Dynamics of toxic algae in the mediterranean

D. Chiron, A. Habbal, P-E Jabin, M. Ribot from the Univ. of Nice

in collaboration with R. Lemée *et al* from the Oceanological Observatory in Villefranche

Which Algae ?

- A subspecies of Ostreopsis Ovata, a microalgae present on the mediterranean coasts for the last 4-5 years.
- Effect on the environment seems to be fortunately limited.
- mildly toxic for human people, provoking small allergic reactions on the skin and more serious respiratory problems when inhaled (though no lethal case up to now).
- Effects when ingested (by eating contaminated seafood for example) are still not completely clear.

$\longrightarrow$ It is mainly a health and economical problem (closing of beaches...)

The behaviour

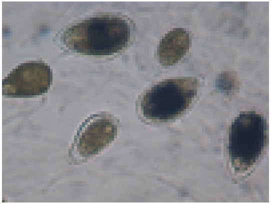
- Algae live in two environments: On the bottom of the sea and dispersed in the sea
- Algae reproduce and develop fixed on a biological film on the bottom
- They colonize new sites by being transported by the currents
- During rough weather, they can be carried in the air for short distances.

$\longrightarrow$ Both environments are important and must be modeled correctly.

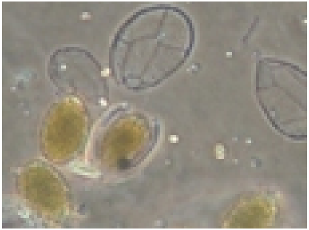
On the bottom

Changes of color indicate the places heavily populated with Ostreopsis.

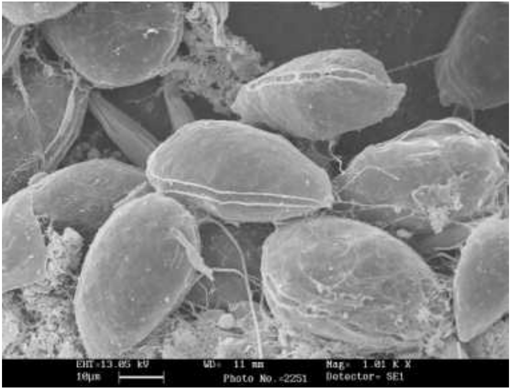
A closer look at the Lab.



And with a different colorant



Coalescence phenomena in the dispersed phase



Those are important as they seem to affect the boyancy of the algae...

The basics

We follow the 4 phases introduced by Anguige, King et Ward and in particular the models by Ambrosi, Preziosi and Clarelli, Di Russo, Natalini, Ribot (see also Zhang, Cogan, and Wang).
The four phases are

- The microalgae themselves with the volume fraction $B(t,x)$
- The dead or inert microalgae with the volume fraction $D(t,x)$
- The extracellular matrix secreted by the algae with $E(t,x)$
- The liquid from the biological film with $L(t,x)$

The basic structure for the equations

We simply write

$$\begin{aligned}\partial_t B + \nabla \cdot (B\mathbf{v_S}) &= \Gamma_B,\\ \partial_t D + \nabla \cdot (D\mathbf{v_S}) &= \Gamma_D,\\ \partial_t E + \nabla \cdot (E\mathbf{v_S}) &= \Gamma_E,\\ \partial_t L + \nabla \cdot (L\mathbf{v_L}) &= \Gamma_L,\end{aligned}$$

With the local conservations of volume and flux

$$B + D + E + L = 1, \quad \Gamma_B + \Gamma_D + \Gamma_E + \Gamma_L = 0.$$

The reaction terms

Very little is known about the exact biological behaviours of those microalgae.
As a first attempt and without additional precise information we choose

$$
\begin{aligned}
\Gamma_B &= k_B L B - k_D B,\\
\Gamma_D &= \alpha k_D B - k_N D,\\
\Gamma_E &= k_E B L - \varepsilon E.\\
\Gamma_L &= -\Gamma_B - \Gamma_D - \Gamma_E
\end{aligned}
$$

All "constants" k_B, k_D, k_N, k_E in fact depend on the temperature and light intensity, in a very sensitive way.

The fluxes

For each phase $\phi = B, D, E, L$, one has

$$\partial_t(\phi \mathbf{v}_\phi) + \nabla \cdot (\phi \mathbf{v}_\phi \otimes \mathbf{v}_\phi) = \mathbf{m}_\phi - \phi \nabla P + \nabla \cdot (\phi \mathbf{T}_\phi) + \Gamma_\phi \mathbf{v}_\phi,$$

with P the common hydrostatic pressure, m_ϕ the force of the phase ϕ on the other phases and T_ϕ the constraint tensor of the phase.

By conservation of forces and mass, one easily gets the equation

$$\partial_t((1-L)\mathbf{v_S}) + \nabla \cdot ((1-L)\mathbf{v_S} \otimes \mathbf{v_S}) = -(1-L)\nabla P + \nabla \cdot (\sum_{\phi \neq L} \phi \mathbf{T}_\phi) - \mathbf{m}_L - \Gamma_L \mathbf{v_L}.$$

A closure is necessary to get an equation on the pressure...

The final model

Following previous works, we choose for Σ a given function of $1 - L$

$$
\begin{cases}
\partial_t B + \nabla \cdot (B\mathbf{v_S}) = k_B BL - k_D B, \\
\partial_t D + \nabla \cdot (D\mathbf{v_S}) = \alpha k_D B - k_N D, \\
\partial_t E + \nabla \cdot (E\mathbf{v_S}) = k_E BL - \varepsilon E, \\
B + D + E + L = 1, \\
\partial_t((1-L)\mathbf{v_S}) + \nabla \cdot ((1-L)\mathbf{v_S} \otimes \mathbf{v_S}) + (1-L)\nabla P = \nabla \Sigma \\
\qquad + (M - \Gamma_L)\mathbf{v_L} - M\mathbf{v_S}, \\
\partial_t(L\mathbf{v_L}) + \nabla \cdot (L\mathbf{v_L} \otimes \mathbf{v_L}) + L\nabla P = -(M - \Gamma_L)\mathbf{v_L} + M\mathbf{v_S}, \\
-\Delta P = \nabla \cdot (\nabla \cdot ((1-L)\mathbf{v_S} \otimes \mathbf{v_S} + L\mathbf{v_L} \otimes \mathbf{v_L})) - \Delta \Sigma.
\end{cases}
$$

Boundary conditions and other modifications

The model has to be supplemented by boundary conditions, which probably depend on the site. We typically choose Neumann for the volume fractions and vanishing flux for the velocities.
In addition an exchange term with the dispersed microalgae should be added. This term should take into account the turbulence of the current (the roughness of the sea). Typically something like

$$-k_S B + \tilde{k}_S \rho$$

with k_S, $\tilde{k}_S$ depending on the meteorological conditions and ρ the density of dispersed algae near the bottom.

Some basics

- The Algae are either alone or may coalesce in large structures.
- They are transported by the sea currents.
- Algae that are alone may swim on their own.
- The buoyancy may depend on the size of the coagulated structure.

The modeling in this environment is considerably more complicated (but with plenty of kinetic equations) so only a sketch is given here.

The starting point

Algae exist in 2 phases

- free algae whose state is determined by their position and velocities hence a distribution $f_l(t, x, v)$.
- Coagulated algae to which the size of the structure should be added so $f(t, x, v, n)$.

One can always write the model in a synthetic manner by defining

$$f(t, x, v, 1) = f_l(t, x, v).$$

The first general equation

Taking all reasonable properties into account,

$$\partial_t f(t,x,v,n) + v\cdot\nabla_x f + \mathrm{div}_v\left(\frac{\mu}{r_n^2}(u(t,x)-v)\,f + m(n)\,g\,f\right)$$
$$= \nu(n)\Delta_x f - \sum_m C(f(n),f(m)) + \frac{1}{2}\sum_{m<n} C_+(f(m),f(n-m))$$
$$+ \sum_{m>n} F(f(m),n) - \frac{1}{2}\sum_m \int_{v'} F(f(n),m)\,dv'$$
$$-(n+1)\,df(t,x,v,n+1) + R(f).$$

The first general equation

Taking all reasonable properties into account,

$$\partial_t f(t,x,v,n) + v \cdot \nabla_x f + \mathrm{div}_v \left(\frac{\mu}{r_n^2} (u(t,x) - v) f + m(n)\, g\, f \right)$$
$$= \nu(n) \Delta_x f - \sum_m C(f(n), f(m)) + \frac{1}{2} \sum_{m<n} C_+(f(m), f(n-m))$$
$$+ \sum_{m>n} F(f(m), n) - \frac{1}{2} \sum_m \int_{v'} F(f(n), m)\, dv'$$
$$-(n+1)\, d f(t,x,v,n+1) + R(f).$$

This term is the advection by the water. r_n represents the effective radius of the “grape”.

The first general equation

Taking all reasonable properties into account,

$$\partial_t f(t,x,v,n) + v\cdot\nabla_x f + \mathrm{div}_v \left(\frac{\mu}{r_n^2}(u(t,x)-v)\, f + m(n)\, g\, f\right)$$
$$= \nu(n)\Delta_x f - \sum_m C(f(n),f(m)) + \frac{1}{2}\sum_{m<n} C_+(f(m),f(n-m))$$
$$+ \sum_{m>n} F(f(m),n) - \frac{1}{2}\sum_m \int_{v'} F(f(n),m)\, dv'$$
$$-(n+1)\, df(t,x,v,n+1) + R(f).$$

This is gravity and buoyancy. The effective mass $m(n)$ may depend on the size of the grape.

The first general equation

Taking all reasonable properties into account,

$$\partial_t f(t,x,v,n) + v\cdot\nabla_x f + \mathrm{div}_v\left(\frac{\mu}{r_n^2}(u(t,x)-v)\,f + m(n)\,g\,f\right)$$
$$= \nu(n)\Delta_x f - \sum_m C(f(n),f(m)) + \frac{1}{2}\sum_{m<n} C_+(f(m),f(n-m))$$
$$+\sum_{m>n} F(f(m),n) - \frac{1}{2}\sum_m \int_{v'} F(f(n),m)\,dv'$$
$$-(n+1)\,df(t,x,v,n+1) + R(f).$$

This takes the own movement of the microalgae into account. Accordingly $\nu(n)=0$ if $n>1$.

The first general equation

Taking all reasonable properties into account,

$$\partial_t f(t,x,v,n) + v\cdot\nabla_x f + \mathrm{div}_v \left(\frac{\mu}{r_n^2}(u(t,x)-v)\, f + m(n)\, g\, f\right)$$

$$= \nu(n)\Delta_x f - \sum_m C(f(n), f(m)) + \frac{1}{2}\sum_{m<n} C_+(f(m), f(n-m))$$

$$+\sum_{m>n} F(f(m), n) - \frac{1}{2}\sum_m \int_{v'} F(f(n), m)\, dv'$$

$$-(n+1)\, df(t,x,v,n+1) + R(f).$$

Those model the coagulation of the algae into larger structure.

The first general equation

Taking all reasonable properties into account,

$$\partial_t f(t,x,v,n) + v\cdot\nabla_x f + \mathrm{div}_v\left(\frac{\mu}{r_n^2}(u(t,x)-v)\,f + m(n)\,g\,f\right)$$
$$= \nu(n)\Delta_x f - \sum_m C(f(n),f(m)) + \frac{1}{2}\sum_{m<n} C_+(f(m),f(n-m))$$
$$+\sum_{m>n} F(f(m),n) - \frac{1}{2}\sum_m \int_{v'} F(f(n),m)\,dv'$$
$$-(n+1)\,df(t,x,v,n+1) + R(f).$$

Those model the possible fragmentation of the grapes. Probably caused by the shearing of the current and hence heavily dependent on the meteorological conditions.

The first general equation

Taking all reasonable properties into account,

$$\partial_t f(t,x,v,n) + v\cdot\nabla_x f + \mathrm{div}_v\left(\frac{\mu}{r_n^2}(u(t,x)-v)\,f + m(n)\,g\,f\right)$$
$$= \nu(n)\Delta_x f - \sum_m C(f(n),f(m)) + \frac{1}{2}\sum_{m<n} C_+(f(m),f(n-m))$$
$$+\sum_{m>n} F(f(m),n) - \frac{1}{2}\sum_m \int_{v'} F(f(n),m)\,dv'$$
$$-(n+1)\,df(t,x,v,n+1) + R(f).$$

Death and reproduction of the algae. As a first step reproduction is assumed not to occur in the dispersed form. However asexual reproduction certainly do and even sexual reproduction cannot be excluded in a grape.

Generic expressions for Coagulation and Fragmentation

As is usual in such models, we may take

$$C_+(f(m), f(n-m)) = \int_{v'} c(v', v^*, m, n-m)\, f(t,x,m,v') \\ f(t,x,n-m,v^*)\, \frac{n^3}{(n-m)^3}\, dv',$$

where by conservation of momentum $n\,v = m\,v' + (n-m)\,v^*$, *i.e.* $v^* = (nv - mv')/(n-m)$.

Similar expressions can be written for C_- and F...

Why the model is not satisfying

Many terms are very large, for instance $1/r_n^2$...
Several length scales also coexist

- Typically longitudinal length (parallel to the beach) is of the order of $1km$.
- Depth is of the order of a few meters (the algae do not live deeper than that).
- Perpendicular length (perpendicular to the beach) is of the order of $10 - 20m$.

The typical time scale is one day so $u(t, x)$ is always of

- Order 1 in the longitudinal coordinates
- Very large in the perpendicular coordinates
- Large or order 1 in the vertical coordinates, depending on the turbulence of the currents.

Homogenization!

In the end, we assume that the distribution f has the structure

$$f(t, x, v, n) = f(t, x_{\|}, v_{\|}, n, Re) \times M(z, v_z, Re)$$

where Re measures the turbulence of the current and M is a known equilibrium function.
Hence the distribution is still stratified if the sea is quiet.
With this it is possible to derive a resonable model, adding boundary conditions

- Exchange with the deep sea
- The original boundary conditions at the bottom and the surface are now included in the equation (by M).

Conclusions

- Complicated model, incorporating many different behaviours.
- The final model is awful but numerically solvable.
- Identification of the constants or known functions is very difficult and probably impossible in a complete way given the sparse knowledge of the algae.

$\longrightarrow$ Only the general behaviour can reasonably be deduced.