Matrix Group Recognition: Final report

Derek F. Holt, Steve A. Linton, Colva M. Roney-Dougal

October 5, 2009

1 Relationship with original proposal

There was no significant deviation from the original proposal.

2 Short report

Computing in finite matrix groups has been an active area of research within Computational Group Theory for about 15 years and remains so today. This was nevertheless the first Workshop that was concerned specifically with that topic.

There are currently two major software packages, written in MAGMA and GAP, which are broadly capable of identifying the composition factors of a matrix group of dimension up to a few hundred over a moderately sized finite field.

The aims of the Workshop were to ascertain the current state of play, to educate and inspire the enthusiasm of younger researchers, to attempt to solve some challenge problems, and to make concrete plans for future research.

The high quality of the talks, the generous allocation of time for collaboration, and the focused theme all featured as highlights in participants' evaluations. Several new research relationships involving younger researchers were developed.

Future plans included: extending the methods to groups defined over infinite fields; determining which algorithms are polynomial; extending available facilities and making them usable by non-specialists; looking for new methods, such as computing symbolically rather than in specific groups. For several of these projects, specific work-plans were drawn up. Overall, the aims of the Workshop were achieved!

3 Detailed report

Computing in finite matrix groups has probably been the single most active and lively area of research within Computational Group Theory for about the past fifteen years, and it still remains so today. This was nevertheless the first Workshop of any kind that was concerned specifically with that topic.

Before the Workshop, the high points in the so-called *matrix group recognition project* (MGRP) had been the design, implementation, and associated theoretical analyses of Leedham-Green & O'Brien's *Composition Tree* [1] and Neunhöffer & Seress's *Recognition Tree* [2] software packages, which are written in MAGMA and GAP respectively. A large number of research papers have been written by a wide range of mathematicians on this topic involving, for example, group theory, representation theory, probability theory, number theory, and the design and complexity analysis of algorithms.

The programs are broadly capable of constructively identifying the composition factors of a matrix group of dimension up to a few hundred over a moderately sized finite field. The general aim is that these programs should run reasonably quickly on all examples within this range, and that they should run in polynomial time assuming the availability of a number of oracles, which almost certainly need to include computing discrete logarithms and factorising integers. These aims have not yet been entirely achieved but encouraging progress has been and continues to be made.

The principal aims of this Workshop were to ascertain the current state of play in the MGRP, to educate and inspire the enthusiasm of younger researchers, to attempt to solve some challenge problems at the meeting, and to make concrete plans to achieve the following within the next few years:

(i) Complete the remaining gaps in the Composition Tree programs, and enable the computation of a chief series passing through a certain series of three characteristic subgroups.

(ii) Use this as a starting point for further algorithm design for finite matrix groups, such as Sylow subgroups, conjugacy classes of elements and maximal subgroups, centralizers, normalizers and intersections of subgroups, ordinary (eventually modular) character tables, subspace stabilisers, tensor decomposition stabilisers, etc.

There were 35 participants, including virtually all of the leading international experts in the area, many experts in related areas, and several younger researchers and research students. There were four 1-hour presentations describing the current situation and the capabilities of the software. These were by Leedham-Green, O'Brien, Seress and Neunhöffer. All participants had been invited to give half-hour talks on their own research, and there were 21 of these. There were also two panel discussions, and about two hours left free each afternoon for collaborative research.

The high quality of the talks, the generous allocation of time for collaborative research, and the unusually focused theme of the Workshop all featured as highlights in participants' evaluations. Since many of the participants already knew each other, there was limited scope for the development of new contacts, but several new and fruitful relationships involving younger researchers were developed, including one instance of a student meeting a future supervisor. Specific plans for writing papers were also made by people who had not previously collaborated.

One problem for which significant new ideas were developed during the meeting was that of deciding whether a matrix group is tensor decomposable. This has appeared to date to be very difficult, but some novel suggestions of Ryba and Parker may render it tractable in the near future. Another achievement during the meeting itself was the computer-assisted proof by Holt and Havas that a certain finitely presented group first investigated by Coxeter is finite; this had been a well-known open problem for many years. A third problem which was solved at the meeting was the development by Leedham-Green and Roney-Dougal of an effective algorithm for producing uniformly-distributed random elements of the derived group of a finite group given by generators and relations.

The progress made during the meeting and future plans were discussed during the final panel meeting and also reported by participants in their evaluations. These included: extending the methods to linear groups over infinite fields and improving them in even characteristic; determining which of the principal algorithms are polynomial; combining matrix and permutation group software, extending the range of available facilities, and making them usable by non-specialists; making more effective use of parallelisation in the MGRP; making greater use of Lie theory; extending databases of representations of quasisimple groups; looking for fundamentally new methods, such as computing symbolically rather than always in specific groups.

For several of these projects, including those described in the aims (i) and (ii) above, and the new ideas for testing for tensor decomposability, specific work-plans were drawn up. For the organisers, this means that one of the most important objectives of the Workshop has been achieved.

Our claims about the success of the conference are strongly supported by the extremely positive feedback that we received from participants. Some comments include: "A very high proportion of the experts worldwide in the area of Matrix Group Recognition and surrounding areas were present at this workshop, which made it possible to make coherent and realistic plans for future work and development"; "the whole week was brilliant, both academically and socially"; and on a more practical note "the organization of the workshop and the facilities were absolutely first rate".

All participants were very satisfied and impressed by the efficiency and helpfulness of the ICMS Workshop co-ordinator Helene Frossling, which ensured an unusually smooth running meeting. The only complaints were that it was not possible for participants to use the Informatics Forum building after about 17.30 each day, and that the toilets in the building were out of order more often than would have been desirable, both of which were unfortunately outside the control of the ICMS.

References

[1] C.R. Leedham-Green. The computational matrix group project. "Groups and Computation III", de Gruyter, Berlin, (2001) 229–248.

[2] M. Neunhöffer and À. Seress. A data structure for a uniform approach to computations with finite groups. "ISSAC'06", ACM, New York (2006) 254–261.