Fish migration, interacting particles, and scaling laws

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Kinetic and mean-field models in the socio-economic sciences

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Joint with Baldvin Einarsson

Background and motivation

- This problem is motivated by an interacting particle model of fish migration
- That model includes circular interaction zones around each particle
- Each particle averages the directional headings of each particle in its interaction zone
- Using this model together with environmental influences, we predicted the migration route of the Icelandic capelin in 2008 and reproduced the routes from 1985 and 1991

Our model

$$\begin{pmatrix} x_k(t+\Delta t) \\ y_k(t+\Delta t) \end{pmatrix} = \begin{pmatrix} x_k(t) \\ y_k(t) \end{pmatrix} + \Delta t \cdot v_k(t+\Delta t) \frac{\mathbf{d}_k(t+\Delta t)}{\parallel \mathbf{d}_k(t+\Delta t) \parallel}$$

- Here, $\mathbf{d}_k$ is the directional heading of particle k
- Δt is the timestep

The directional heading of the particles (based on interactions among particles) is determined as follows:

$$\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix} = \frac{\mathbf{d}_k(t+\Delta t)}{\| \mathbf{d}_k(t+\Delta t) \|}$$

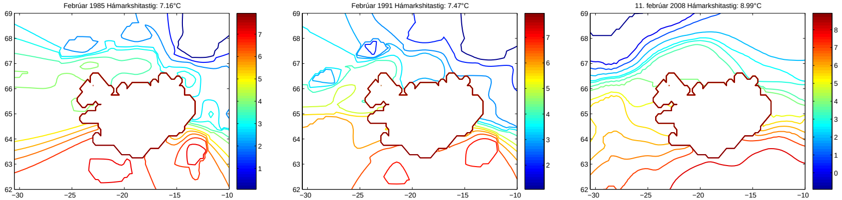
where

$$\mathbf{d}_k(t+\Delta t) := \frac{1}{|R_k| + |O_k| + |A_k|} \left(\sum_{r \in R_k} \frac{\mathbf{p}_k(t) - \mathbf{p}_r(t)}{\| \mathbf{p}_k(t) - \mathbf{p}_r(t) \|} + \sum_{o \in O_k} \begin{pmatrix} \cos(\phi_o(t)) \\ \sin(\phi_o(t)) \end{pmatrix} + \sum_{a \in A_k} \frac{\mathbf{p}_a(t) - \mathbf{p}_k(t)}{\| \mathbf{p}_a(t) - \mathbf{p}_k(t) \|} \right).$$

The problem of superindividuals

- In the real migrations which we are trying to accurately capture, it is safe to assume there are around $5 \cdot 10^{10}$ fish
- In our simulations, we use roughly $5 \cdot 10^{4}$ particles
- This means each particle represents 10^{6} fish
- Each particle must therefore be thought of as a superindividual
- With these superindividuals, we captured the migration

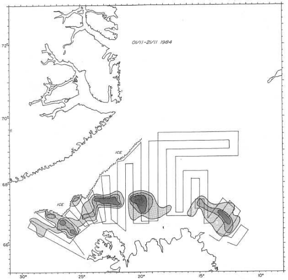
Temperature fields used in the simulations



February, 1985 (left), February 1991 (middle), February 2008[1] (right)

[1] http://www.wetterzentrale.de/topkarten/fsfaxsem.html

1984–1985

(a)


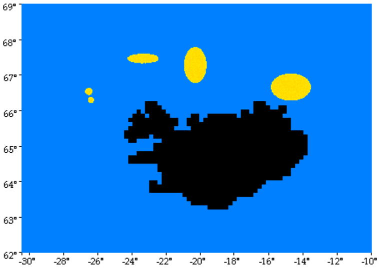
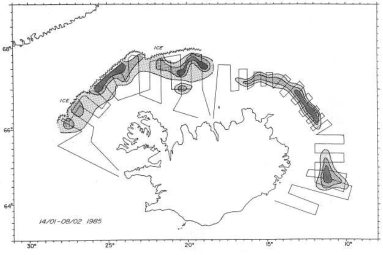
(b)


Figure: The distribution of capelin in November of 1984. a) Acoustic data from November 1 to November 21. b) Initial distribution for the simulation.

1984–1985

(a)


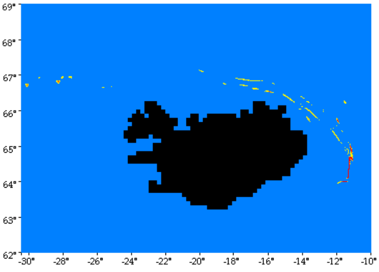
(b)


Figure: The distribution of capelin in mid-January to early February of 1985. a) Acoustic data from January 14 to February 8. b) Simulated distribution in mid-January, day 65 of the simulation.

1984–1985

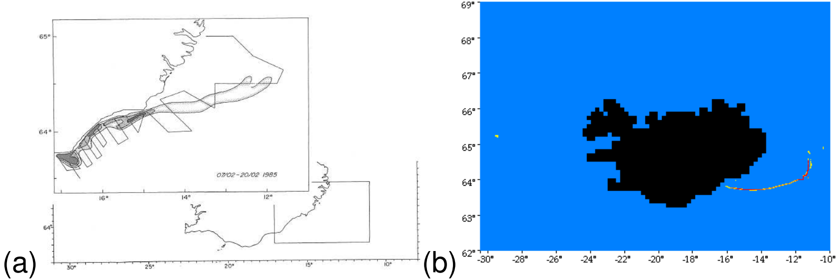
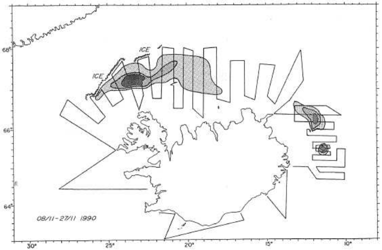


Figure: a) Close up of the distribution of capelin from February 7 to February 20 of 1985. b) Simulated distribution in late February, day 109.

1990–1991

(a)


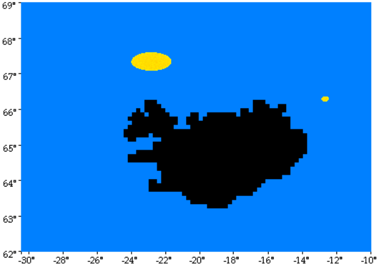
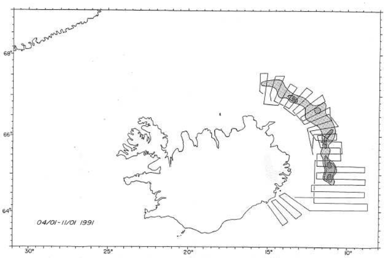
(b)


Figure: The distribution of capelin in November of 1990. a) Acoustic data from November 8 to November 27. b) Initial distribution for the simulation.

1990–1991

(a) 

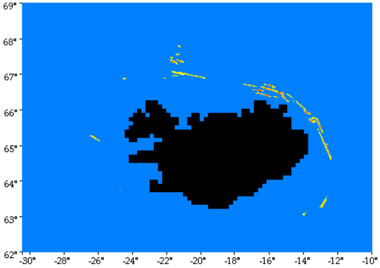
(b) 

Figure: The distribution of capelin in January of 1991. a) Acoustic data from January 4 to January 11. b) Simulated distribution in early January, day 44 of the simulation.

1990–1991

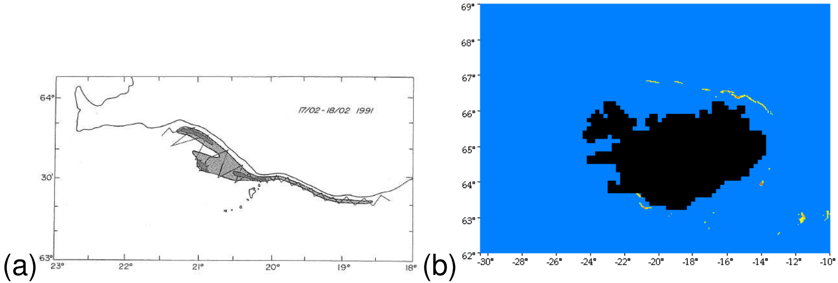


Figure: The distribution of capelin southwest of Iceland in February of 1991. a) Acoustic data from February 17 to February 18. b) Simulated distribution in mid-February, day 72.

2008

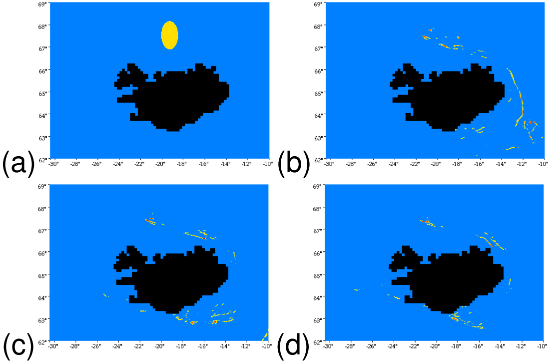


Figure: Simulation of the 2007-2008 spawning migration. a) Early January, day 0 b) Mid-February, day 47 c) Late February, day 59 d) Early March, day 65.

Acoustic data from 2008

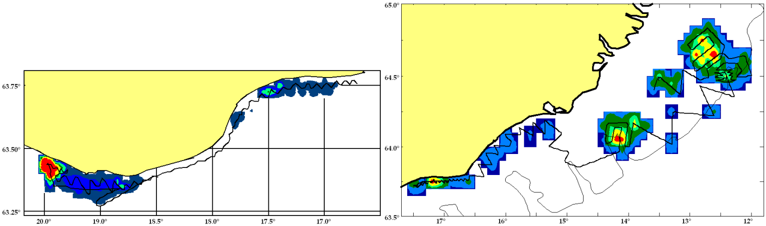


Figure: a) Measured distribution of capelin near the south coast of Iceland from February 26 to February 27 of 2008. b) Measured distribution of capelin near the southeast coast of Iceland from February 29 to March 3 of 2008.

The goal

- One fish per particle
- Then we could more confidently justify our behavioral rules, since they are based on data obtained from interactions among individual fish

So this leads to a question: how does the system change as we change the number of particles?

Assumptions

- We assume uniform density of particles and fish in the schools
- The interaction length of the particles should be much less than the size of the school
- We further assume the velocities of the particles are equal

Propagation of information through the school

- If sufficiently dense, local interactions between particles allows information to propagate through a school
  - Temperature information
  - Information about predators
  - Information about food
- Our scaling should preserve the speed at which this information propagates through the school

Relating time and spatial scales

- Let q_k denote the position and u_k denote the unit vector in the direction of the velocity of the k^{th} particle
- $\Delta q_k = \Delta t \cdot v \cdot u_k \Rightarrow \Delta q_k \propto \Delta t$
- To ensure particle k does not move beyond its range of vision in one timestep, we need a scaling between the radius of interaction, r, and the timestep
  - $r \propto \Delta t$
- We can think of Δq as defining a spatial scale

Constant density of actual fish

- In the actual migration, there are a given number of fish within a given area
- Each simulation needs to relate back to this real situation
- Schematic:
  - $\frac{fish}{region} = (\frac{particles}{interaction \cdot zone})(\frac{fish}{particle})(\frac{interaction \cdot zone}{region})$
- Let N denote the total number of particles in a simulation, F denote the number of fish in the migration, and A_w denote the total area of the region
- Let M denote the number of particles per interaction zone
  - Constant across interaction zones due to constant density assumption
  - For computational intensity, need M is constant across different simulations (so the number of neighbors for each particle remains constant)

Relating time and space to the number of particles

- Then for a given simulation indexed by i,
  $\frac{F}{A_w} = (M)(\frac{F}{N_i})(\frac{A_w}{\pi r_i^2}) \Rightarrow \frac{1}{A_w^2} = \frac{M}{(\pi r_i^2)N_i}$
- For two different simulations:
  - $\frac{M}{(\pi r_0^2)N_0} = \frac{M}{(\pi r_1^2)N_1} \Rightarrow (\frac{r_1}{r_0})^2 = \frac{N_0}{N_1}$
- $r_1 = r_0\sqrt{\frac{N_0}{N_1}}$
- Considering r_0 and N_0 to have come from a reference simulation:
  - $\Delta t \propto r \propto \sqrt{\frac{1}{N}}$

Our parameters for the migrations

- $\Delta t = 0.05$ days
- Initial speed $v_k \simeq 4 - 8$ km/day
- $r_r = 0.01$ or about ~ 120 m
- $r_o = r_a = 0.1$ or about ~ 1.2 km
- Number of particles is roughly $5 \cdot 10^4$

Scaling down to an individual level

How do the particles scale as we take N^s to 1? A rough estimate for the total number of fish in a migration is $F \simeq 5 \cdot 10^{10}$.

- $N_0 \simeq 5 \cdot 10^4$ and $N_1 \simeq 5 \cdot 10^{10}$
- $\frac{\Delta q_0}{\frac{1}{\sqrt{N_0}}} = \frac{\Delta q_1}{\frac{1}{\sqrt{N_1}}}$ and $\Delta q_0 \simeq 1.2$ km $\Rightarrow \Delta q_1 \simeq 1.2$ meters
- $\Delta t_0 = 0.05$ days and $\frac{\Delta t_0}{\Delta q_0} = \frac{\Delta t_1}{\Delta q_1} \Rightarrow \Delta t_1 = 4.32$ seconds
- Radii scale with Δq, so
  - $r_{r_0} \simeq 120$ meters $\Rightarrow r_{r_1} \simeq 12$cm
  - $r_{o_0} = r_{a_0} \simeq 1.2$ km $\Rightarrow r_{o_1} = r_{a_1} \simeq 1.2$m

  These are all biologically reasonable!

Future work

- Verify that these scaling laws allow us to reproduce solutions with different numbers of particles
- Use this scaling to derive a mean field equation for the original discrete model
- Compare the mean field model with the original model to see under what circumstances it is a good approximation

Thank you!