Stability Characteristics Of Large Amplitude Internal Solitary Waves

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Outline

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Motivation

ISWs in the Laboratory

- Generation & visualisation
- Measurment techniques
- Configuration - the two regimes

Stability Characteristics

- Results for the 3 layer regime
- Results for the 2 layer regime

Future Work

Outline

(a) (b)

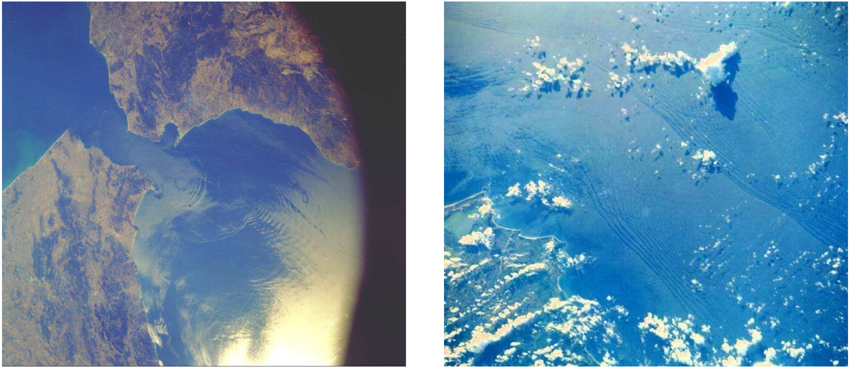


FIG.: NASA satellite photos of (a) the strait of Gibraltar and (b) Hainan Island in the South China Sea.

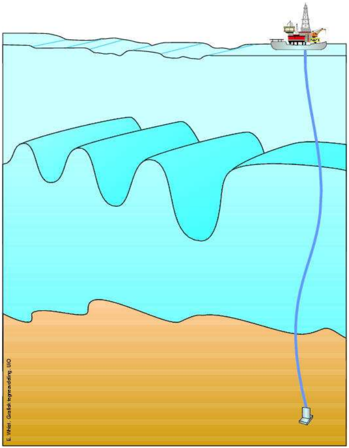
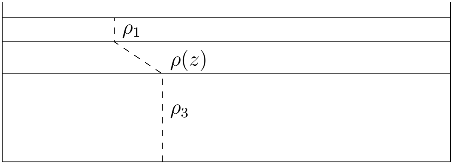
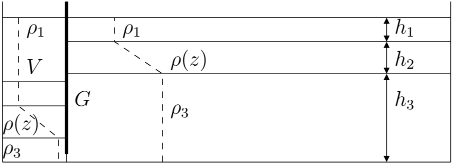


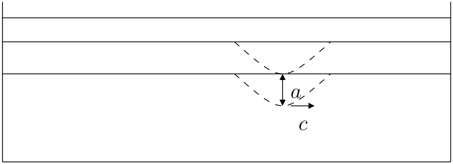
FIG.: Schematic diagram of an internal wave.



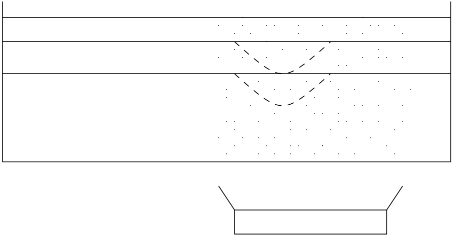
- Fix ρ_1 and ρ_3, such that $(\rho_3 - \rho_1)/\rho_3 << 1$. $\rho(z)$ is a linear function of the depth, z.



- Free variables are V, h_1, h_2 and h_3.



- Flow is seeded with 'Pliolite' particles and illuminated from below.



- Particle Image Velocimetry (PIV) - continuous synoptic velocity field data.
- Laser Doppler Anemometry (LDA) - point velocity measurements.
- Synthetic Schlieren - continuous synoptic density data.
- Micro-conductivity probes - high precision traverse of density field.

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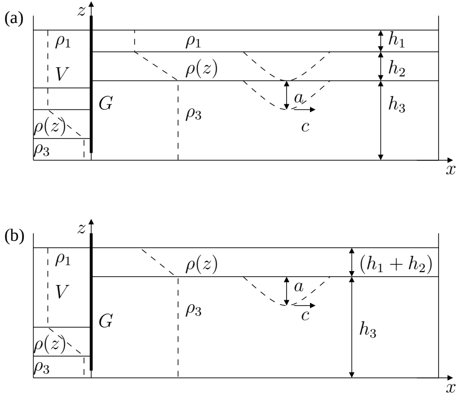
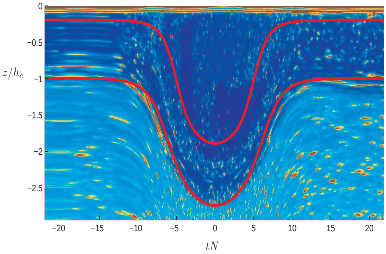


FIG.: Schematic diagram of the experimental set up (a) 3 layer configuration (b) 2 layer configuration.

Time series

02/06/05 - Stable



13/06/05 - Unstable

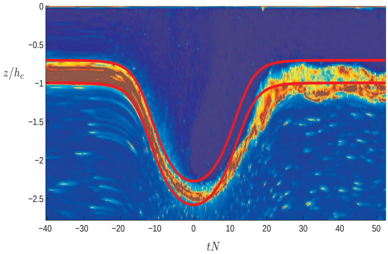


FIG.: Reconstructed experimental wave profile (background) and the corresponding fully nonlinear numerical solution (thick red lines).

Critical amplitude

$$a_{1crit} = 2.24\sqrt{h_1 h_2}(1 + h_2/h_1), \quad h_2/h_1 \leq 1.$$

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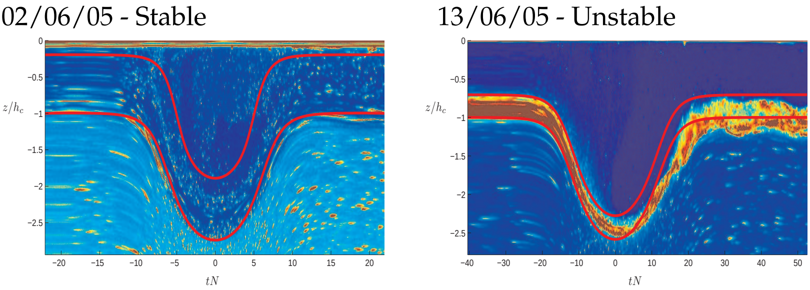

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Nonlinear Computation

- Integral Equation Method - Fructus & Grue 2004, *J. Fluid. Mech.*
- Fully Nonlinear Method - solves the Laplace equation in the homogeneous layers & Helmholtz equation in the stratified layer.

$$\nabla^2\psi + \frac{N_\infty}{c^2}\psi = 0$$

- Wave frame of reference - stationary.
- Green function, Bessel funtion & Fourier Transform.
- Compare velocity fields to infer data.

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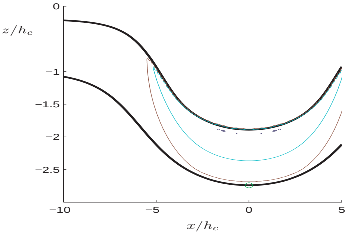
$$Ri = \frac{g\beta}{(\partial u/\partial z)^2}, \quad \text{where} \quad \beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial z} \quad \text{or} \quad Ri = \frac{c(c-u)}{N_\infty^2 \delta^2}.$$

02/06/05 - Stable, $Ri_{min} = 0.06$.



13/06/05 - Unstable $Ri_{min} = 0.11$.

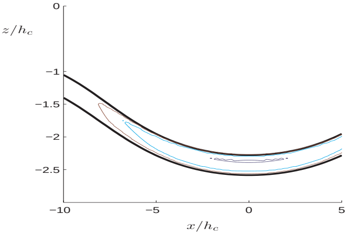


FIG.: Numerical wave profile and ISO-Ri lines ($Ri = 0.5$ and $Ri = 0.25$).

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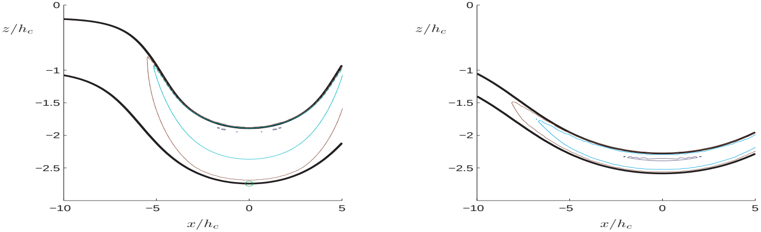

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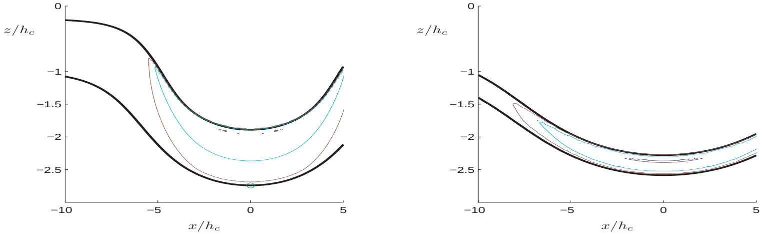

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Stability Criteria

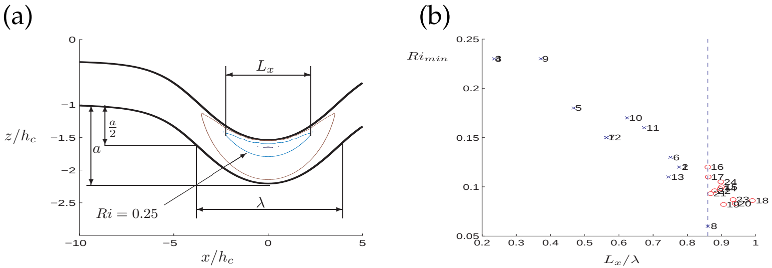

FIG.: (a) Schematic of key parameters (b) L_x/λ versus Ri_{min} for stable (x) and breaking (○) cases.

The critical parameter for shear instability is

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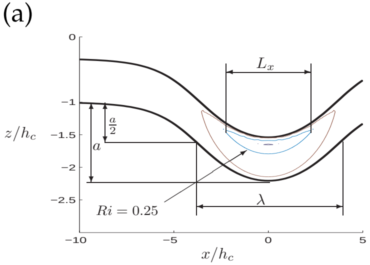

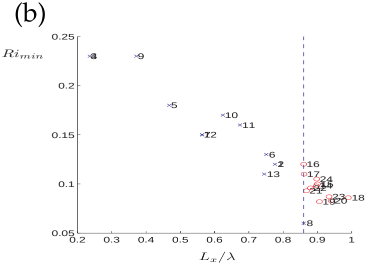

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3 Layer - Synthetic Schlieren Data

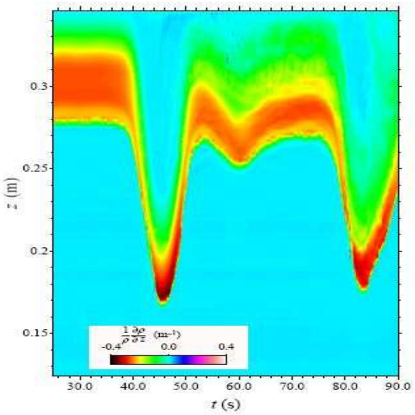

FIG.: Variation of the vertical density gradient for a stable wave in the three layer regime.

Recall

$$Ri = \frac{g\beta}{(\partial u/\partial z)^2}, \quad \text{where} \quad \beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial z}.$$

The buoyancy frequency $g\beta$ is a measure of stabilising effects while the velocity gradient $\partial u/\partial z$ is destabilising.

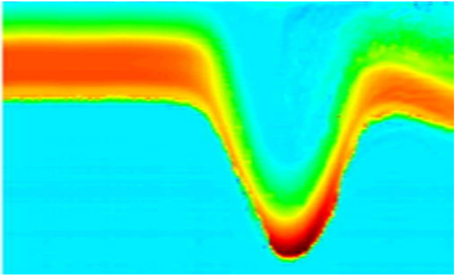

FIG.: Time series of the vertical density gradient, $-\beta$, for a stable wave in the three layer diffuse regime.

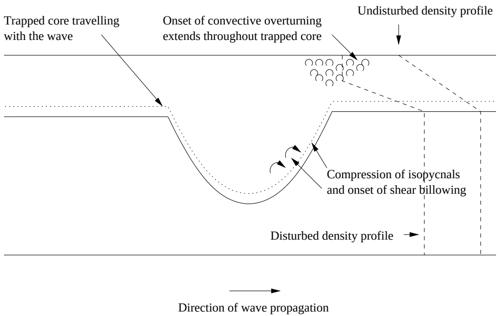

FIG.: Schematic diagram of the unstable motion seen in the 2 layer regime.

Ri and $u/c > 1$ plot

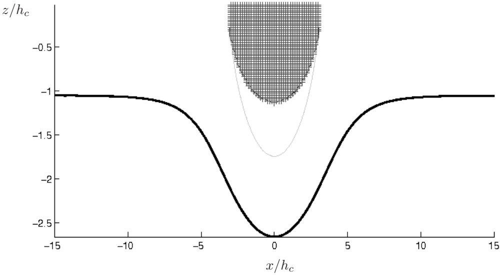

FIG.: 24/04/06 (2 layer movie shown). Shaded area marks convectively unstable region in which $u/c > 1$, the light grey contour marks $Ri = 0.25$ and the thick black line indicates the interface.

Summary

- Background stratification has a dramatic effect on the onset and type of instability.
- The critical parameter for shear instability is L_x/λ as opposed to Ri_{min}.
- Convective instability may induce shear in the two layer regime (on the leading face).
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Future Work

- Growth, decay and dissipation of ISW induced instabilities. ($\lambda = 7.9h_2, \quad c_r = 0.09c, \quad F = 3.3 - 3.7$)
- The effect of the upper boundary condition on stability.
- The effect of instabilities on benthic dynamics.
- Stability characteristics of ISWs of elevation.

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