

We construct uniformly bounded solutions of the equations $\operatorname{div}(U)=f$ and $\operatorname{curl}(U)=f$, for general f 's in the critical regularity spaces $L^d(\mathbb{R}^d)$ and, respectively, $L^3(\mathbb{R}^3)$. The study of these equations was motivated by recent results of Bourgain and Brezis. The equations are linear but construction of their solutions is not. Our constructions are, in fact, special cases of a rather general framework for solving linear equations, $L(U)=f$, covered by the closed range theorem. The solutions are realized in terms of nonlinear hierarchical representations, $U=\sum(u_j)$, which we introduced earlier in the context of image processing. The u_j s are constructed recursively as proper minimizers, yielding a multi-scale decomposition of the solutions U .