

# New phases of black holes in higher dimensions

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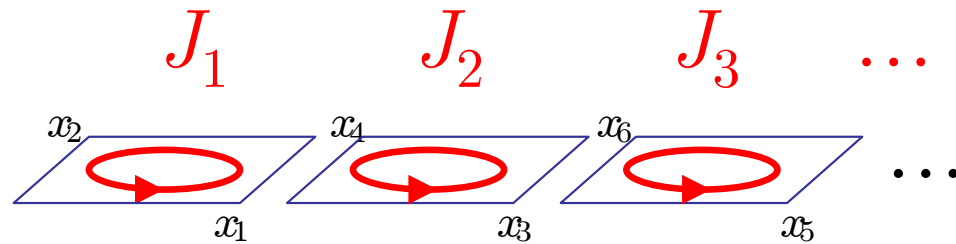
- Review w/ H. Reall: 0801.3471 [hep-th]
- M.Caldarelli, RE, M.J.Rodríguez: 0806.1954 [hep-th]
- RE, T.Harmark, V.Niarchos, N.Obers to appear

- Recently (~7 yrs ago) we've fully realized how *little* we know about black holes (even **classical** ones) and their dynamics in **D>4**
- Progress will have impact on:
  - String / M theory
  - Large Extra Dimensions & TeV gravity
  - AdS/CFT
  - Mathematics of Lorentzian geometry
  - **General understanding of spacetime at its most extreme**
- Activity launched initially by:
  - **GL-instability**, its endpoint, and inhomogeneous phases
  - **Black rings**, non-uniqueness, non-spherical topologies
- This talk:
  - First: vacuum,  $R_{\mu\nu} = 0$ , asymptotically flat solutions
  - Then, AdS etc

# FAQ's

- Why is  $D > 4$  richer?

- More degrees of freedom
- Rotation:



- more rotation planes
- gravitational attraction  $\Leftrightarrow$  centrifugal repulsion

- $\exists$  extended black objects: black p-branes

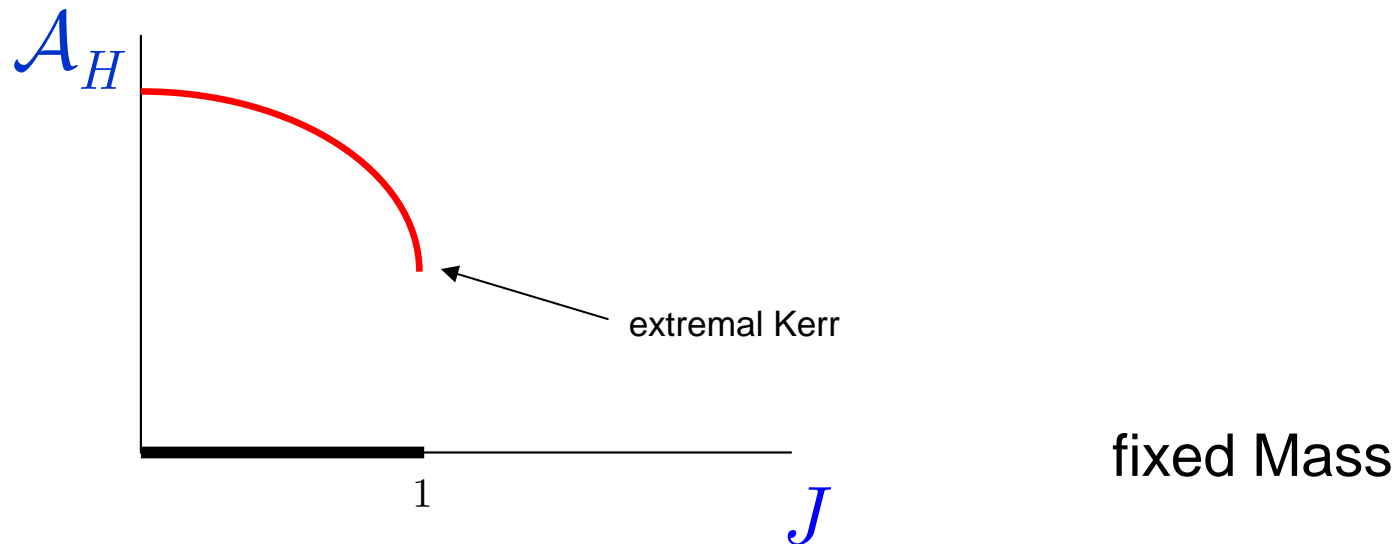
$$-\frac{GM}{r^{D-3}} + \frac{J^2}{M^2 r^2}$$

- Why is  $D > 4$  harder?

- More degrees of freedom
- Axial symmetries:  $U(1)$ 's at asymptotic infinity appear only every 2 more dimensions -- not enough to reduce to 2D  $\sigma$ -model if  $D > 5$

# Phases of 4D black holes

- Just the Kerr black hole: Uniqueness thm

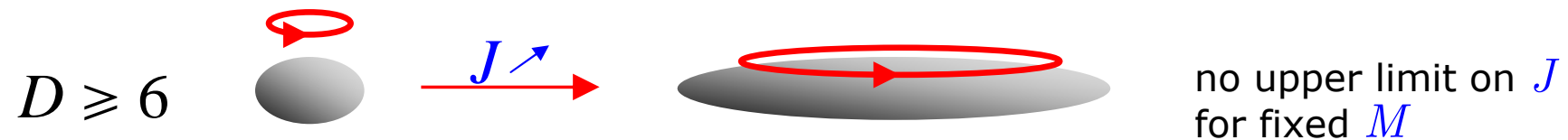
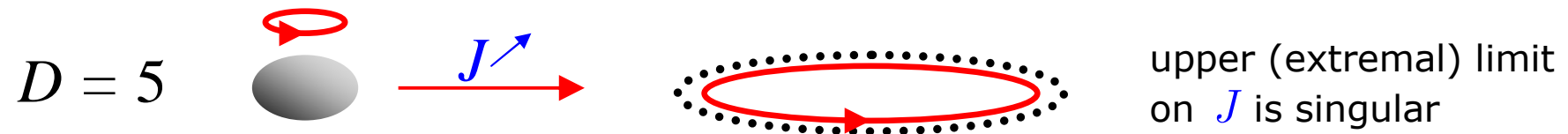


- **End of the story!**

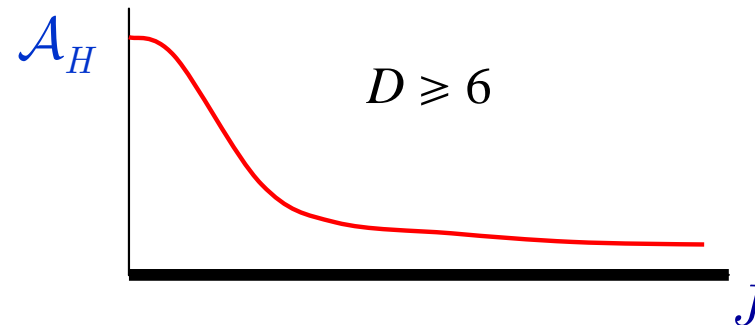
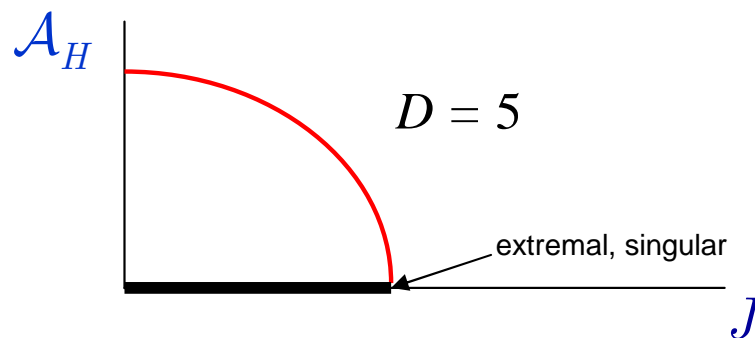
Multi-bhs not rigorously ruled out, but physically unlikely to be stationary (eg multi-Kerr can't be balanced)

# Myers-Perry black holes in $D$ dimensions

- They all have spherical topology  $S^{D-2}$
- Consider a **single spin**:

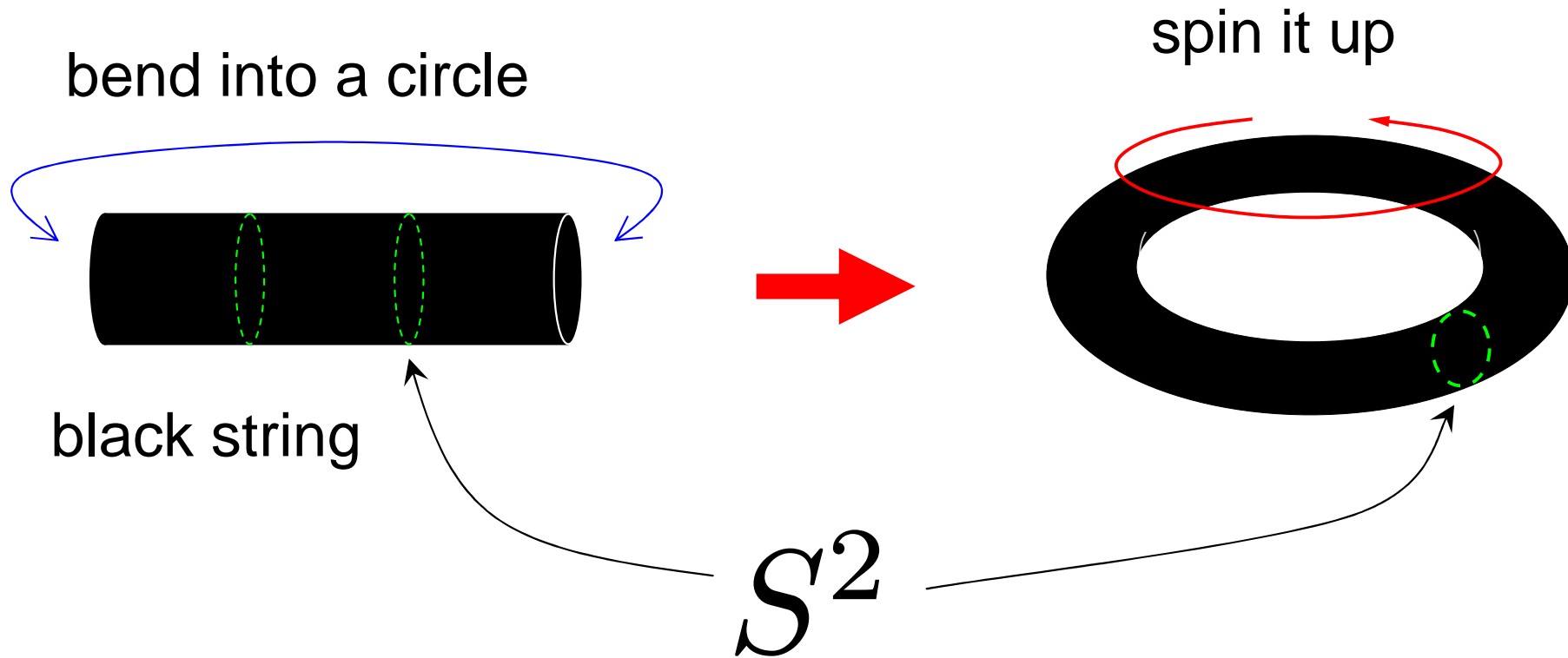


ultraspinning bh  $\rightarrow$  black membrane



But there's  
much more...

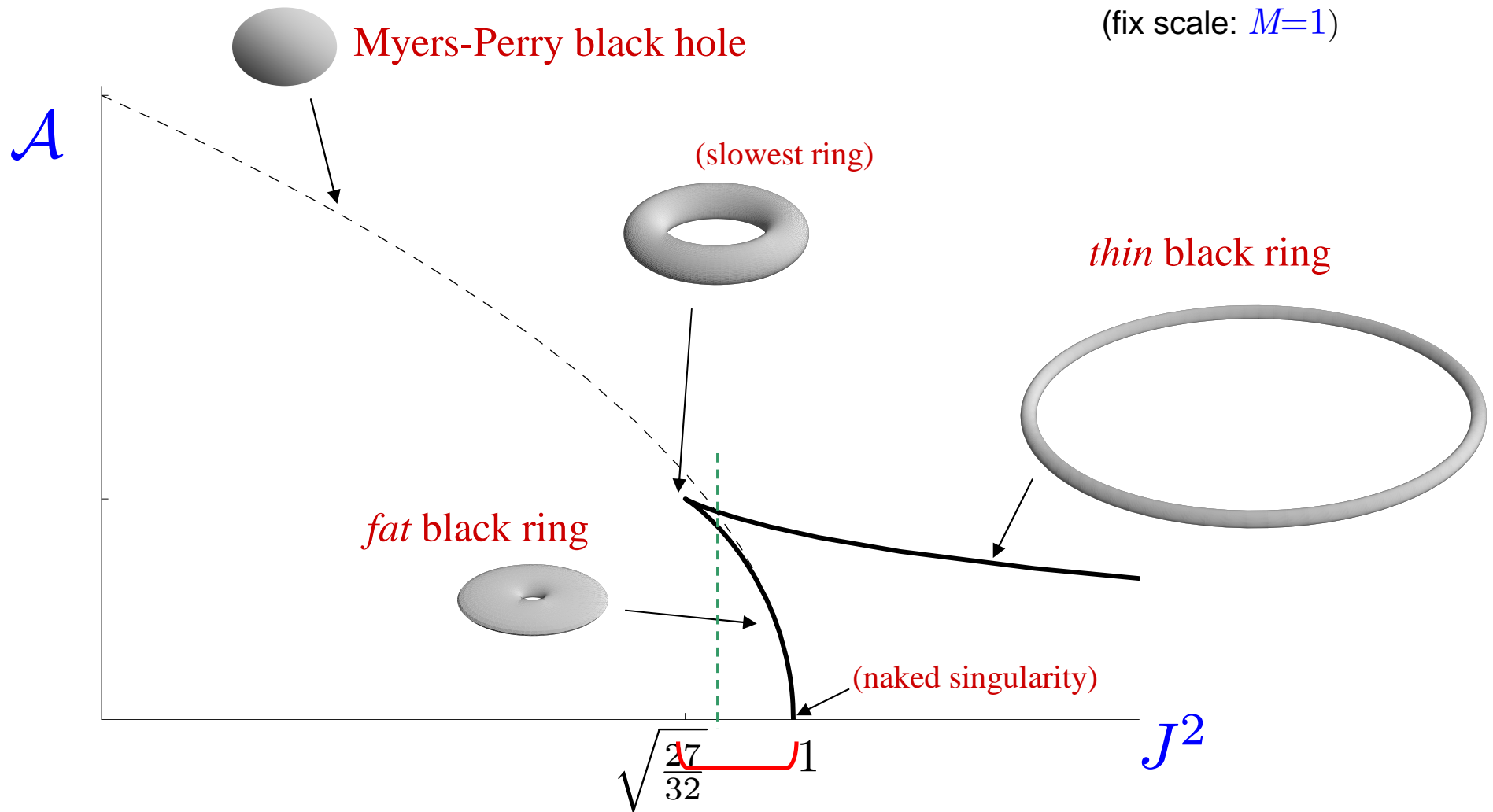
# The forging of the ring (in $D=5$ )



There's explicit solution, and fairly simple

*RE + Reall 2001*

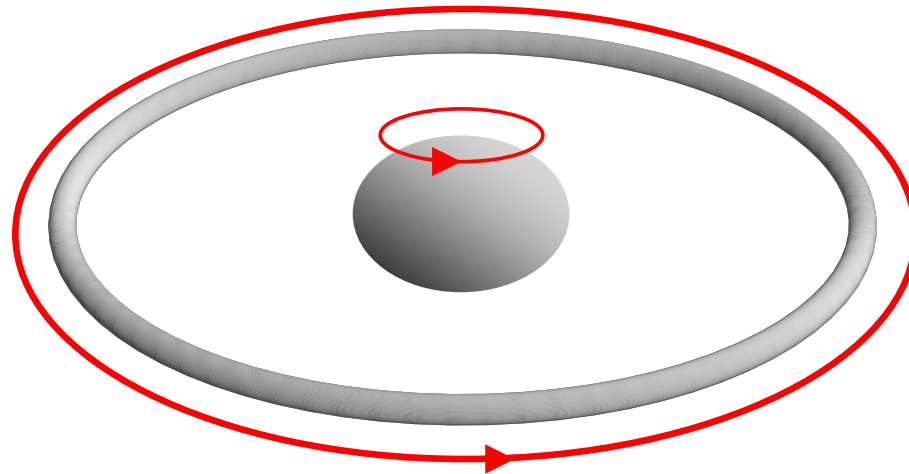
# 5D: one-black hole phases



**3** different black holes with the same value of  $M, J$

# Multi-black holes

- *Black Saturn:*



- Exact solutions available
- Co- & counter-rotating, rotational dragging...

*Elvang+Figueras*

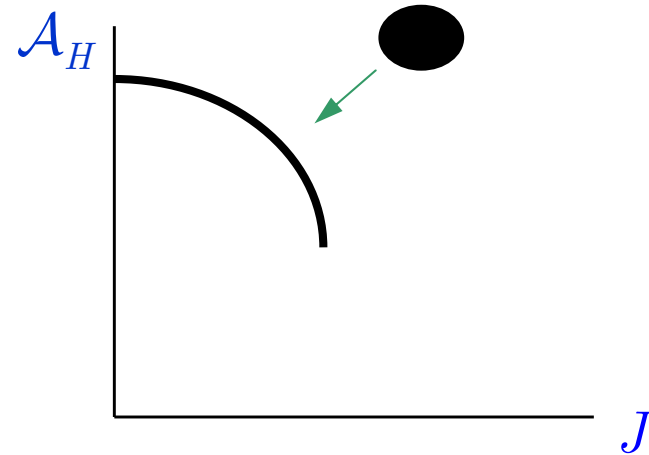
- Up to iterations of multi-rings, and addition of second angular momentum (not discussed today), this *might* be all...
- ...if two rotational symmetries are required:

$$\mathbb{R}_t \times U(1)_{\phi_1} \times U(1)_{\phi_2} \rightarrow \text{complete integrability}$$

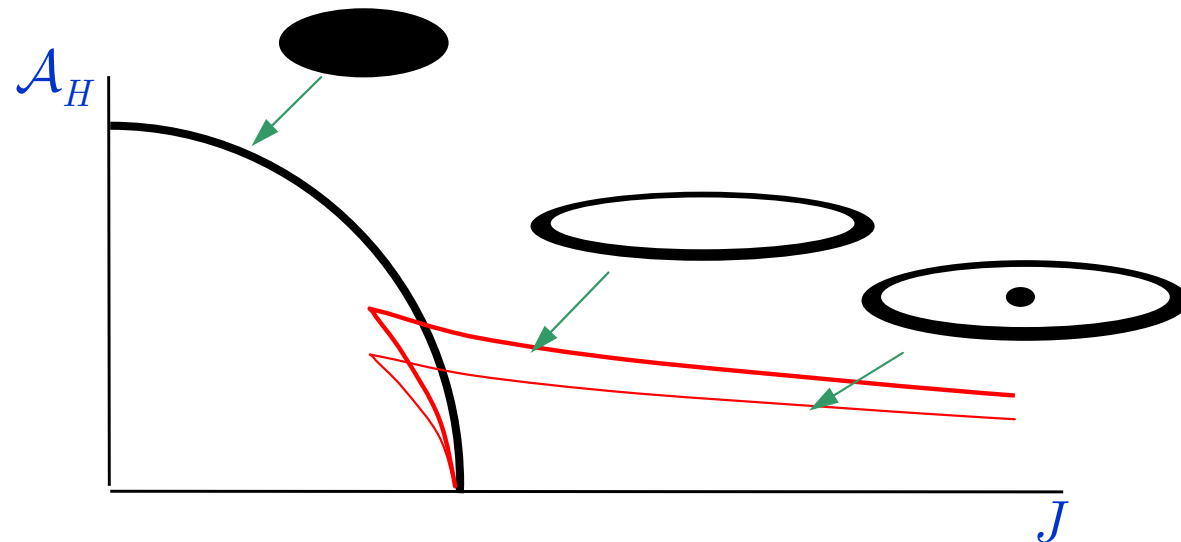
- But: stationarity  $\rightarrow$  one axial  $U(1)$ , but not (yet?) necessarily two *Hollands et al*  
(new solutions with broken  $U(1)$ ? where/how?)
- This remains the **main open issue** in the classification of 5D black holes (vacuum)

# The plot thickens...

$D=4$



$D=5$



# Road blocks in $D \geq 6$

## No solution-construction techniques

- Newman-Penrose formalism: unwieldy in  $D > 4$
- Integrability of Weyl class w/  $\mathbb{R}_t \times U(1)^{D-3}$  symm:  
only helps in  $D=4, 5$  :

AF black holes have at most

$$\mathbb{R}_t \times O(D-1) \supset \mathbb{R}_t \times U(1)^{\lfloor (D-1)/2 \rfloor} \quad \left\lfloor \frac{D-1}{2} \right\rfloor = D-3$$

- Kerr-Schild class:  $\Rightarrow D = 4, 5$

MP black holes are K-S, but black rings are not

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H(x)k_\mu k_\nu$$

# Horizon topology

- Hawking's 4D theorem relies on Gauss-Bonnet thm:

$$\int_H R^{(2)} > 0 \Rightarrow H = S^2$$

- $D=5$ : Galloway+Schoen:  $S^3, S^1 \times S^2$  (& quotients): +ve Yamabe
- $D=6$ : Helfgott et al:  $S^4, S^2 \times S^2, S^1 \times S^3, \Sigma_g \times S^2$   
so far:  $S^4$  exactly (MP, but possibly others too)  
 $S^1 \times S^3$  approximately
- $D > 6$ : essentially unknown (in progress)

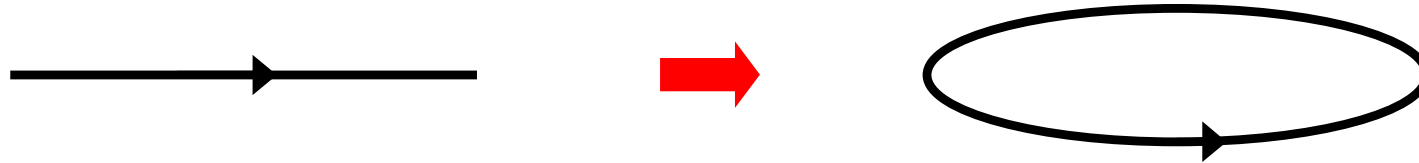
# ***Blackfolds***

*RE+Harmark+Niarchos+Obers*

- **Black** branes w/ worldvolume = curved submanifold of spacetime
- Analogy: D-branes w/ non-trivial worldvolume geometry arise as solutions of DBI action
- Can this be extended to black branes?
- Yes: we can have a *general theory of classical brane dynamics*, including ***thin black branes*** w/ regular horizons

- Simplest blackfold:

thin black ring = circular (boosted) black string



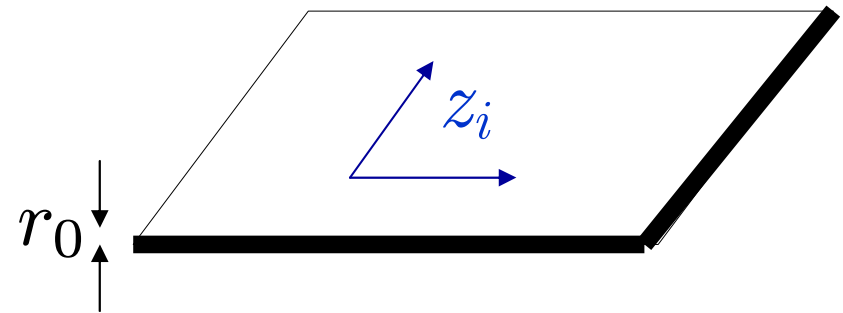
- Take a black p-brane, and bend it onto a p-diml submanifold: when is this possible?
- Approx:
  - **thin** brane: thickness  $\ll$  curvature of submanifold
    - perturbative solution via matched asymptotic expansion
  - equivalent to flat black brane, up to local Lorentz transform (**boost** & possibly rotation)
    - determines local stress tensor
  - **blackness condn**: temp and angular velocity uniform over blackfold
    - constraint on eqns to yield black objects

- Black p-brane

$$ds^2 = - \left( 1 - \frac{r_0^n}{r^n} \right) dt^2 + \sum_{i=1}^p dz_i^2 + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}^2$$

$$T_{tt} = r_0^n (n + 1)$$

$$T_{ii} = -r_0^n$$

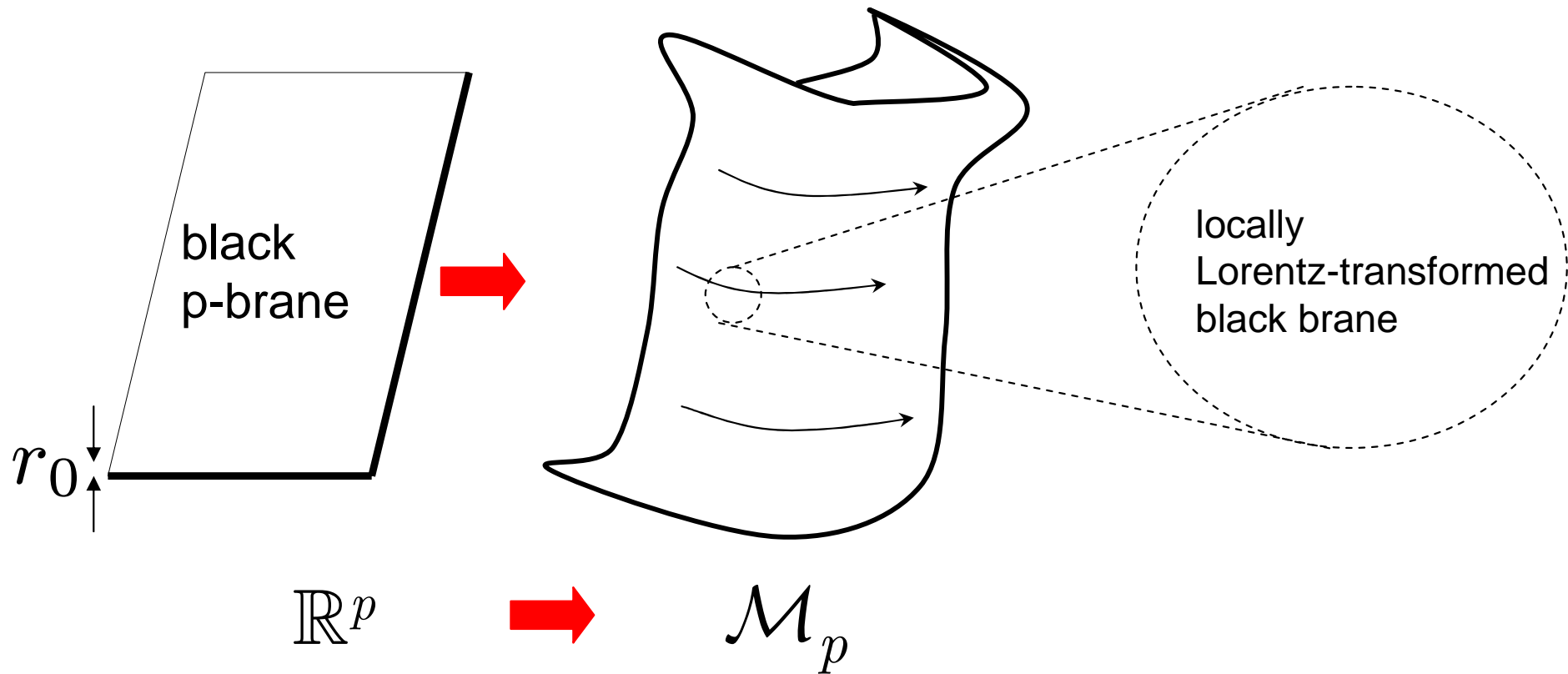


Lorentz-transform:

$$(t, z_i) = \sigma^\mu, \quad \sigma^\mu \rightarrow a_\nu^\mu \sigma^\nu, \quad a_\nu^\mu \in O(1, p)$$

$$T_{\mu\nu} \rightarrow T_{\mu\nu} = r_0^n \left[ (n + 1) a_\mu^t a_\nu^t - \sum_{i=1}^p a_\mu^i a_\nu^i \right]$$

E.g., black string  $\rightarrow$  boosted black string



Brane in  $\mathcal{M}_p$  is (extrinsically) curved, and has tension  
 $\rightarrow$  generically not in mechanical equilibrium

# • Classical black brane dynamics

$$\nabla_{\mu} T^{\mu\nu} = 0 \quad \Rightarrow \quad K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$

extrinsic curvature

Carter

or, with external force:  $F^{\rho} = K_{\mu\nu}{}^{\rho} T^{\mu\nu}$

– Newton's force law:  $F=ma$

– Nambu-Goto-Dirac eqns:  $T_{\mu\nu} = T g_{\mu\nu} \rightarrow K^{\rho}=0$ : minimal surface

– Generic eom's for *given*  $T_{\mu\nu}$

## • Blackfolds:

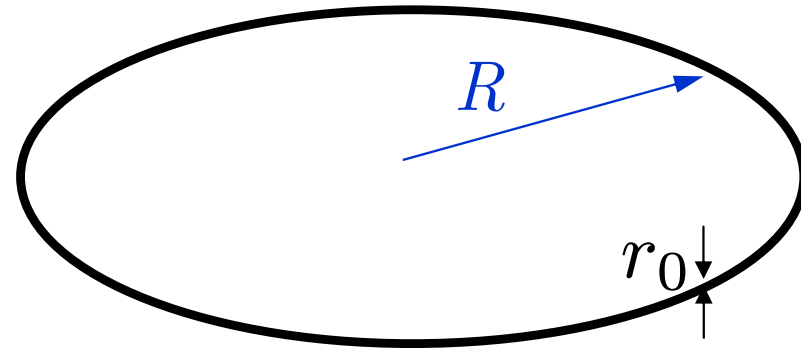
–  $T_{\mu\nu}(\sigma_{\mu})$  locally Lorentz equiv to black p-brane:  $r_0(\sigma), a^{\mu}_{\nu}(\sigma)$

– Uniform surface gravity & angular velocities: eliminate *thickness*, and *boost* parameters in favor of geometric parameters

- Simplest example: black rings in  $D \geq 5$

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$

➔  $\frac{T_{11}}{R} = 0$



$$T_{11} = r_0^{D-3} [(D-3) \sinh^2 \sigma - 1]$$

Tune boost to equilibrium  $\Rightarrow \sinh^2 \sigma = \frac{1}{D-3}$

Horizon  $S^1 \times S^{D-3}$

↖ "small" transverse sphere  $\sim r_0$

- $p$ -brane curved into  $\mathbf{T}^p$ :

$$K_{\mu\nu}{}^\rho T^{\mu\nu} = 0$$

→  $\frac{T_{11}}{R_1} = \frac{T_{22}}{R_2} = \dots = 0$

$$T_{ii} = r_0^n (n(a_i^t)^2 - 1) \quad n = D - p - 3$$

Tune boost to equilibrium  $\Rightarrow a_i^t = -\frac{1}{\sqrt{n}}, \quad a_t^t = \sqrt{\frac{n+p}{n}}$

Horizon  $\mathbf{T}^p \times \mathcal{S}^{n+1}$

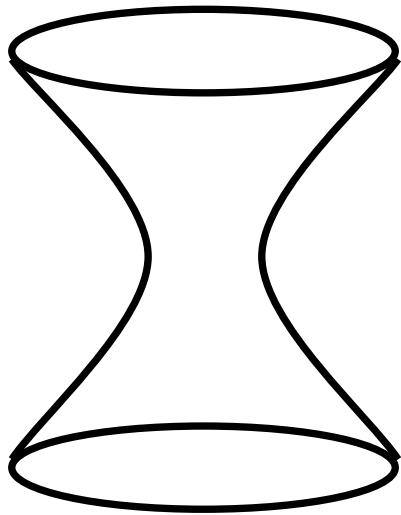
↑ "small" transverse sphere  $\sim r_0$

- **Static minimal blackfolds:**

If no boost (no local Lorentz transf)

→  $T_{ij} = -P g_{ij}$  (spatial)

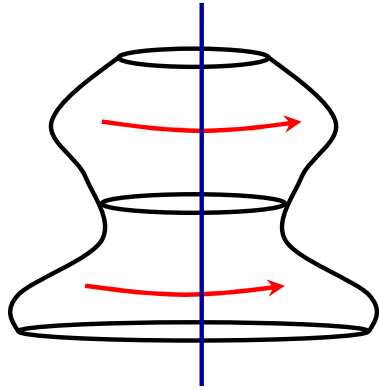
→  $K^{\rho}_{ij}=0$ : minimal submanifold



E.g.: hyperboloid

→ non-compact static blackfold

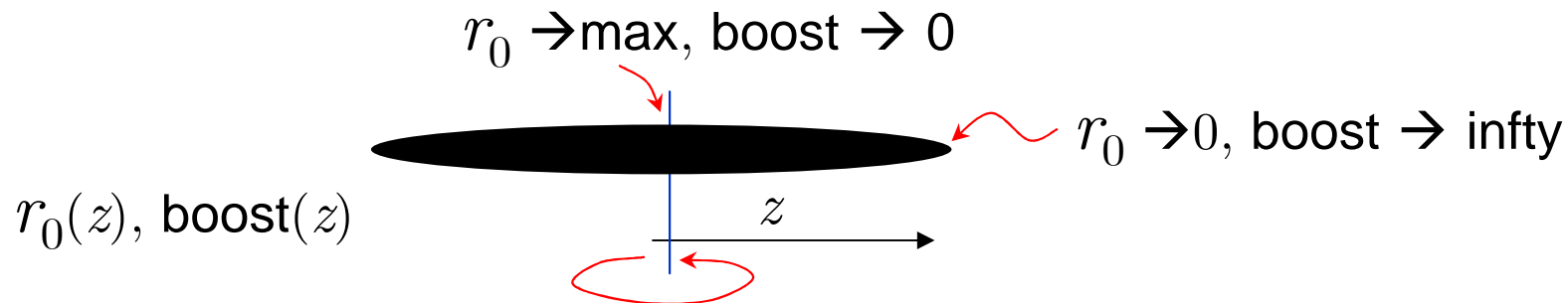
- Axisymmetric blackfolds



(possibly rotations along all axes)

- Simple analytic solutions:

- odd  $p$  : round  $S^p$ , with all  $(p+1)/2$  rotations equal
- even  $p$  : **ultraspinning MP bh**, with  $p/2$  ultraspins



- In progress: blackfolds with horizon topology

$$\prod_{p_i \in \text{odd}} S^{p_i} \times S^{n+1} \quad ?$$

$$D = \sum_i p_i + n + 3$$

- But: does p-brane horizon remain *regular* after bending?
  - Perturb black p-brane into a slightly curved surface
 
$$r_0 \ll R_{\text{curvature}}$$
  - Regularity checked in several cases – hard work, not most general yet

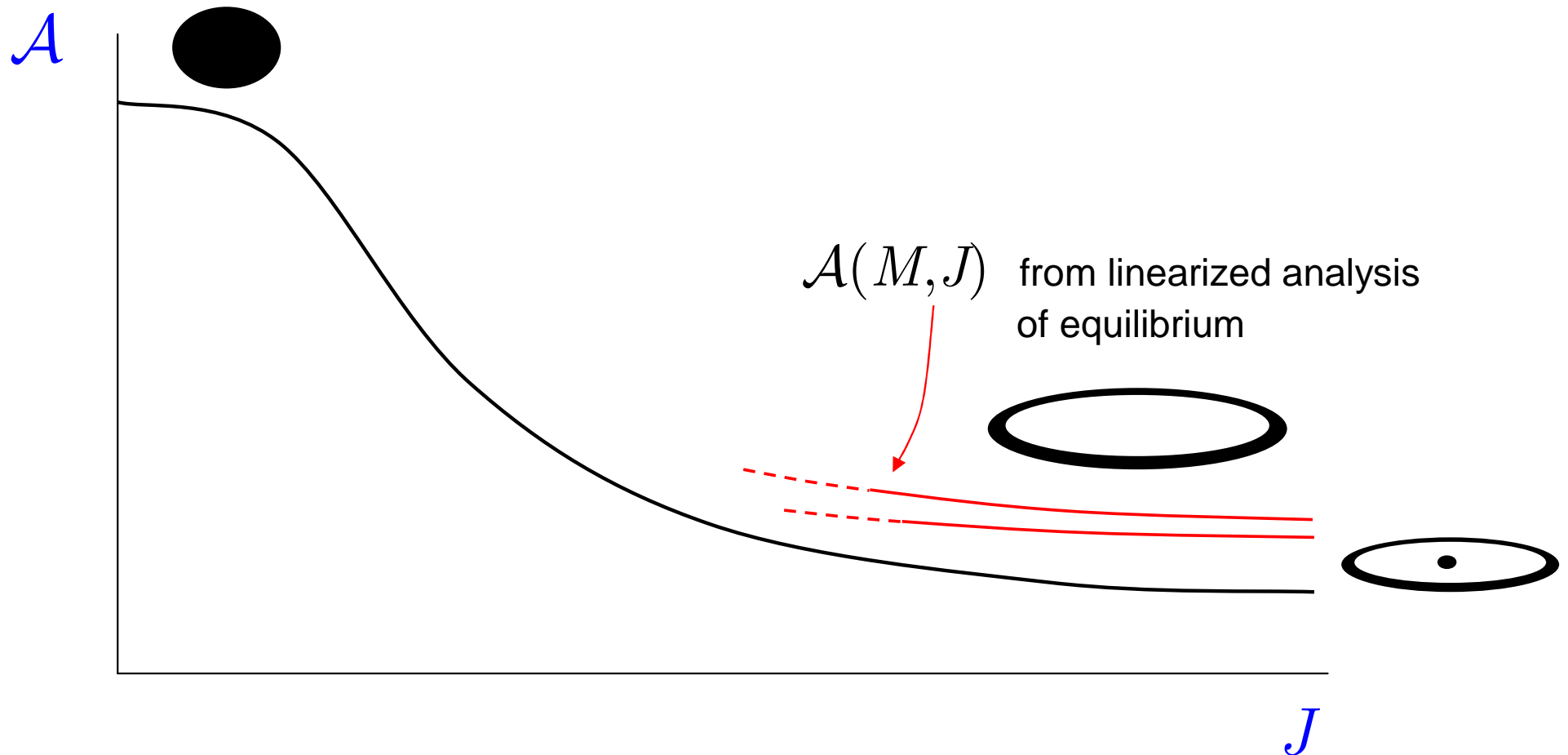
# Other blackfold applications

- Curved strings & branes in gravitational potentials: Caldarelli+RE+Rodriguez
  - black rings in AdS:  $\exists$  w/ arbitrarily large radius
  - black rings in dS:  $\exists$  static ones
  - black saturns, charged and susy blackfolds...
- **Very powerful and general** approach to hi-d bh dynamics, extending (& including) DBI approach to D-branes

*Caveat emptor*: blackfold *may not* remain stationary beyond linear approx -- use with caution

# $D \geq 6$ phase diagram

(fix  $M$ )

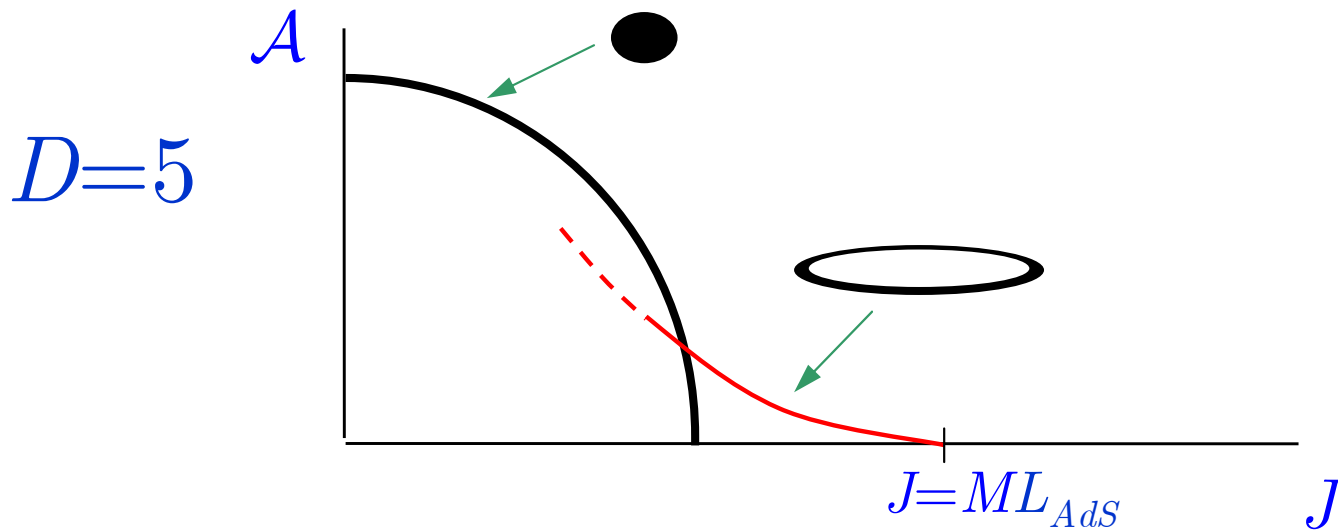


→ Black rings dominate the entropy at large  $J$

# AdS phase diagram

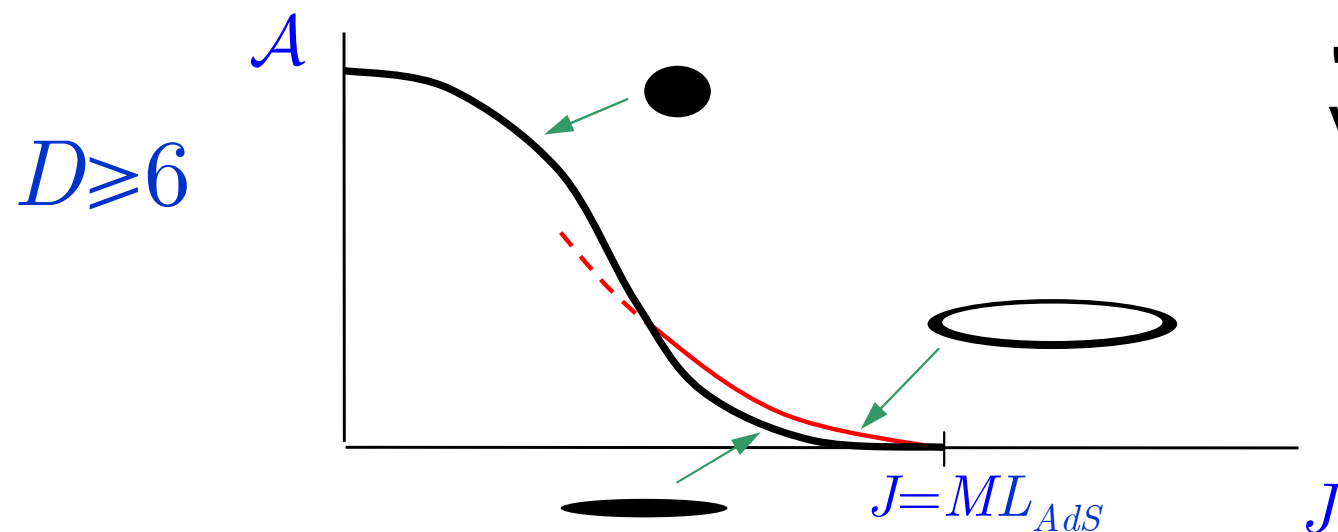
"BPS" bound  $J=ML_{AdS}$

*Chrusciel et al*



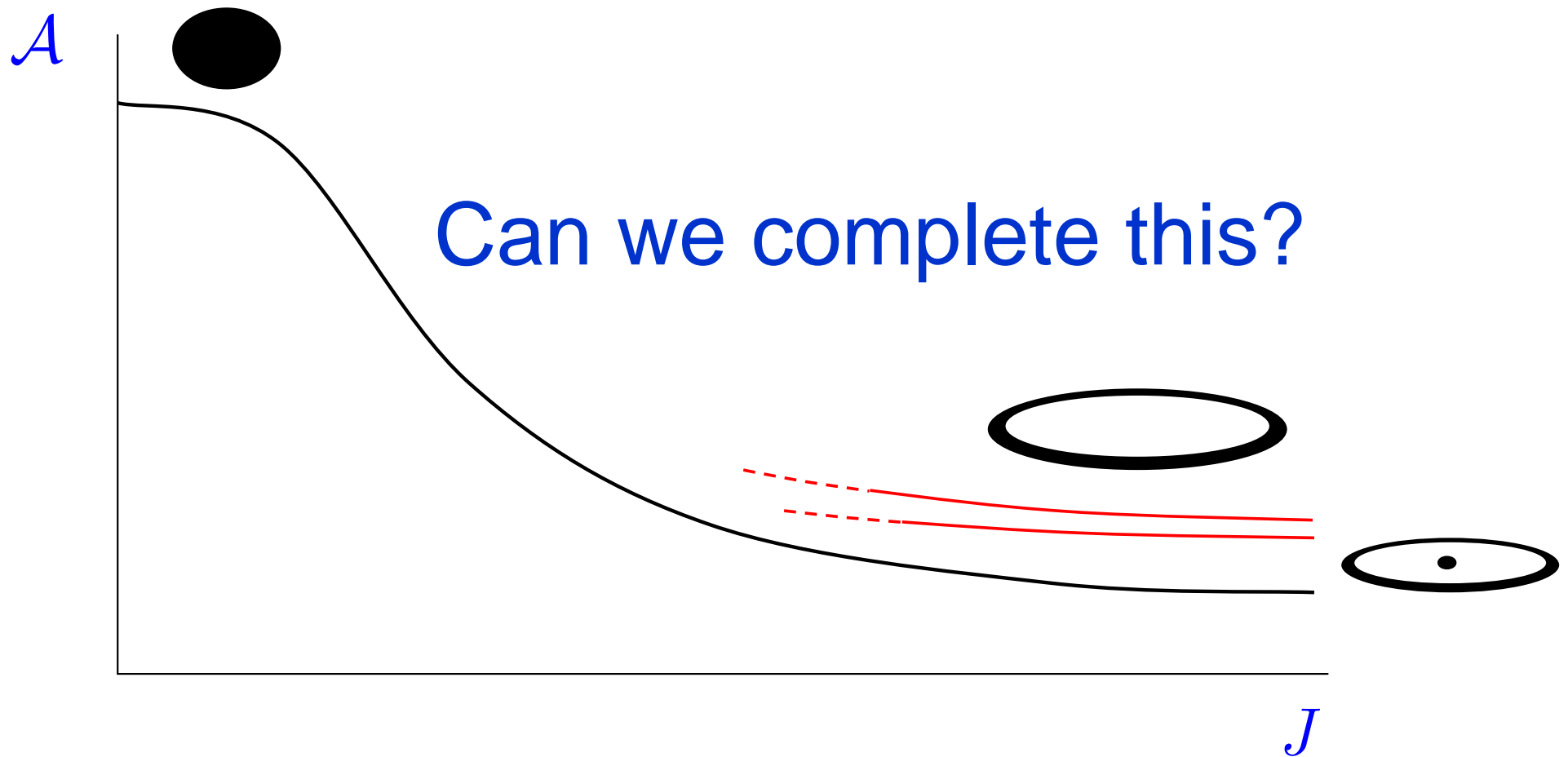
rot AdS bh:  
*Hawking+Hunter+Taylor*  
*Gibbons et al*

fixed  $M$



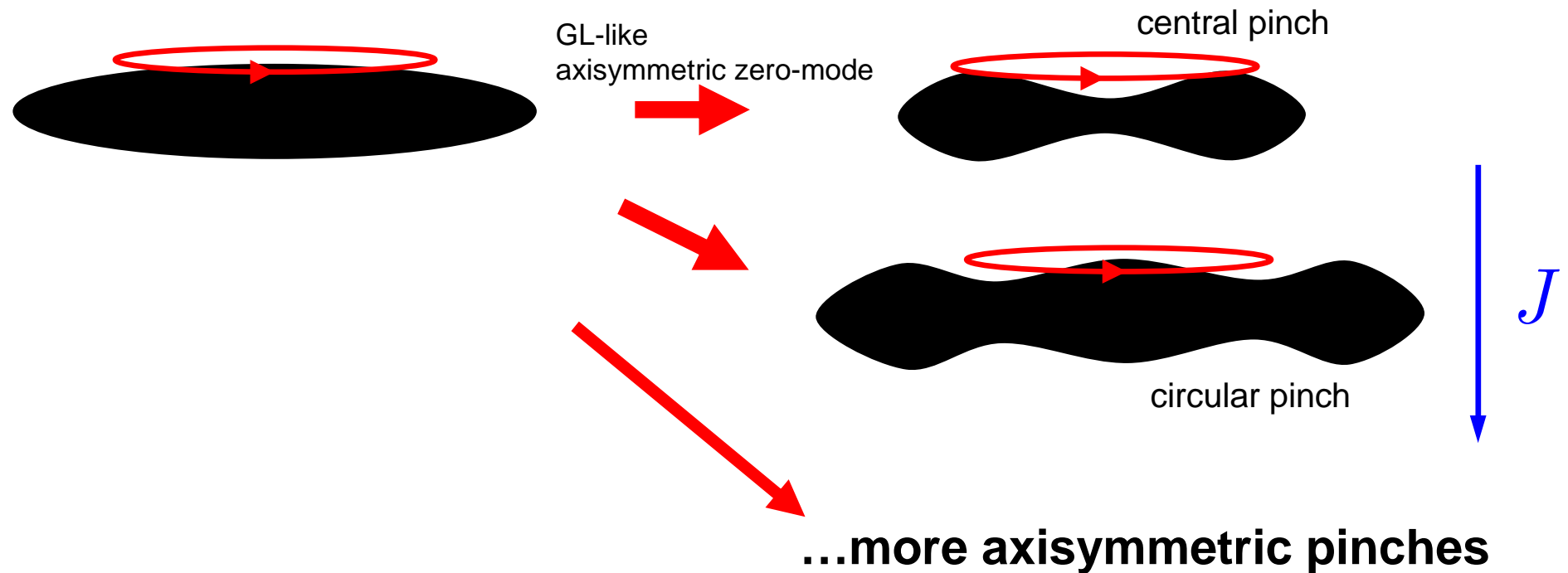
"compressed"  
versions of  $\Lambda=0$

# Back to AF $D \geq 6$ phase diagram



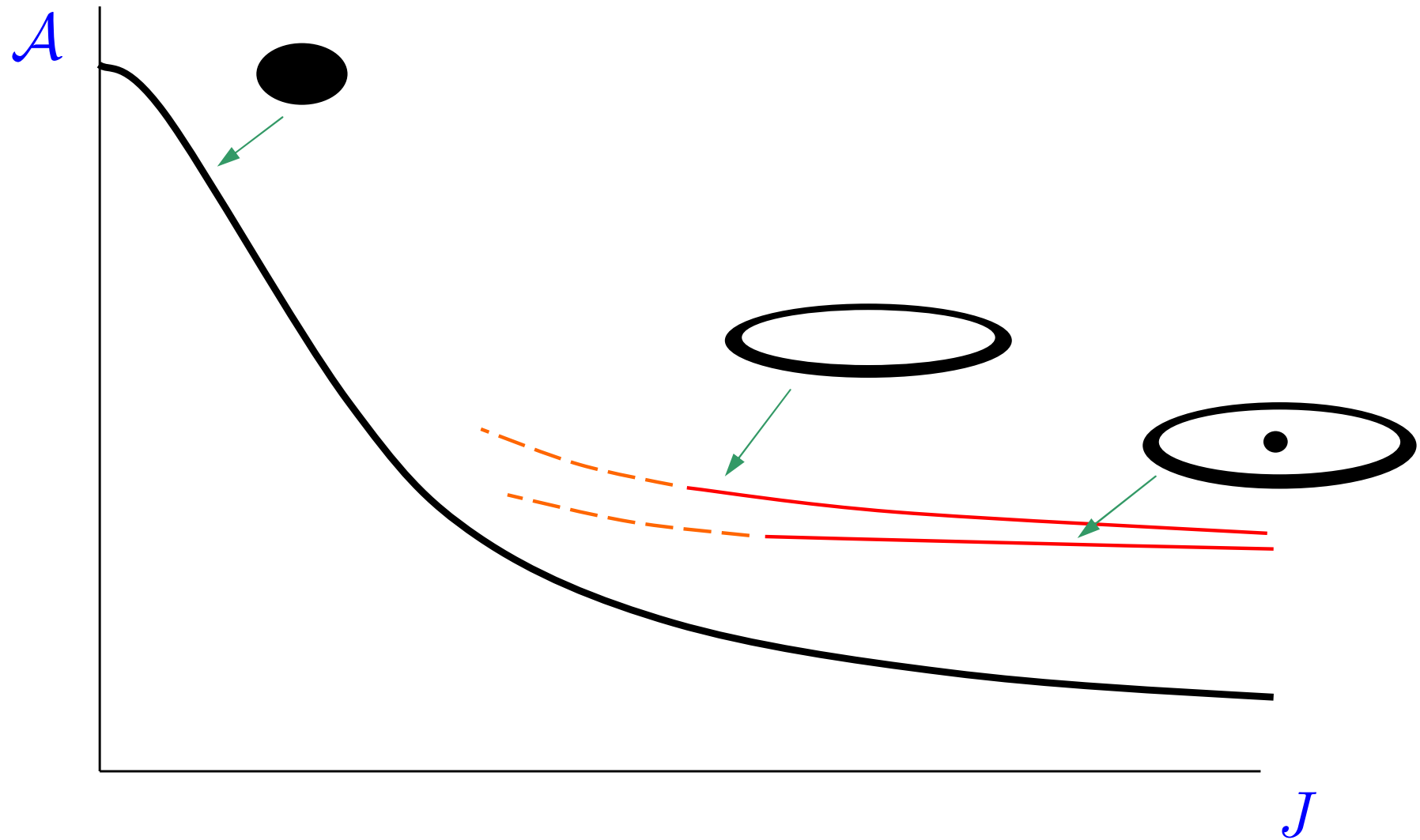
# Multiply pinched black holes from axisymmetric zero-modes:

RE+Myers

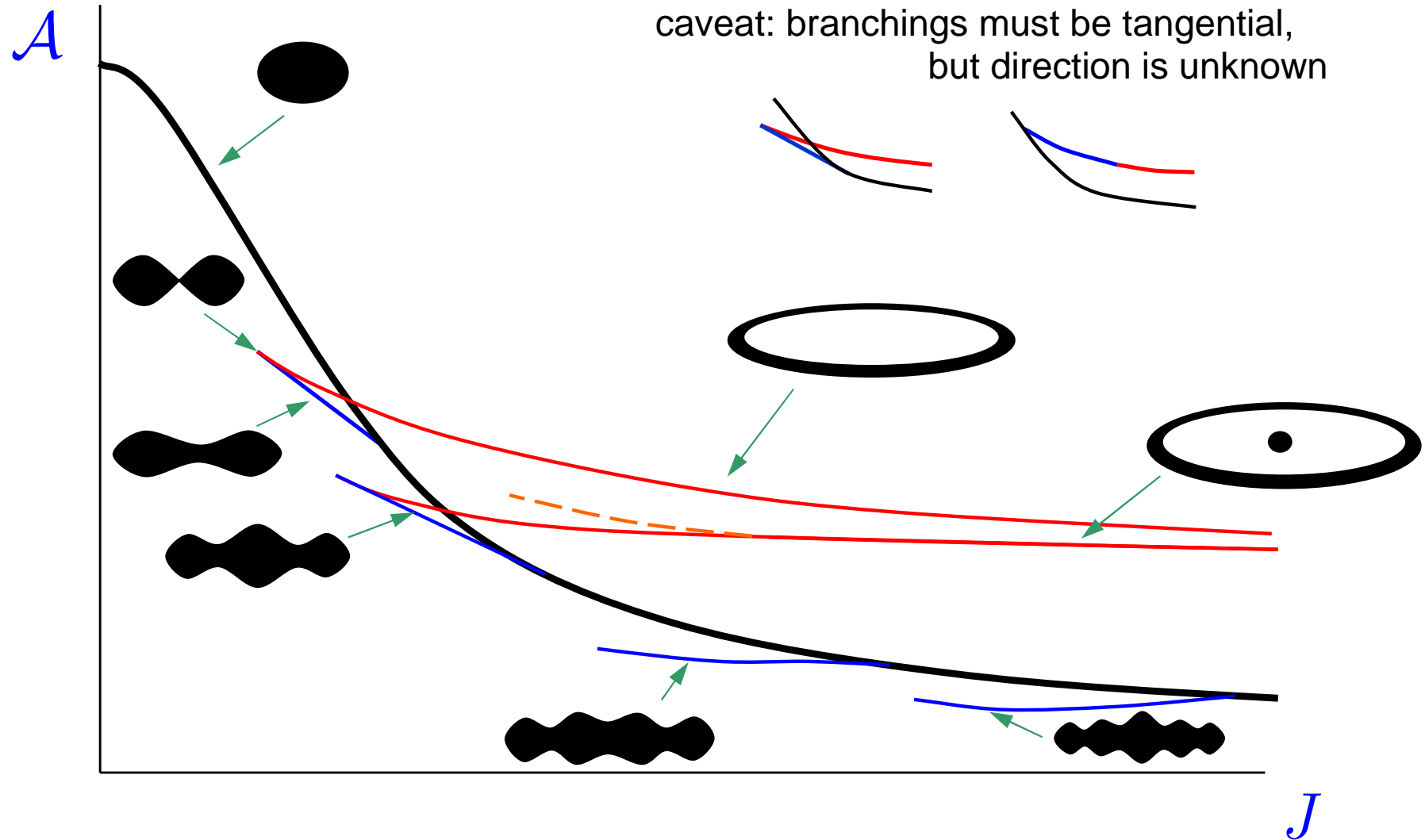


- Not yet found --- presumably numerically or approximately
- *Pinched plasma balls* found by *Lahiri+Minwalla*:  
dual to pinched black holes at IR in AdS w/ confining vacuum

# $D \geq 6$ phase diagram: a proposal



# $D \geq 6$ phase diagram: a proposal



# Conclusions: *More is different*

Vacuum gravity  $R_{\mu\nu} = 0$  in

- $D=3$  has no black holes
  - $GM$  is **dimensionless** → can't construct a length scale  
( $\Lambda$ , or  $h$ , provide length scale)
- $D=4$  has **one** black hole
  - but no 3D bh → no 4D black strings → no 4D black rings
- $D=5$  has **three** black holes (two topologies); black strings → black rings, infinitely many multi-bhs...
- $D \geq 6$  seem to have **infinitely many blackfolds, lumpy black holes**... (& possibly others that lie away from black brane limit)