Real-time gauge/gravity duality

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Edinburgh
20 June 2008

Introduction

The main question I would like to address in this talk is:

- How do we set up the gravity/gauge theory duality in real-time?

One would like to set up a prescription as general as the Euclidean one. In particular, it should

- apply to any n-point function, including correlators in non-trivial states.
- apply to all QFTs with a holographic dual.
- the prescription should be fully holographic, i.e. only boundary data and regularity should suffice.
- Within the supergravity approximation, all information should be encoded in classical bulk dynamics.

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Motivation

Euclidean techniques suffice for many applications. However, it is clear that there are many reasons to set up the holographic prescription directly in Lorentzian signature. To mention a few:

1. holography for time-dependent backgrounds,
2. holographic description of non-equilibrium QFT,
3. computation of correlators in non-trivial states,
4. Holography vs causality,
5. Understanding the physics of black hole horizons,
6. etc. etc.

The development of a real-time formalism is also becoming urgent, as actual application, for example the modeling of the quark-gluon plasma in RHIC and LHC, require real-time techniques. Actually some of the previous work on the subject was driven by such applications [Son, Starinets], [Herzog, Son](2002)

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Outline

1. Review of Euclidean prescription
2. Lorentzian prescription
3. Examples
4. Conclusions

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References

The discussion here is based on

- KS, Balt van Rees, 0805.0150 and to appear

Previous work includes

- Balasubramanian, Kraus, Lawrence, 1998
- Maldacena, 2001
- Son, Starinets, 2002
- Herzog, Son, 2002
- Marolf, 2005

Basic Dictionary

Let us start by briefly reviewing the basics of holography. In the low energy approximation, where the bulk theory is approximated by supergravity the basic holographic dictionary is [GKP,W (1998)]:

1. There is 1-1 correspondence between local gauge invariant operators $\mathcal{O}$ of the boundary QFT and bulk supergravity modes Φ.
2. The fields $\phi_{(0)}$ parametrizing the boundary conditions of the bulk fields Φ are identified with the sources of dual operators.

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3. The fundamental relation between the bulk and boundary theories in **Euclidean signature** within the supegravity approximation is

$$Z_{SUGRA}[\phi_{(0)}] = \int_{\Phi \sim \phi_{(0)}} \mathcal{D}\Phi \exp\left(-S[\Phi]\right) = \langle \exp(-\int_{\partial M} \phi_{(0)} \mathcal{O}) \rangle_{QFT}$$

To leading order

$$S_{on-shell}[\phi_{(0)}, ...] = -W_{QFT}[\phi_{(0)}, ...]$$

on-shell SUGRA action = generating functional of QFT connected graphs

Such a relation is however formal as both sides diverge. On the QFT side these are the usual UV divergences, dealt with by standard renormalization techniques. On the gravitational side, the infinities are due to the infinite volume of the spacetime. This issue is dealt with by the formalism of holographic renormalization, which is the precise gravitational analogue of QFT renormalization. [Henningson, KS (1998)], ...

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Holographic renormalization

To understand holographic renormalization one needs to know some facts about asymptotically (locally) AdS spacetimes.

- These spacetimes solve the Einstein equations with a negative cosmological constant and have the following asymptotic (Fefferman-Graham) form

  $$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x,r) dx^i dx^j$$

  where

  $$g_{ij}(x,r) = \mathbf{g_{(0)ij}} + r^2 g_{(2)ij} + \ldots + r^d \left(\log r^2 h_{(d)ij} + g_{(d)ij}\right) + \ldots$$

  This is an expansion in r (the conformal boundary of the spacetime is located at $r = 0$).
- Matter fields, e.g. scalar fields, have a similar asymptotic expansion

$$\Phi(x,r) = r^{d-\Delta}\left(\phi_{(0)} + r^2 \phi_{(2)} + \ldots + r^{2\Delta-d}\left(\log r^2 \psi_{(2\Delta-d)} + \phi_{(2\Delta-d)}\right) + \ldots\right)$$

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- The asymptotic solution is determined by solving the Einstein equations perturbatively in r. This procedure does not depend on the spacetime signature and yields algebraic equations that can be solved to determine the asymptotic coefficients.
- The coefficients $g_{(2n)}$ with $2n < d$, $\phi_{(2k)}$ with $2k < 2\Delta - d$ and $h_{(d)}, \psi_{(2\Delta-d)}$ are determined locally in terms of $g_{(0)}, \phi_{(0)}$.
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Holographic Renormalization

Renormalized correlators can now be obtained as follows:[de Haro, KS, Solodukhin (2000)]

1. Regulate the divergences by restricting the radial coordinate to have a finite range.
2. Evaluate the action on the asymptotic solution.
3. Subtract the infinite terms by adding suitable local covariant counterterms.
4. Compute the holographic 1-point functions in the **presence of sources**.

→ This leads to a precise relation between correlation functions and asymptotics

$$\langle T_{ij} \rangle = \frac{2}{\sqrt{g_{(0)}}} \frac{\delta S^{ren}_{SUGRA}}{\delta g^{ij}_{(0)}} = \frac{d}{16\pi G} [g_{(d)ij} + X^{(d)}_{ij}(g_{(0)})].$$

where $X^{(d)}_{ij}(g_{(0)})$ are local functions of $g_{(0)}$.

$$\langle O_\Delta \rangle = \frac{1}{\sqrt{g_{(0)}}} \frac{\delta S^{ren}_{SUGRA}}{\delta \phi_{(0)}} = (2\Delta - d)\phi_{(2\Delta-d)}$$

→ Correlators satisfy all expected Ward identities,

$$\nabla^i \langle T_{ij} \rangle = \langle O_\Delta \rangle \partial_j \phi_{(0)}, \qquad \langle T^i_i \rangle = -(d-\Delta)\phi_{(0)} \langle O_\Delta \rangle + \mathcal{A}$$

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Holographic Renormalization: higher point functions

5 Since the first variation of the on-shell action was performed in complete generality, one may obtain higher-point functions by differentiating the 1-point functions w.r.t. sources and then set the sources to zero

$$\langle O_\Delta(x_1) O_\Delta(x_2) \cdots O_\Delta(x_n) \rangle \sim \left. \frac{\delta^{(n-1)} \phi_{(2\Delta-d)}(x_1)}{\delta\phi_{(0)}(x_2) \cdots \delta\phi_{(0)}(x_n)} \right|_{\phi_{(0)}=0}$$

6 Thus to solve the theory we need to know $\phi_{(2\Delta-d)}, g_{(d)}$ as a function of $\phi_{(0)}, g_{(0)}$.

→ In absence of more powerful techniques we proceed perturbatively: 2-point functions are obtained by solving linearized fluctuations, 3-point functions by solving quadratic fluctuations etc.

→ For this procedure to be well-posed these equations should have a unique solution given boundary data. This is indeed the case in the Euclidean set up, but not in the Lorentzian case. We will return to this issue later.

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Radial Hamiltonian formalism

- The method of holographic renormalization used so far is conceptually simple, but computationally inefficient as it does not exploit the underlying conformal structure.
- For most explicit computations, it is better to use the radial Hamiltonian formalism, a Hamiltonian formulation in which the radius plays the role of time.
- One relates the regularized holographic 1-point of an operator $\mathcal{O}_\Phi$ to the radial canonical momentum π_Φ of the corresponding bulk field Φ [de Boer,Verlinde^2], [Papadimitriou, KS].

$$\begin{aligned} \delta S &= \int dr \left(\frac{\partial L}{\partial \Phi} - \partial_r \frac{\partial L}{\partial(\partial_r \Phi)} \right) \delta\Phi + \left[\frac{\partial L}{\partial(\partial_r \Phi)} \delta\Phi \right]_r, \qquad L \equiv \int d^d x \sqrt{G} \mathcal{L} \\ &\Rightarrow \frac{\delta S_{on-shell}}{\delta \Phi} = \frac{\partial L}{\partial(\partial_r \Phi)} \equiv \pi_\Phi \end{aligned}$$

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Radial Hamiltonian formalism: renormalization [Papadimitriou, KS (2004)]

One still has to renormalize ....

- A fundamental property of asymptotically locally AdS spacetimes is that scale transformations are part of the asymptotic symmetries and therefore every covariant quantity can be decomposed into a sum of terms each having a definite scaling.
- Thus the canonical momenta of a field dual to a dimension k operator are asymptotically expanded as

$$\pi^k = \pi^k_{(d-k)} + \cdots + \pi^k_{(k)} + \tilde{\pi}^k_{(k)} \log r + \cdots$$

  with each coefficient $\pi^k_{(n)}$ having weight n.
- Each coefficient can be expressed (non-linearly) in terms of the asymptotic expansions, but the holographic 1-point functions are more naturally expressed in terms of the coefficients of π^k,

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Lorentzian Issues

Let us summarize the special issues that arise in the Lorentzian set up:

1. In the Lorentzian case one has to specify initial and final conditions as well $\phi_\pm$. So the on-shell action, $S_{onshell}[\phi_{(0)}, \phi_\pm]$, depends not only $\phi_{(0)}$ but also of $\phi_\pm$.
2. The variation of the on-shell supergravity action appears to pick up additional contributions from $t = \pm\infty$,

$$\delta S_{onshell} = [\pi_r \delta\Phi]_r + [\pi_t \delta\Phi]_{t=\infty} - [\pi_t \delta\Phi]_{t=-\infty}$$

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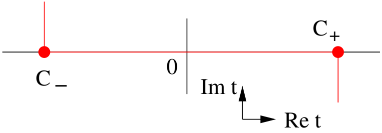
QFT interlude

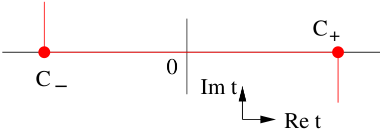
Consider the QFT path integral

$$\int_{\Psi(\vec{x},t=\pm T)=\psi_\pm(\vec{x})} [\mathcal{D}\Psi] e^{iS[\Psi]}$$

This computes the transition amplitude $\langle \psi_+(\vec{x}), T | \psi_-(\vec{x}), -T \rangle$. To compute vacuum-to-vacuum amplitudes we multiply with the wavefunctions $\langle \psi_-(\vec{x}), -T | 0 \rangle$, $\langle 0 | \psi_+(\vec{x}), T \rangle$ and integrate over $\psi_\pm$. The insertions of these wavefunctions is equivalent to extending the fields in the path integral to live along the red contour in the complex time plane:



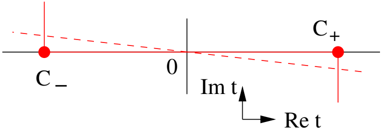
Remarks



- The infinite vertical segments represent the wavefunctions $\langle \psi_-(\vec{x}), -T|0\rangle$, $\langle 0|\psi_+(\vec{x}), T\rangle$ as Euclidean path integrals,

$$\langle \psi_-(\vec{x}), -T|0\rangle = \lim_{\beta \to \infty} \langle \psi_-(\vec{x}), -T|e^{-\beta H}|\psi\rangle$$

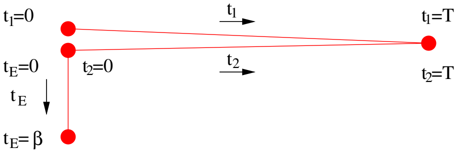
- These wavefunctions are ultimately lead to $i\epsilon$ factors in the Feynman propagator.



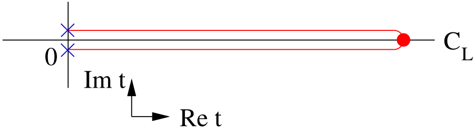
Remarks

Correlators in non-trivial states, thermal ensembles etc. can be obtained by using different time contours. E.g.

- the real-time real contror is



- the in-in contour, used to calculate correlators $\langle in|O\cdots O|in\rangle$, is



Lorentzian prescription

The holographic prescription is now to use "piece-wise" holography:

- Real segments are associated with Lorentzian solutions,
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- Solutions are matched at the corners.

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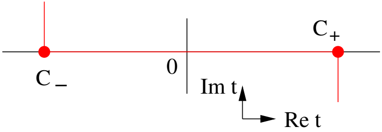
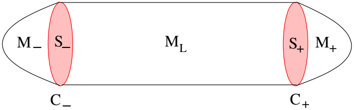
Vacuum-to-vacuum amplitudes

To illustrate the prescription, consider vacuum-to-vacuum amplitudes for CFT_d. Corresponding to the time-contour

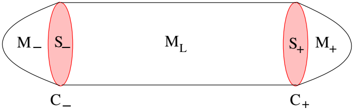


we consider the following solution:



Here M_L is Lorentzian AdS_{d+1} and the two caps $M_\pm$ are half of Euclidean AdS_{d+1} spaces.

Matching conditions



- Induced values of the bulk fields are continuous across $S_\pm$.
- The combined on-shell supergravity actions should be stationary w.r.t. variations with respect to $\phi_\pm$:

$$\frac{\delta}{\delta\phi_\pm}\Big(iI_L[\phi_{(0)},\phi_-,\phi_+] - I_E[\phi_{(0,-)},\phi_-] - I_E[\phi_{(0,+)},\phi_+]\Big) = 0$$

→ The matching conditions are equations for $\phi_\pm$.

→ Using the Hamilton-Jacobi relation the last condition becomes the standard Israel matching condition

$$i\pi_t|_{S_-} = \pi_\tau|_{S_-}, \qquad i\pi_t|_{S_+} = \pi_\tau|_{S_+}$$

Fundamental bulk-boundary relation

The fundamental relation between bulk and boundary quantities reads

$$\langle 0|T\exp\left(i\int_{M_L} d^dx\sqrt{-g}\,\phi_{(0)}\mathcal{O}\right)|\mathbf{0}\rangle = \exp\left(iI_L[\phi_{(0)},\phi_-,\phi_+]-I_E[0,\phi_+]-I_E[\mathbf{0},\phi_-]\right)$$

- In this expesssion $\phi_\pm$ are the values determined via the matching conditions.
- We have set $\phi_{(0,-)}=\phi_{(0,+)}=0$ since we are interested in vacuum-to-vacuum correlators. One can consider non-trivial *in* and *out* states by turning on these sources.

→ This is a fully holographic prescription. Everything is determined by boundary conditions and regularity in the interior.

→ The Euclidean caps can be also be thought of as Hartle-Hawking wavefunctions.

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Correlators

- Having set up the prescription one can verify that there are no additional ambiguities.
- A well known problem in the computation of 2-point functions is that the linearized field equations do not have a unique solution with Dirichlet boundary conditions.
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Massive scalar field in AdS_3

As the simplest yet illustrative example we consider a free massive scalar field in AdS_3,

$$S = \frac{1}{2} \int d^3x \sqrt{|G|} (-\partial_\mu \Phi \partial^\mu \Phi - m^2 \Phi^2).$$

The dimension of $\mathcal{O}$ is $\Delta = 1 + \sqrt{1 + m^2} = 1 + l$ with $l \in \{0, 1, 2, \ldots\}$.

We want to solve

$$(\Box - m^2) \Phi(t, \phi, r) = 0$$

in the AdS$_3$ background,

$$ds^2 = -(r^2 + 1) dt^2 + \frac{dr^2}{r^2 + 1} + r^2 d\phi^2 \,,$$

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Solution of $(\Box - m^2)\Phi(t,\phi,r) = 0$

The solution to this equation is well known:

- Non-normalizable modes: $\Phi(t,\phi,r) \sim e^{-i\omega t + ik\phi} f(\omega, \pm k, r)$ with

$$f(\omega,k,r) \sim r^{l-1} + \ldots + r^{-l-1}\alpha(\omega,k,l)[\ln(r^2) + \beta(\omega,k,l)] + \ldots$$

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Most general solution with prescribed boundary data

Thus the most general solution that is regular in the interior and whose leading asymptotics ($\sim r^{l-1}$ as $r \to \infty$) contain an arbitrary source $\phi_{(0)}(t,\phi)$ for the dual operator is

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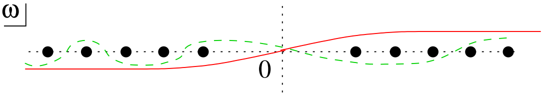
Choice of contour



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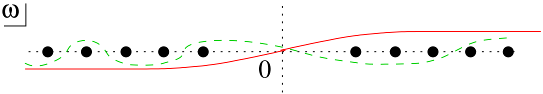

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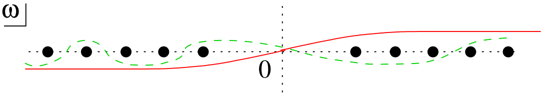

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Euclidean solutions

We will now show that the matching conditions determine $c^{\pm}_{nk}$.

- Consider the solution on the 'initial cap', so on the space specified by the metric,

  $$ds^2 = (r^2+1)d\tau^2 + \frac{dr^2}{r^2+1} + r^2 d\phi^2$$

  with $-\infty < \tau \leq 0$, so that we have half of Euclidean AdS space.
- Had the bulk been the entire Euclidean AdS space, the Klein-Gordon equation would have a unique regular solution given boundary data. In particular, with zero sources the unique regular solution is identically equal to zero.
- In our case the sources are zero but we only consider half of the space, so solutions that would be excluded are now allowed because they are only singular at the other half of the space,

  $$\Phi(\tau,\phi,r) = \sum_{n,k} d^{-}_{nk} e^{-\omega^{-}_{nk}\tau + ik\phi} g(\omega_{nk}, |k|, r)\,,$$

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2-point function

- Following our earlier discussion we can now extract the 2-point function from r^{-l-1} term in the asymptotic expansion of full solutions. This leads to

$$\langle 0|T\mathcal{O}(t,\phi)\mathcal{O}(0,0)|0\rangle = \frac{l+1}{4\pi^2 i}\sum_k \int_{\mathbf{C}} d\omega e^{-i\omega t+ik\phi}\alpha(\omega,|k|,l)\beta(\omega,|k|,l).$$

  with the contour C being the same as for the bulk solution, which was completely fixed by the matching to the caps. This is the standard Feynman prescription leading to time-ordered correlators.

- Performing the ω integral leads to

$$\langle 0|T\mathcal{O}(t,\phi)\mathcal{O}(0,0)|0\rangle = \frac{C_l}{[\cos(t-i\epsilon t)-\cos(\phi)]^{\Delta}},$$

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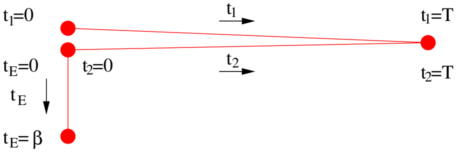
Thermal 2-point function from Thermal AdS

A fairly straightforward extension is the computation of the thermal 2-point function using a scalar field in thermal AdS. The relevant time contour is



and this implies the following matching conditions

$$\begin{aligned}
\Phi_1(0,\phi,r) &= \Phi_E(\beta,\phi,r)\\
\partial_{t_1}\Phi_1(0,\phi,r) &= i\partial_{t_E}\Phi_E(\beta,\phi,r)\\
\Phi_1(T,\phi,r) &= \Phi_2(T,\phi,r)\\
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\Phi_2(0,\phi,r) &= \Phi_E(0,\phi,r)\\
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\end{aligned}$$

Thermal 2-point function from thermal AdS

Carrying out the computation for both operators inserted in the first real segment leads to

$$\langle 0|T\mathcal{O}(t,\phi)\mathcal{O}(0,0)|0\rangle_{\beta} = \sum_{n\in\mathbb{Z}} \frac{C_l}{[\cos(t+in\beta)-\cos(\phi)]^{\Delta}} \ ,$$

- This is a sum over images in imaginary time of the zero temperature result, as it should be, since thermal AdS is obtained by identification in the time direction of global AdS.
- It satisfies the Kubo-Martin-Schwinger (KMS) condition.
- Considering operators inserted on both real segments results in the 2×2 matrix of Schwinger-Keldysh propagators.

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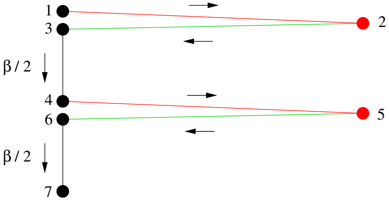
Thermal 2-point function from the BTZ black hole

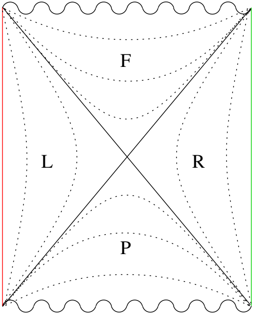
A more challenging example is the computation of the thermal propagator using a scalar field in the non-rotating massive BTZ black hole. It is more convenient to use the following thermal contour:

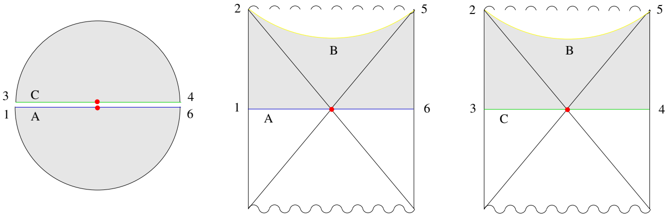


This a more convenient choice because it is easier to solve the various matching conditions. To fill in this contour we need two copies of half of the Lorentzian eternal BTZ and two copies of half of the Euclidean BTZ.

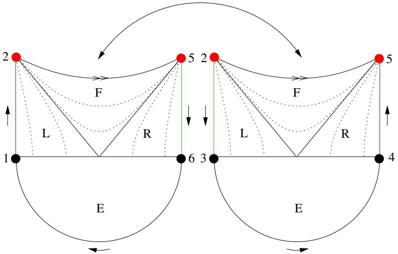
Bulk solution corresponding to time contour

The Penrose diagram for the eternal BTZ black is





Left figure: $\phi = 0$ slice: $\tau = 0$ at points 1,3 and $\tau = \beta/2$ at points 4,6.



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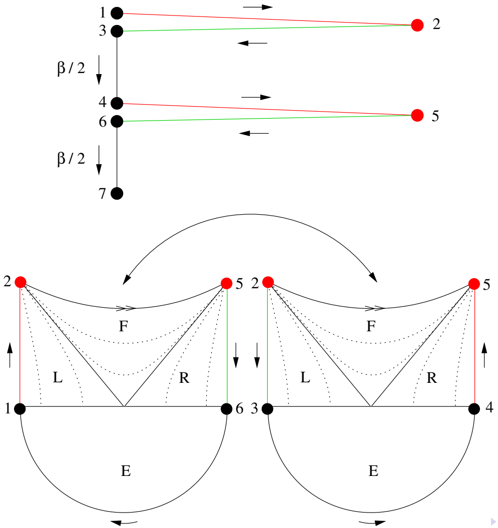

2-point function

- The 2-point function is

$$\langle T\mathcal{O}(t,\phi)\mathcal{O}(0,0)\rangle_{\beta} \sim \sum_{m\in\mathbb{Z}} \frac{1}{[\cosh(t)-\cosh(\phi+2\pi\sqrt{M}m)]^{l+1}}$$

  where M is the mass of the BTZ black hole.
- This is also a sum over images reflecting the fact that the BTZ is a quotient of AdS_3.
- The result agrees with results in the literature (obtained using the fact that BTZ is the quotient of AdS_3) and obeys the KMS condition.
- The matching conditions imply the "natural boundary conditions" at the horizon, namely positive frequency are in-going and negative frequency modes are out-going at the horizon in the R quadrant, which was the starting point in the analysis of [Herzog, Son](2002).

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Concluding remarks

- We have present a general prescription for holographic computation in real time.
- The prescription amounts to "filling-in" the complex time contour with bulk solution: real segments with Lorentzian solutions and imaginary segments with Euclidean solutions.
- This prescription fulfils all requirements described earlier: it allows for computation of n-point functions in any holographic QFT and in non-trivial states. It is fully holographic and all information is encoded in classical bulk dynamics.
- This prescription also offers a new perspective on the holographic encoding of bulk spacetimes, since the state or density matrix corresponding to a given geometry is directly related to the Euclidean parts of the solution. This may allow us to understand how regions beyond bulk horizons are 'encoded' in the QFT data.

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