AdS Solutions Through Transgression

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Supersymmetric AdS/CFT Solutions

Programme: characterise general classes of supersymmetric warped product solutions of D=11 or type IIB supergravity of the form: $AdS_{d+1} \times_w M$:

$$ds^2 = e^{2A(y)}[ds^2(AdS_{d+1}) + ds^2(M)(y)]$$

with fluxes preserving isometries of AdS.

Solutions are invariant under $SO(d,2) \Leftrightarrow$ characterise the most general SCFTs in d spacetimes that have a supergravity description.

Motivation

⋆ SCFTs are interesting for many reasons.

⋆ AdS_3, AdS_2 solutions $\leftrightarrow$ blackholes.

⋆ Can analytically continue $\rightarrow$ "BPS bubbles" dual to smooth BPS states of SCFTs (LLM) Also: Wilson Lines Lunin, D'Hoker, Gutperle

Programme

1. Use G-Structure techniques JPG, Martelli, Pakis, Waldram ("Invariant Tensor Method", "Spinorial Geometry", ...) to give a precise characterisation of geometry ($ds^2(M)$, A, fluxes). Prove Theorems.

2. In favourable cases construct explicit solutions and try to identify the SCFTs.

e.g. $AdS_5 \times SE_5$ solutions in type IIB and N=1 SCFTs.

Special Cases

AdS_3 in Type IIB sugra:

$$\begin{aligned} ds^2 &= e^{-B/2}[ds^2(AdS_3) + ds^2(Y_7)] \\ F_5 &= (1+*)Vol(AdS_3) \wedge F_2 \end{aligned}$$

Dual to N=(0,2) SCFTs in D=2: D3-branes on 2-cycles in CY_4.

AdS_2 in D=11 sugra:

$$\begin{aligned} ds^2 &= e^{-2B/3}[ds^2(AdS_2) + ds^2(Y_9)] \\ G_4 &= Vol(AdS_2) \wedge F_2 \end{aligned}$$

Dual to N=2 SCQM: M2-branes wrapped on 2-cycles in CY_5.

Geometry given by $ds^2(Y_{2n+1}), F_2, B$.

Also: 1/8 BPS Bubbles with $SO(4)$ or $SO(3)$ symmetry that generalise LLM .

There is always a Killing vector - dual to R symmetry

Locally, metrics can be written Kim

$$ds^2_{2n+1} = \frac{(n-2)^2}{4}(dz+P)^2 + e^B ds^2(B_{2n})$$

where B_{2n} is Kähler and satisfies

$$\Box R - \frac{1}{2}R^2 + R_{ij}R^{ij} = 0 \qquad (*)$$

which arises from the Bianchi identity for the five-form $dF_5 = 0$.

Furthermore, e^B and F_2 are fixed by Kähler geometry and $dP = \mathcal{R}$.

Many similarities to $AdS_5 \times SE_5$ solutions of type IIB and $AdS_4 \times SE_7$ solutions of D=11.

$$ds^2(SE_{2n+1}) = (dz + P)^2 + ds^2(B_{2n})$$

where B_{2n} is Kähler and Einstein and $dP = \mathcal{R}$.

Can we switch on additional fluxes? Recall Klebanov and Strassler: start with $AdS_5 \times T^{1,1}$ solution of type IIB. Observe that the cone metric C over $T^{1,1}$ is the Calabi-Yau conifold metric. Then observe that there is a $\mathbb{R}^{1,3} \times_w C$ solution with $G \neq 0$ provided that $dG = 0$, G is $(1,2)$ and $J \wedge G = 0$. Solution boils down to solving an equation for a single function that arises from the Bianchi identity with "transgression terms"

$$dF_5 = \frac{i}{2} G \wedge G^*$$

"Brane Resolution Through Transgression" Cvetic, Lu, Pope

AdS_3 in Type IIB sugra:

$$\begin{aligned} ds^2 &= e^{-B/2}[ds^2(AdS_3) + ds^2(Y_7)] \\ F_5 &= (1+*)Vol(AdS_3) \wedge F_2 \\ G &\neq 0 \end{aligned}$$

Dual to N=(0,2) SCFTs in D=2:

$$ds_7^2 = \frac{1}{4}(dz + P)^2 + e^B ds^2(B_6)$$

Require G to be a closed, primitive, $(1,2)$ form on B_6. If such a form exists the entire solution boils down to solving

$$\Box R - \tfrac{1}{2}R^2 + R_{ij}R^{ij} + \frac{2}{3}G^{ijk}G^*_{ijk} = 0 \qquad (*)$$

with $dP = \mathcal{R}$. which arises from solving the modified Bianchi identity $dF_5 = \frac{i}{2}G \wedge G^*$.

AdS_2 in D=11 sugra:

$$
\begin{aligned}
ds^2 &= e^{-2B/3}[ds^2(AdS_3) + ds^2(Y_7)] \\
G_4 &= Vol(AdS_2) \wedge F_2 + F_4
\end{aligned}
$$

Dual to N=(0,2) SCFTs in D=2:

$$ds_9^2 = (dz + P)^2 + e^B ds^2(B_8)$$

Require F_4 to be a closed, primitive, $(2,2)$ form on B_8. If such a form exists the entire solution boils down to solving

$$\Box R - \frac{1}{2}R^2 + R_{ij}R^{ij} + \frac{1}{4!}F_4^{ijkl}F_{4ijkl} = 0 \qquad (*)$$

with $dP = \mathcal{R}$. which arises from solving the modified Bianchi identity $dG_4 = \frac{1}{2}G_4 \wedge G_4$.

Explicit Solutions

Adapt constructions of Sasaki-Einstein metrics $Y^{p,q}$ JPG, Martelli, Sparks, Waldram

Use Page, Pope to build Kähler bases by considering line bundles over Kähler-Einstein manifolds and then try and solve (*).

In more detail

$$ds^2_{2n} = \frac{1}{U}d\rho^2 + U\rho^2 D\phi^2 + \rho^2 ds^2(KE_{2n-2})$$

where $(D\phi) = 2J_{KE}$. Demanding that it solves $(*)$ leads to solving a second order non-linear ODE for U. Remarkably it admits polynomial solutions!

Type IIB: $G = 0$

Infinite classes of solutions when we use ds^2_6 built from $KE_4 = CP^2$, $KE_4 = S^2 \times S^2$ or $B_4 = S^2 \times H^2$.

Type IIB: $G \neq 0$

Let $ds^2_6 = ds^2_4 \times T^2$ where ds^2_4 is built from a line bundle over S^2. Then set

$$G = d\bar{u} \wedge d(\frac{q}{\rho^2} D\phi)$$

Leads to infinite new solutions parametrised by q.

For these solutions with a T^2 factor we can T-dualise on one leg and then on the other leg of the T^2 to get new type IIB solutions. After a further S-duality we find that the solutions are constructed from NS fields only!

$$\begin{aligned} ds^2 &= ds^2(AdS_3) \times_w ds^2(X_7) \\ e^{2\phi} &\neq 0 \\ H_3 &\neq 0 \end{aligned}$$

Furthermore $X^7 = S^3 \times S^3 \times S^1$.

Final Comments

$\star$ Studied general class of AdS_3 solutions of type IIB and AdS_2 solutions of D=11 supergravity.

$\star$ Many similarities with Sasakian Einstein Geometry.

$\star$ Motivated generalisations with $G \neq 0$ and magnetic F_4.

$\star$ Motivated construction of explicit solutions - infinite new examples both for type IIB and D=11.

$\star$ For type IIB there are duality frames where some of the solutions are constructed from NS field only: perhaps we can make progress on constructing the bulk CFT?

$\star$ What are dual $d = 2$ (0,2) SCFTs? Are they related to SCFTs that are dual to SE? In some cases, it appears that switching on the three-form corresponds to exactly marginal deformations (there are also additional beta deformations).

$\star$ Bubble solutions.

$\star$ Extensive classification of supersymmetric AdS_n geometries, for various n in type IIB and $D = 11$. but more to be done. For some classes there are analogously rich classes of explicit solutions - to do with the fact that the solutions are complex?