Beyond local QFT: non-locality, quantum group symmetries and κ-quantum fields

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Outline

- **Introduction: non-locality and quantum group symmetries in QFT**
- **The model: κ-Poincaré symmetries and deformed quantum fields**
- **The "fine structure" of the Fock space at the Planck scale**
- **Conclusions and outlook**

Hopf algebras and field quantization

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$$\rho \equiv (\rho_1 \otimes \rho_2)\Delta$$

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- Using **path integral** techniques a set of relations involving $j^\mu(x)$, $\phi(x)$ and G can be established, known as Ward identities

$$\partial_\mu \langle 0|\,T(j^\mu(x)\phi(x_1)...\phi(x_n))|0\rangle = -i\sum_{i=1}^{n}\delta(x-x_i)\langle 0|\,T(\phi(x_1)...G \triangleright \phi(x_i)...\phi(x_n))|0\rangle$$

“Mutant” charges from 2d QFT and quantum group symmetries

Integrating the Ward identity

$$\langle 0|G \triangleright \phi(x')|0\rangle = \int_V dx\, \partial_\mu \langle 0|T(j^\mu(x)\phi(x'))|0\rangle = \int_{\partial V} d^3\vec{x}\, \langle 0|T(j^0(x)\phi(x'))|0\rangle$$

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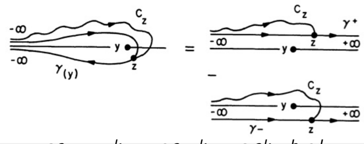

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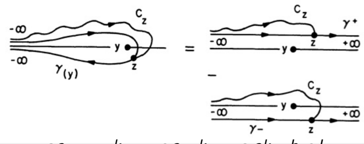

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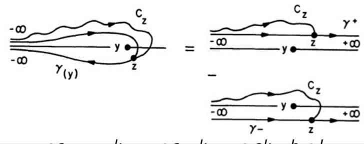

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Use the same tools to introduce an effective deformation of standard QFT which models the effects of a "geometric" non-locality?

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$$< 0|[\hat{Q}, A_\Psi]|0 > \neq 0$$

looks like *spontaneous symmetry breaking*, but now, by construction, $\hat{Q}|0 >= 0$...

Symmetries, quantum groups and non-locality

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Quantum groups appear to be natural generalizations of Lie-algebra **symmetries** in QFT in the presence of **non-locality**

MA, Phys. Rev. D **77**, 025013 (2008) [arXiv:0710.1083 [hep-th]].

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$\kappa \rightarrow \infty$ recover the *trivial* Hopf algebra naturally associated to the Poincaré algebra

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Relation between κ-**Poincaré and** κ-**Minkowski NCST:**

$$[x_m, t] = \frac{i}{\kappa} x_m \ , \qquad [x_m, x_l] = 0$$

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NOTE: $q_1 \dot{+} q_2 \neq q_2 \dot{+} q_1$

We have a **non-commutative algebra of functions**

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spanned by an orthonormal set of **plane waves** on the deformed mass-shell $\{\phi_{\vec{p}}\}$,

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> given n-different modes one has $n!$ different n-particle states, one for each permutation of the n modes p_1 , p_2 ... p_n

- the different states are orthogonal for every finite κ, e.g.

$$\langle p_1 p_2 \,|\, p_2 p_1\rangle_\kappa \simeq \frac{1}{2}\delta^{(3)}\left(\epsilon_2\vec{p}_1\right)\delta^{(3)}\left((\epsilon_1(1-\epsilon_2)^{-1}-1)^{-1}\vec{p}_2\right) + 1 \leftrightarrow 2\ .$$

MA and A. Marciano, Phys. Rev. D **76**, 125005 (2007); MA and D. Benedetti, in progress see also D. Benedetti's talk.

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$Q_{\rm vac}$ is finite and the charges obey a deformed dispersion relation!

$$|\vec{Q}| = \kappa \tanh\left(\frac{Q_0}{\kappa}\right)$$

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$$|E\rangle \otimes |0\rangle = |p_1 p_2\rangle_\kappa$$
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- e.g. the state superposition of two total "classical" energies $E_A = E(\vec{p}_A) + E(\vec{q}_A)$ and $E_B = E(\vec{p}_B) + E(\vec{q}_B)$ can be entangled with the additional hidden modes e.g.

$$|\Psi\rangle = 1/\sqrt{2}(|E_A\rangle \otimes |0\rangle + |E_B\rangle \otimes |1\rangle)$$

(MA, A. Hamma and S. Severini, arXiv:0806.2145.)

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The possibility of mode entanglement also suggests a neat proposal for a resolution of the **BH info paradox** (more details in arXiv:0806.2145)

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  we didn't introduce anything new, we just deformed!