

Canonical NC spacetime is NOT invariant under the action of the classical Poincaré algebra (Lorentz sector):

$$M_{\mu\nu}[\hat{x}_\alpha, \hat{x}_\beta] \begin{cases} = \theta_{\beta[\mu}\eta_{\nu]\alpha} - \theta_{\alpha[\mu}\eta_{\nu]\beta} \\ \neq M_{\mu\nu}(i\theta_{\alpha\beta}) = 0 \end{cases}$$

$\Rightarrow$ It is known a deformation of this algebra which leaves invariant the commutators $[\hat{x}_\mu, \hat{x}_\nu]$.

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Weyl maps

Symmetric map (full symmetrization of the coordinates):

$$\Omega_w\left(x_1 x_2^2\right) = \frac{1}{3}\left(\hat{x}_1\hat{x}_2^2 + \hat{x}_2\hat{x}_1\hat{x}_2 + \hat{x}_2^2\hat{x}_1\right)$$

$$\Omega_w\left(e^{ikx}\right) := e^{ik\hat{x}}$$

$$\Downarrow$$

$$f(\hat{x}) \equiv \Omega_w(f_w(x)) = \int d^4k\, \tilde{f}_w(k)\Omega_w(e^{ikx}) \equiv \int d^4k\, \tilde{f}_w(k) e^{ik\hat{x}}$$

Also other ordering prescriptions are allowed:

→Example: Map "x_1 *on the right*":

$$\Omega_1\left(e^{ikx}\right) := e^{i(k^2\hat{x}_2 + k^3\hat{x}_3 + k^0\hat{x}_0)} e^{i\,k^1\hat{x}_1} \qquad \Omega_w\left(x_1 x_2^2\right) = \hat{x}_2^2\hat{x}_1$$

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Definition of the deformed algebra

$$\hat{G}f(\hat{x}) := \Omega_w\left(Gf(x)\right)$$

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- The commutators between generators remain unchanged:

$$[G_i, G_j] = \sum_k \alpha_k G_k \Rightarrow \left[\hat{G}_i, \hat{G}_j\right] f(\hat{x}) = \Omega_w\left([G_i, G_j] f(x)\right) = \Omega_w\left(\sum_k \alpha_k G_k f(x)\right) = \sum_k \alpha_k \hat{G}_k f(\hat{x})$$

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$$\hat{P}_\alpha \hat{x}_\mu = \Omega_w(P_\alpha x_\mu) = \Omega_w(i\eta_{\mu\alpha}) = i\eta_{\mu\alpha}, \qquad \hat{M}_{\alpha\beta}\hat{x}_\mu = \Omega_w\left(i x_{[\alpha}\eta_{\beta]\mu}\right) = i\hat{x}_{[\alpha}\eta_{\beta]\mu}$$

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$$\hat{M}_{\mu\nu}(fg) = \left(\hat{M}_{\mu\nu} f\right) g + f\left(\hat{M}_{\mu\nu} g\right) + \frac{1}{2}\theta_{\beta[\mu}\left[\left(\hat{P}_{\nu]} f\right)\left(\hat{P}^\beta g\right) - \left(\hat{P}^\beta f\right)\left(\hat{P}_{\nu]} g\right)\right]$$

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Twisting

With the language of Hopf algebras the deformed action on products of functions is formalized specifying the *coproduct*:

$$\begin{aligned}\Delta\hat{P}_\mu &= \hat{P}_\mu \otimes 1 + 1 \otimes \hat{P}_\mu \\ \Delta\hat{M}_{\mu\nu} &= \hat{M}_{\mu\nu} \otimes 1 + 1 \otimes \hat{M}_{\mu\nu} + \frac{1}{2}\theta_{\beta[\mu}\left[\hat{P}_{\nu]} \otimes \hat{P}^\beta - \hat{P}^\beta \otimes \hat{P}_{\nu]}\right]\end{aligned}$$

$\rightarrow$ The algebra of symmetries of canonical NCST results to be a deformation of the classical Poincarè Lie algebra by the *twist element*:

$$\mathcal{F} = e^{\frac{i}{2}\theta^{\mu\nu}P_\mu \otimes P_\nu}$$

$$\left[\Rightarrow \Delta(\hat{G}) = \mathcal{F}\Delta(G)\mathcal{F}^{-1}\right]$$

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Differential of the field

Classical Noether analysis: differential variation of the field associated to a certain transformation described by the generators G_A in terms of the generators themselves and infinitesimal transformation parameters:

$$d\phi(x) = i\epsilon^A G_A \, \phi(x)$$

We use a “minimal” extension of this tool, defining the differential of the field as above, but with noncommutative transformation parameters

Requirements on the differential

- $df(\hat{x}) = i\epsilon^A \hat{G}_A f(\hat{x})$
- Leibniz rule must be satisfied by the differential:

$$d\left(f(\hat{x})g(\hat{x})\right) = \left(df(\hat{x})\right)g(\hat{x}) + f(\hat{x})\left(dg(\hat{x})\right)$$

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Characterization of the transformations (I)

- Translations: $d_P f(\hat{x}) = i\epsilon^\mu \hat{P}_\mu f(\hat{x})$

  Leibniz: $[\epsilon^\mu, f(\hat{x})] = 0$

- Lorentz sector: $d_L f(\hat{x}) = i\omega^{\mu\nu} \hat{M}_{\mu\nu} f(\hat{x})$

  Leibniz:

$$[\omega^{\mu\nu}, f(\hat{x})]\hat{M}_{\mu\nu} g(\hat{x}) = \frac{1}{2}\omega^{\mu\nu}\left[\theta_{\sigma[\mu}\eta_{\nu]\rho} - \theta_{\rho[\mu}\eta_{\nu]\sigma}\right](\hat{P}^\rho f(\hat{x}))\hat{P}^\sigma g(\hat{x})$$

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Characterization of the transformations (II)

We define a differential containing all the generators of the algebra:

$$df(\hat{x}) = i\left[\epsilon^{\mu}\hat{P}_{\mu} + \omega^{\mu\nu}\hat{M}_{\mu\nu}\right] f(\hat{x})$$

Commutation rules of the parameters (enforcing the LR):

$$[\hat{x}^{\rho}, \omega^{\mu\nu}] = 0$$

$$[\hat{x}^{\rho}, \epsilon^{\alpha}] = -\frac{i}{2}\omega^{\mu\nu}(\theta_{[\mu}{}^{\alpha}\delta_{\nu]}{}^{\rho} + \theta^{\rho}{}_{[\mu}\delta_{\nu]}{}^{\alpha})$$

- Pure translation are allowed: it is always possible to set $\omega^{\mu\nu} = 0$
  $\Rightarrow [\hat{x}^{\rho}, \epsilon^{\alpha}] = 0 \rightarrow$ pure translations are classical.
- Rotations/Boosts: $\omega^{\mu\nu} \neq 0 \Rightarrow [\hat{x}^{\rho}, \epsilon^{\alpha}] \neq 0$
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Scalar field theory

The first Casimir of the deformed algebra is classical in form:

$$\hat{\Box} = \hat{P}_\mu \hat{P}^\mu$$

$\Rightarrow$ Klein-Gordon-like scalar field theory:

$$\hat{\Box}\phi(\hat{x}) = 0$$

Action:

$$S = \frac{1}{2} \int d^4\hat{x} \, \left(\hat{P}^\alpha \phi(\hat{x}) \right) \hat{P}_\alpha \phi(\hat{x}).$$

Action variation and conserved charges

$$\delta S = \frac{1}{2}\int d^4\hat{x}[\hat{P}_\alpha(\delta\phi)\hat{P}^\alpha\phi + (\hat{P}_\alpha\phi)\hat{P}^\alpha(\delta\phi) + d((\hat{P}_\alpha\phi)\hat{P}^\alpha\phi)]$$

$$[\ \delta\phi = -d\phi\ ]$$

$$\Rightarrow \quad \delta S = -\frac{i}{2}\int d^4\hat{x}\ \epsilon^\mu \hat{P}_\alpha T^\alpha_\mu + \omega^{\mu\nu}\hat{P}_\alpha J^\alpha_{\mu\nu}$$

where:

$$T^\alpha_\mu = \phi(\hat{x})\hat{P}^\alpha\hat{P}_\mu\phi(\hat{x}) - \left(\hat{P}^\alpha\phi(\hat{x})\right)\hat{P}_\mu\phi(\hat{x})$$

$$J^\alpha_{\mu\nu} = \phi(\hat{x})\hat{P}^\alpha\hat{M}_{\mu\nu}\phi(\hat{x}) - \left(\hat{P}^\alpha\phi(\hat{x})\right)\hat{M}_{\mu\nu}\phi(\hat{x}) + -\frac{1}{2}\Upsilon^{\sigma\rho}_{\mu\nu}\left[\left(\hat{P}_\rho\phi(\hat{x})\right)\hat{P}^\alpha\hat{P}_\sigma\phi(\hat{x}) - \left(\hat{P}_\rho\hat{P}^\alpha\phi(\hat{x})\right)\hat{P}_\sigma\phi(\hat{x})\right]$$

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$$Q_\mu \equiv \int d^3\hat{x}\; T^0_\mu \;, \qquad K_{\mu\nu} \equiv \int d^3\hat{x}\; J^0_{\mu\nu}$$

On-shell field: $\phi(\hat{x}) = \int d^4k\; \tilde{\phi}_{(w)}(k)\delta(k^2)e^{ik^\beta \hat{x}_\beta}$

$$Q_\mu = \frac{1}{2}\int \frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_\mu \left\{\tilde{\phi}_{(w)}(-\vec{q},|\vec{q}|)\left(q^0+|\vec{q}|\right) + +\tilde{\phi}_{(w)}(-\vec{q},-|\vec{q}|)\left(q^0-|\vec{q}|\right)\right\}$$

$$K_{\rho\sigma} = \frac{i}{2}\int \frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_{[\rho}\left\{(q^0+|\vec{q}|)\frac{\partial\tilde{\phi}_{(w)}(-\vec{q},|\vec{q}|)}{\partial q^{\sigma]}} + +(q^0-|\vec{q}|)\frac{\partial\tilde{\phi}_{(w)}(-\vec{q},-|\vec{q}|)}{\partial q^{\sigma]}}\right\}$$

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$$Q_\mu = \frac{1}{2}\int \frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_\mu \left\{\tilde{\phi}_{(w)}(-\vec{q},|\vec{q}|)\left(q^0+|\vec{q}|\right) + +\tilde{\phi}_{(w)}(-\vec{q},-|\vec{q}|)\left(q^0-|\vec{q}|\right)\right\}$$

$$K_{\rho\sigma} = \frac{i}{2}\int \frac{d^4q}{2|\vec{q}|}\delta(q^2)\tilde{\phi}_{(w)}(q)q_{[\rho}\left\{(q^0+|\vec{q}|)\frac{\partial\tilde{\phi}_{(w)}(-\vec{q},|\vec{q}|)}{\partial q^{\sigma]}} + + (q^0-|\vec{q}|)\frac{\partial\tilde{\phi}_{(w)}(-\vec{q},-|\vec{q}|)}{\partial q^{\sigma]}}\right\}$$

Conserved charges (I)

$$Q_\mu \equiv \int d^3\hat{x}\ T^0_\mu\ , \qquad K_{\mu\nu} \equiv \int d^3\hat{x}\ J^0_{\mu\nu}$$

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$$\begin{aligned}
Q_\mu &= \frac{1}{2}\int \frac{d^4q}{2|\vec{q}|}\,\delta(q^2)\tilde{\phi}_{(w)}(q)q_\mu \Big\{\tilde{\phi}_{(w)}(-\vec{q},|\vec{q}|)\left(q^0+|\vec{q}|\right)+ \\
&\quad +\tilde{\phi}_{(w)}(-\vec{q},-|\vec{q}|)\left(q^0-|\vec{q}|\right)\Big\} \\
K_{\rho\sigma} &= \frac{i}{2}\int \frac{d^4q}{2|\vec{q}|}\,\delta(q^2)\tilde{\phi}_{(w)}(q)q_{[\rho}\Big\{(q^0+|\vec{q}|)\frac{\partial\tilde{\phi}_{(w)}(-\vec{q},|\vec{q}|)}{\partial q^{\sigma]}}+ \\
&\quad +(q^0-|\vec{q}|)\frac{\partial\tilde{\phi}_{(w)}(-\vec{q},-|\vec{q}|)}{\partial q^{\sigma]}}\Big\}
\end{aligned}$$

On the choice of the Weyl map (I)

It is possible to define a different basis of θ-Poincaré with classical action through the $\hat{x}_1$*-on-the-right* Weyl map Ω_1:

$$\hat{M}^{(1)}_{\mu\nu}(f\,g) = \left(\hat{M}^{(1)}_{\mu\nu} f\right) g + f\left(\hat{M}^{(1)}_{\mu\nu} g\right) - \frac{1}{2}\theta^{\alpha\beta}\left[\eta_{\alpha[\mu}\left(\hat{P}_{\nu]} f\right)\hat{P}_\beta g + \right.$$
$$+ \left.\left(\hat{P}_\alpha f\right)\eta_{\beta[\mu}\hat{P}_{\nu]} g\right] + \frac{1}{2}\theta^{A1}\left[\eta_{A[\mu}\left(\hat{P}_{\nu]} f\right)\hat{P}_1 g + \right.$$
$$\left. + \eta_{1[\mu}\left(\hat{P}_{\nu]} f\right)\hat{P}_A g + \left(\hat{P}_1 f\right)\hat{P}_{[\nu} g \eta_{\mu]A} + \left(\hat{P}_A f\right)\hat{P}_{[\nu} g\,\eta_{\mu]1}\right]$$

Relation between the bases:

$$\hat{P}^{(1)}_\mu = \hat{P}_\mu$$
$$\hat{M}^{(1)}_{\mu\nu} = \hat{M}^{(w)}_{\mu\nu} + \frac{1}{2}\theta^{A1}\left[\eta_{1[\mu}\hat{P}_{\nu]}\hat{P}_A + \eta_{A[\mu}\hat{P}_{\nu]}\hat{P}_1\right]$$

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On the choice of the Weyl map (II)

$\Rightarrow$ The differential of scalar functions results to be different from the previous one:

$$d^{(1)}f(\hat{x})=i\left[\epsilon^{\alpha}_{(1)}P^{(1)}_{\alpha}+\omega^{\mu\nu}_{(1)}M^{(1)}_{\mu\nu}\right]f(\hat{x})\Rightarrow\begin{cases}\left[f(\hat{x}),\epsilon^{\alpha}_{(1)}\right]=-\frac{1}{2}\omega^{\mu\nu}_{(1)}\Upsilon^{\alpha\rho}_{\mu\nu}P^{(1)}_{\rho}f(\hat{x})\\ \left[f(\hat{x}),\omega^{\mu\nu}_{(1)}\right]=0\end{cases}$$

where $\Upsilon^{\alpha\rho}_{\mu\nu}=\left(\theta^{\alpha}_{[\mu}\delta^{\rho}_{\nu]}+\theta^{\rho}_{[\mu}\delta^{\alpha}_{\nu]}\right)-\theta^{A1}\left[\eta_{A[\mu}\delta^{\rho}_{\nu]}\delta^{\alpha}_{1}+\eta_{1[\mu}\delta^{\rho}_{\nu]}\delta^{\alpha}_{A}+\eta_{A[\mu}\delta^{\alpha}_{\nu]}\delta^{\rho}_{1}+\eta_{1[\mu}\delta^{\alpha}_{\nu]}\delta^{\rho}_{A}\right]$

Independence on the choice of the Weyl map

Currents associated to the new transformations:

$$
\begin{aligned}
J^{\mu\,(1)}_{\rho\sigma} &= \frac{1}{2}\left(\phi(\hat{x})P^{\mu}_{(1)}M^{(1)}_{\rho\sigma}\phi(\hat{x}) - P^{\mu}_{(1)}\phi(\hat{x})M^{(1)}_{\rho\sigma}\phi(\hat{x})\right) + \\
&- \frac{1}{4}\Upsilon^{\nu\lambda}_{\rho\sigma}\left[P^{(1)}_{\lambda}\phi(\hat{x})P^{\mu}_{(1)}P^{(1)}_{\nu}\phi(\hat{x}) - P^{\mu}_{(1)}P^{(1)}_{\lambda}\phi(\hat{x})P^{(1)}_{\nu}\phi(\hat{x})\right]
\end{aligned}
$$

Charges:

$$
\begin{aligned}
K_{\mu\nu} &= i\int \frac{d^3q}{2|\vec{q}|}\Big\{\left(q_i\delta^j_{[\mu}\delta^i_{\nu]} - |\vec{q}|\delta^j_{[\mu}\delta^0_{\nu]}\right)\tilde{\phi}_{(1)}(\vec{q},-|\vec{q}|) \\
&\frac{\partial\tilde{\phi}_{(1)}(-\vec{q},|\vec{q}|)}{\partial q^j}e^{-i|\vec{q}|q^1\theta_{01}} - \left(q_i\delta^j_{[\mu}\delta^i_{\nu]} + |\vec{q}|\delta^j_{[\mu}\delta^0_{\nu]}\right) \\
&\tilde{\phi}_{(1)}(\vec{q},|\vec{q}|)\frac{\partial\tilde{\phi}_{(1)}(-\vec{q},-|\vec{q}|)}{\partial q^j}e^{i|\vec{q}|q^1\theta_{01}}\Big\}e^{-iq^i\delta^A_i q^1\theta_{A1}}.
\end{aligned}
$$

where: $\tilde{\phi}_{(1)}(k) = \tilde{\phi}_{(w)}(k)e^{-\frac{i}{2}k^A k^1\theta_{A1}}$

Independence on the choice of the Weyl map

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J^{\mu\,(1)}_{\rho\sigma} &= \frac{1}{2}\left(\phi(\hat{x})P^{\mu}_{(1)}M^{(1)}_{\rho\sigma}\phi(\hat{x}) - P^{\mu}_{(1)}\phi(\hat{x})M^{(1)}_{\rho\sigma}\phi(\hat{x})\right) + \\
&- \frac{1}{4}\Upsilon^{\nu\lambda}_{\rho\sigma}\left[P^{(1)}_{\lambda}\phi(\hat{x})P^{\mu}_{(1)}P^{(1)}_{\nu}\phi(\hat{x}) - P^{\mu}_{(1)}P^{(1)}_{\lambda}\phi(\hat{x})P^{(1)}_{\nu}\phi(\hat{x})\right]
\end{aligned}
$$

Charges:

$$
\begin{aligned}
K_{\mu\nu} &= i\int\frac{d^3q}{2|\vec{q}|}\left\{\left(q_i\delta^j_{[\mu}\delta^i_{\nu]} - |\vec{q}|\delta^j_{[\mu}\delta^0_{\nu]}\right)\tilde{\phi}_{(1)}(\vec{q},-|\vec{q}|)\right. \\
&\quad \frac{\partial\tilde{\phi}_{(1)}(-\vec{q},|\vec{q}|)}{\partial q^j}e^{-i|\vec{q}|q^1\theta_{01}} - \left(q_i\delta^j_{[\mu}\delta^i_{\nu]} + |\vec{q}|\delta^j_{[\mu}\delta^0_{\nu]}\right) \\
&\quad \left.\tilde{\phi}_{(1)}(\vec{q},|\vec{q}|)\frac{\partial\tilde{\phi}_{(1)}(-\vec{q},-|\vec{q}|)}{\partial q^j}e^{i|\vec{q}|q^1\theta_{01}}\right\}e^{-iq^i\delta^A_i q^1\theta_{A1}}.
\end{aligned}
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where: $\tilde{\phi}_{(1)}(k) = \tilde{\phi}_{(w)}(k)e^{-\frac{i}{2}k^A k^1\theta_{A1}}$

Conclusions

- Algebra of symmetries of canonical noncommutative spacetime
- Scalar field transformation (differential)
- Charges: conserved and independent on the ordering choice

Open issues

- Observables of the theory?
- Comparison with classical theory
- Interacting field theory

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