Report on the Workshop:

Large Amplitude Internal Waves

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Submitted by:

Professor Roger Grimshaw[1] & Professeur Jean-Claude Saut[2]

[1] Department of Mathematical Sciences, Loughborough University
Email: R.H.J.Grimshaw@lboro.ac.uk

[2] Département Mathématique, Université de Paris-Sud 11
Email: jean-claude.saut@math.u-psud.fr

Background

Internal waves occur in density-stratified fluids. They are ubiquitous in the ocean and atmosphere, and often have large amplitudes. Of particular concern in this workshop were internal solitary waves, where nonlinearity is essential. In their simplest representation internal solitary waves are nonlinear waves of quasi-permanent form which owe their existence to a balance between nonlinear wave-steepening effects and linear wave dispersion. They are a commonly occurring feature of coastal seas, straits, fjords and lakes, and also can occur as spectacular roll clouds in the atmospheric boundary layer. On the theoretical side, nonlinear evolution equations of the Korteweg-de Vries (KdV) type are usually used as a first-order basis for qualitative modelling and prediction.

These ubiquitous internal solitary waves are a major factor in the flow over the oceanic continental shelf and slope. Their associated currents and pycnocline displacements have profound implications for the design and placement of offshore structures, the safety of submersibles, the coherence of underwater sound signals, the movement of nutrients and biological matter, and sediment transport. Further, they can produce microstructure and localized turbulent patches, and in general are a sink for the barotropic tidal energy.

Although these waves are now well documented, the associated theory and modelling are not well developed, certainly by contrast to the corresponding state of affairs for water waves. Although model equations of the KdV-type are widely used, validation has largely been confined to comparisons with numerical simulations and laboratory experiments, and there are very few rigorous theoretical results available, even for steady-state solitary waves. Hence the aim of this workshop is to bring together physical oceanographers and engineers concerned with the observation and measurements of internal waves, applied mathematicians and physicists involved in modelling and

numerical simulations, with mathematicians concerned to develop a rigorous understanding of the dynamics of large-amplitude internal waves. While the focus is on internal waves, the general principles involved have relevance for the study of nonlinear waves in other physical contexts.

Structure:

There were forty participants, fifteen from UK, twelve from EU, eleven from US, and two from Russia. They included a broad cross-section of mathematicians interested in large-amplitude waves, together with some theoretically minded physicists and engineers concerned with modelling and simulation aspects, and scientists with observational and experimental concerns. A further eighteen people were invited, but were either unable to come, or contacted us too late to be included as we were at the maximum capacity. The programme consisted of four keynote lectures of one hour each, and twenty-nine half-hour contributions. Each day concluded with an open discussion session. There were no significant deviations from the original accepted proposal.

Topics covered:

The talks ranged widely over many aspects of internal waves, with a heavy and natural emphasis on solitary waves. Given the mix of mathematicians, theoretical and numerical modelers, experimentalists and observationalists who made up the workshop, the wide coverage of the talks and the discussion was inevitable and desirable. The talks were structured around four sub-topics, and the timetable was structured so that these were grouped appropriately. In this list a "*" denotes a keynote lecture, and otherwise the speakers are listed in alphabetical order; however, note that the allocation to topics is somewhat arbitrary, as many talks covered more than one of these sub-topics.

- **Observations and experiments:** The occurrence of internal waves in the ocean and atmosphere is well documented, both from in situ observations and satellite images, and there have been several significant laboratory experiments. These were reviewed by the experts in the area, with an emphasis on some of the more recent studies (e.g. experiments in the South China sea) and brought out some of the key issues that modellers and theoreticians need to consider. [Carr, Farmer, Ramp, da Silva, Scotti]

- **Numerical modelling:** Numerical simulations based on the full Euler equations have mainly been two-dimensional, and fall into two categories. First, there those numerical studies which have focussed on the structure of the steady solitary waves. Second, there are those studies which have attempted to simulate the propagation of internal solitary waves in realistic situations. These were reviewed by experts, with the twin aims of establishing what is known from such studies and determining what aspects can be further explored by numerical means. [Choi, Davies, *Dias, Grue, *Helfrich, Lamb, Staschuk, Vanden-Broeck, Vlasenko]

- **Analytical modelling:** The KdV equation, usually extended to take account of cubic as well as quadratic nonlinearity, and the variability of the background medium, is widely used with reasonable qualitative success. However, it has not been rigorously validated, and there is a compelling need to establish more firmly its

domain of validity. At the same time, there is a need to extend or replace this basic model in two directions, first to take better account of large-amplitude effects in a more systematic way, and second to take account of wave reflection and refraction, for instance by developing suitable Boussinesq model equations. Further, the emphasis up to the present time has been on the fundamental solitary wave solutions, and while this has certainly been appropriate as a first step, there is a need to extend analysis to the study of wave trains and undular bores. The workshop reviewed the current state-of-the-art, and identified areas where improvements can be made. [Akylas, Bona, Camassa, Chazel, *Grimshaw, Khusnutdinova, Nguyen, Osborne, Ostrovsky, Shrira, Slunyaev, Talipova]

- **Theory**: Compared to the situation for water waves, there are very few rigorous results available for the existence, structure and stability of internal solitary waves. Even for the simplest case of interfacial waves on the interface between two homogeneous layers, where there is a close analogy with water waves, some basic issues are still to be resolved. One such key issue is the systematic derivation of long waves models for internal waves, together with the rigorous comparison of their long time dynamics with the full Euler system. The workshop brought in experts in the theory of nonlinear waves, and identified some of the problem areas of immediate importance where progress can be made. [Bridges, Haragus, Iooss, *Lannes, Saut, Sun, Varvaruca]

Outcomes:

Our primary aims were two-fold. First, we wanted to focus the attention of mathematicians onto this important and challenging topic. Second, we aimed to provide a forum where mathematicians and modellers could mix constructively with oceanographers and experimentalists, with the outcome that clear directions for future research in this topic were established. The main thrust of the workshop was on identifying where significant modelling improvements and theoretical underpinning are needed. Although the main emphasis was on internal waves, the concepts involved have relevance for studies of nonlinear waves in other physical contexts.

Overall, the workshop was successful in bringing together a mix of theoreticians, analytical and numerical modellers with experimentalists and observationalists, with a good representation of some of the key people in each area. While clearly each participant will have their own view of what are now the key issues to be addressed, from the discussions and the responses to the questionnaire we can identify several such key topics. These are:

- While two-layer models for large-amplitude waves are reasonably well-developed in theory and application, the same cannot be said for models using continuous stratification.
- Stability of large-amplitude waves, and whether or not instability leads to mixing.
- Three-dimensional aspects, currently largely unexplored.
- Validity of higher-order models such as the Gardner equation, and the Choi-Camassa equation.

- A better understanding of forcing mechanisms.
- The role of background rotation needs further work, both in relation to the long-time behaviour of solitary waves, and in the generation process.
- While isolated internal solitary waves have been quite intensively studied, there is now a need to look at wave trains in more detail.
- Continued close collaboration between theoreticians and modellers on the one hand, and experimentalists and observationalists on the other hand.

Dissemination:

This will take place primarily through the ICMS web-site, where each contributor has been asked to post either their presentation or an extended abstract, with links to related publications and to their home page.

Organisation:

All the participants expressed great satisfaction with the logistical work of the ICMS staff, the overall organization and the stimulating experience of being in the "Maxwell" house.