Compartmental Fires : Vertical Spread

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joint work with

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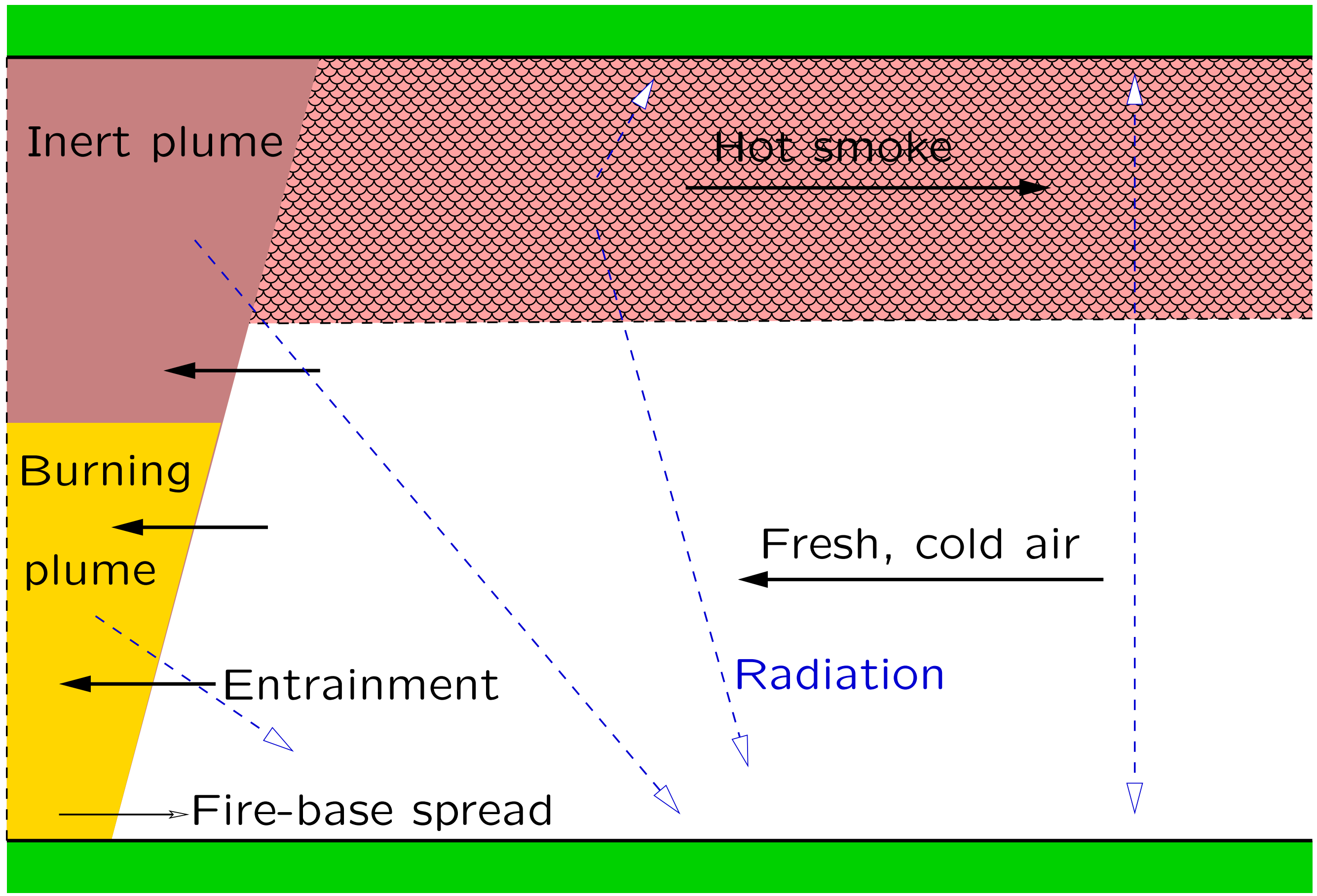
Other compartmental models for flashover ...



- Empirical plume modelling

- Two-layer gas flow: u_{fresh}, u_{smoke}, height

- Heat equations for plume, fluid, floor, ceiling, fluid

- Fire/plume base radius growth in terms of temperature

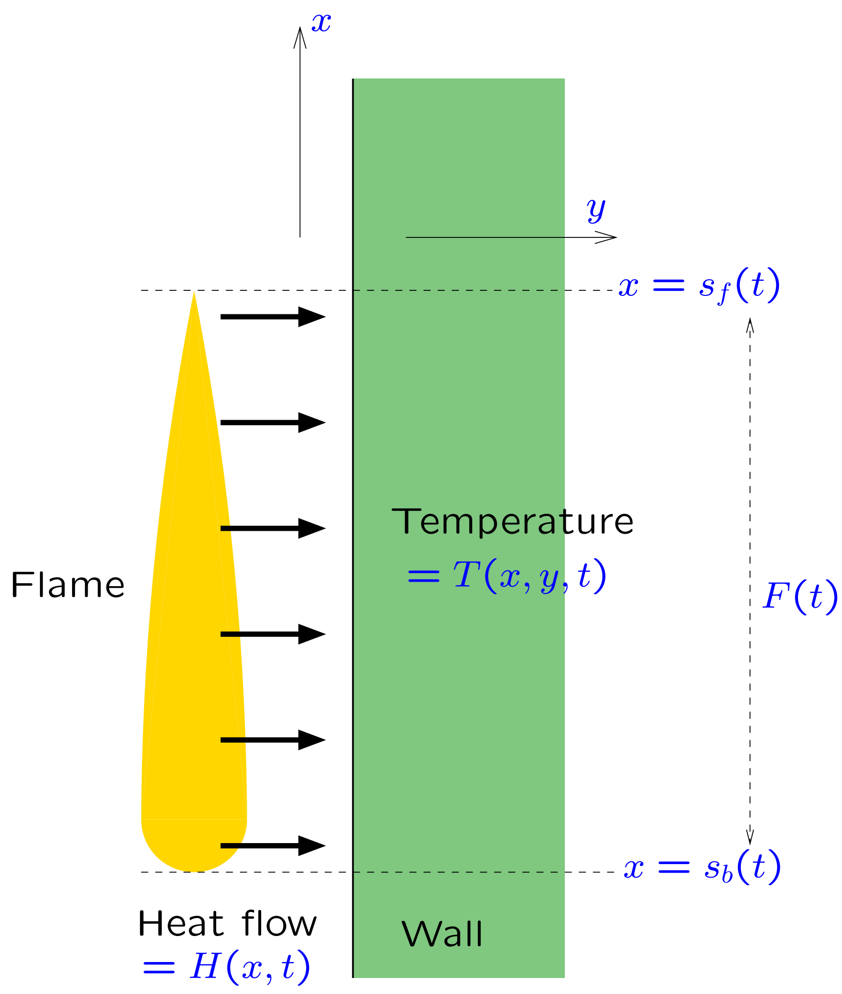
Vertical Fire Spread

Fuel-release rate =
$Q(t) \;=\; \int_{s_b}^{s_f} P(x,t)\,\mathrm{d}x$ or
$\int_0^{s_f} P(x,t)\,\mathrm{d}x$

Heat flux =
$H(t) = \mathcal{H}(Q(t))$

Flame extent =
$F(t) = \mathcal{F}(Q(t))$

Burn-out condition for s_b.



Special cases

Modelling ...

UH: Uniform heat flux: $H(x,t) = \mathcal{H}(Q)(t)$ is independent of x for $s_b < x < s_f$.

CH: Constant heat flux: $H = \mathcal{H}$ is a constant.

TT: Thermally thin wall: $T = T(x,t)$.

PF: Pyrolysis front: $P = \dfrac{\partial Y}{\partial t}$.

SP: Small heat of pyrolysis.

Solution ...

TW: Steady motion: $s_b = Vt = s_f - \mathcal{F}(Q)\,,\ Q =$ constant.

"Burn-out time" $= \tau_b = \tau_b(H)$ with $H = \mathcal{H}(Q)$: $s_f(t) = s_b(t{+}\tau_b)$.

$t = \omega_f(x) \Leftrightarrow x = s_f(t)\ :\qquad \tau = t - \omega_f(x)$.

$P = P(\tau) = P(\tau; H)\ :\qquad Q = \int_{s_b}^{s_f} P(t{-}\omega_f(x))\,\mathrm{d}x;\qquad V = \mathcal{F}(Q)/\tau_b(\mathcal{H}(Q))\,.$

Q *etc.* given by $Q = \dfrac{\mathcal{F}(Q)}{\tau_b(\mathcal{H}(Q))} \displaystyle\int_0^{\tau_b(\mathcal{H}(Q))} P(t - \omega_f(x); \mathcal{H}(Q))\,\mathrm{d}\tau\,.$

Base Conditions

1. $x = s_b$ where P falls to zero.

2. $x = s_b$ where P falls to P_c.

3. $x = s_b$ where $\int^x P \, \mathrm{d}x$ falls to Q_c.

Constant $\mathcal{H}$, Low Heat of Pyrolysis,

Precise Pyrolysis Temperature, Thermally Thin

Pyrolysis at base, $x = s_b(t)$.

Pyrolysis rate $\propto$ speed of flame base.

$s_f(t) = s_b(t + \tau_b)$.

Scaled problem: $s(t+1) - s(t) = \mathcal{F}\left(\frac{\mathrm{d}s(t)}{\mathrm{d}t}\right)$ or

$$\frac{\mathrm{d}s}{\mathrm{d}t}(t) = \mathcal{G}(s(t+1) - s(t)).$$

Types of Model for the Wall

Pyrolysis front: $T(x, y, t)$ given by heat equation.

(Possibly with a Stefan condition.)

Reaction kinetics: Thermally thin

$\Rightarrow T = T(x, t)\,, \quad Z = Z(x, t) =$ remaining reactant,

$\partial Z/\partial t = -P(Z, T)\,, \quad \partial T/\partial t = K(H + LP(Z, T)).$

Models Allowing Analytic Results

1. CH: $H = \mathcal{H}(Q) \equiv H^* =$ constant.

N.B. Case of thermally thick wall with pyrolysis front and low heat of pyrolysis.

$$P = P(t - \omega_f(x))\,, \quad Q(t) = \int_0^{\tau_b} P(\tau)\frac{\mathrm{d}s_f}{\mathrm{d}t}(t-\tau)\,\mathrm{d}\tau\,.$$

2. TT + SP + PF + linear $\mathcal{H}$:

Pyrolysis at $x = s_b(t)$ where $T = T_p$; $\quad H \propto Q \propto \dfrac{\mathrm{d}s_b}{\mathrm{d}t}$.

$$s_f - s_b = \mathcal{F}(\mathrm{d}s_b/\mathrm{d}t) \quad \Rightarrow \quad s_f(\omega_b(x)) - x = \mathcal{F}\left(\frac{1}{\mathrm{d}\omega_b/\mathrm{d}x}\right).$$

$$\text{const.} = \int_{\omega_f(s_b(t))}^{t} \frac{\mathrm{d}s_b}{\mathrm{d}t}(\tau)\,\mathrm{d}\tau \quad \Rightarrow \quad x - s_b(\omega_f(x)) = C\,.$$

$$\therefore \mathcal{F}\left(\frac{1}{\mathrm{d}\omega_b/\mathrm{d}x}\right) = C\text{: Travelling wave.}$$

3. TT + SP + PF + constant $\mathcal{F}$: $F = \mathcal{F}(Q) \equiv F^*$.

Pyrolysis at $x = s_b(t)$ where $T = T_p$; $Q = \dfrac{\mathrm{d}s_b}{\mathrm{d}t}$.

$$\text{const.} = \int_{\omega_f(x)}^{\omega_b(x)} \mathcal{H}\left(\frac{\mathrm{d}s_b}{\mathrm{d}t}\right) \mathrm{d}t = \int_{x-F^*}^{x} \mathcal{H}\left(\frac{1}{\mathrm{d}\omega_b/\mathrm{d}x}\right) \frac{\mathrm{d}\omega_b}{\mathrm{d}x}\, \mathrm{d}x\,.$$

$$\Rightarrow \mathcal{H}\left(\frac{1}{\mathrm{d}\omega_b/\mathrm{d}x}\right) \frac{\mathrm{d}\omega_b}{\mathrm{d}x}(x) = \mathcal{H}\left(\frac{1}{\mathrm{d}\omega_b/\mathrm{d}x}\right) \frac{\mathrm{d}\omega_b}{\mathrm{d}x}(x - F^*)\,.$$

$\therefore$ Periodic travelling wave (if $\mathcal{H}(Q)/Q$ invertible).

Linear Stability of Travelling Waves for CH

CH, $\tau_p = 1 \quad \Rightarrow \quad s_f(t) - s_f(t-1) = \mathcal{F}\left(\int_0^1 P(\tau)\frac{\mathrm{d}s_f}{\mathrm{d}t}(t-\tau)\,\mathrm{d}\tau\right)$.

TW $\quad \Rightarrow \quad V = \mathcal{F}(Q)\,, \quad Q = V\int_0^1 P(\tau)\,\mathrm{d}\tau = \mathcal{F}(Q)\int_0^1 P(\tau)\,\mathrm{d}\tau\,.$

N.B. Significance of linear $\mathcal{F}$!!

Linearise about TW: $s_f(t) \sim Vt + \delta \mathrm{e}^{\lambda t}$:

$$1 - \mathrm{e}^{-\lambda} = \lambda \mathcal{F}'(Q)\int_0^1 P(\tau)\,\mathrm{d}\tau\,.$$

Particular case: $P(\tau) \equiv 1$ for $\tau_p = 1 - c < \tau < 1\,, \quad 0$ otherwise.

$(\int P\,\mathrm{d}\tau = c\,, \quad Q = cV = c\mathcal{F}(Q)\,.)$

$$\mathrm{e}^{\lambda} - \mathcal{F}'\mathrm{e}^{c\lambda} + (\mathcal{F}' - 1) = 0\,.$$

Instability for $|z| > 1$ in $z - \mathcal{F}'z^{c} + (\mathcal{F}' - 1) = 0\,.$

"Generic case" (??): Instability for $\mathcal{F}' > 1$; Stability for $\mathcal{F}' < 1$.

Power-law models: $\mathcal{F}(Q) = AQ^{\alpha} \Rightarrow Q^{1-\alpha} = Ac\,.$

$$\mathcal{F}'(Q) = \frac{\alpha}{c}\,.$$

Bad Assumptions: $s_f(t)$ not necessarily increasing!

$P \neq P(\tau)$.

Lack of simple models to be treated analytically.

Next guess: $P = P(Z)$, $\quad Z = Z(x,t)$,

$$\frac{\partial Z}{\partial t} = \begin{cases} -Z_1 & \text{where} \quad H = H^* \\ 0 & \text{where} \quad H = 0 \end{cases} .$$

Unrealistic!!

A toy model.

$Z(x,t) =$ remaining combustibles, $\xi(x,t) =$ heat supplied so far.

$$\frac{\partial \xi}{\partial t} = H\,,$$

$$P = \text{pyrolysis rate } = -\frac{\partial Z}{\partial t} = (\xi - 1)^+ \Big(Z - (\xi_c - \xi)^+ \Big)\ .$$

Heat needed for any pyrolysis $= 1$.

Heat needed for total pyrolysis $= \xi_c$.

Some numerical simulations with pilots.

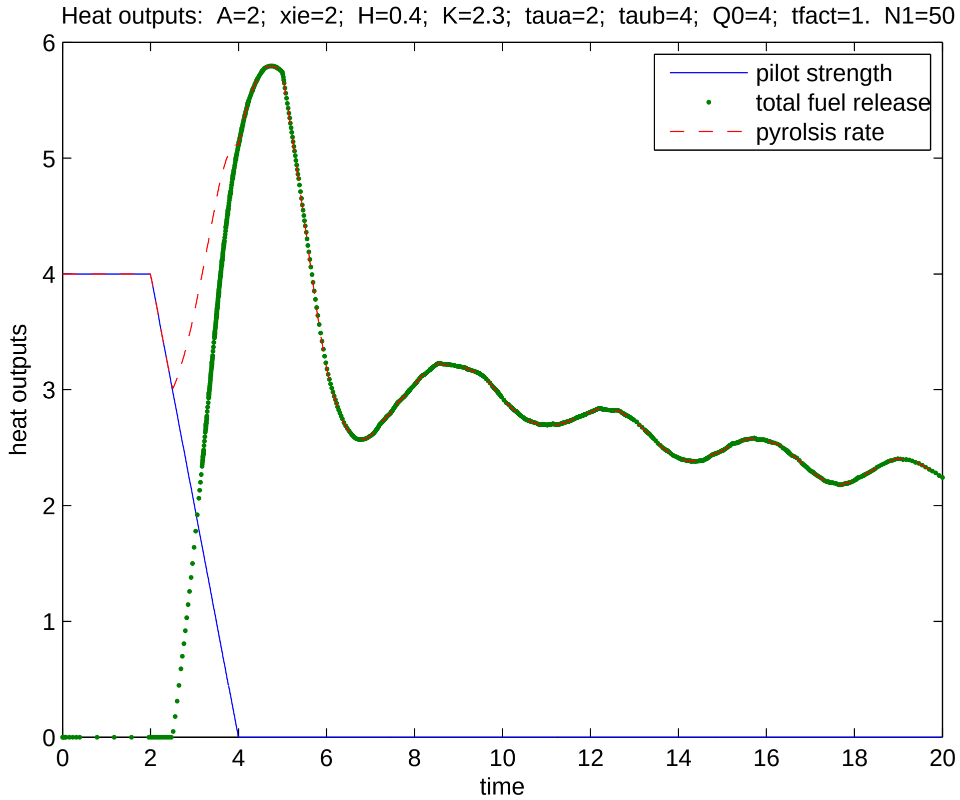
$Z(x, 0) = A =$ initial fuel.

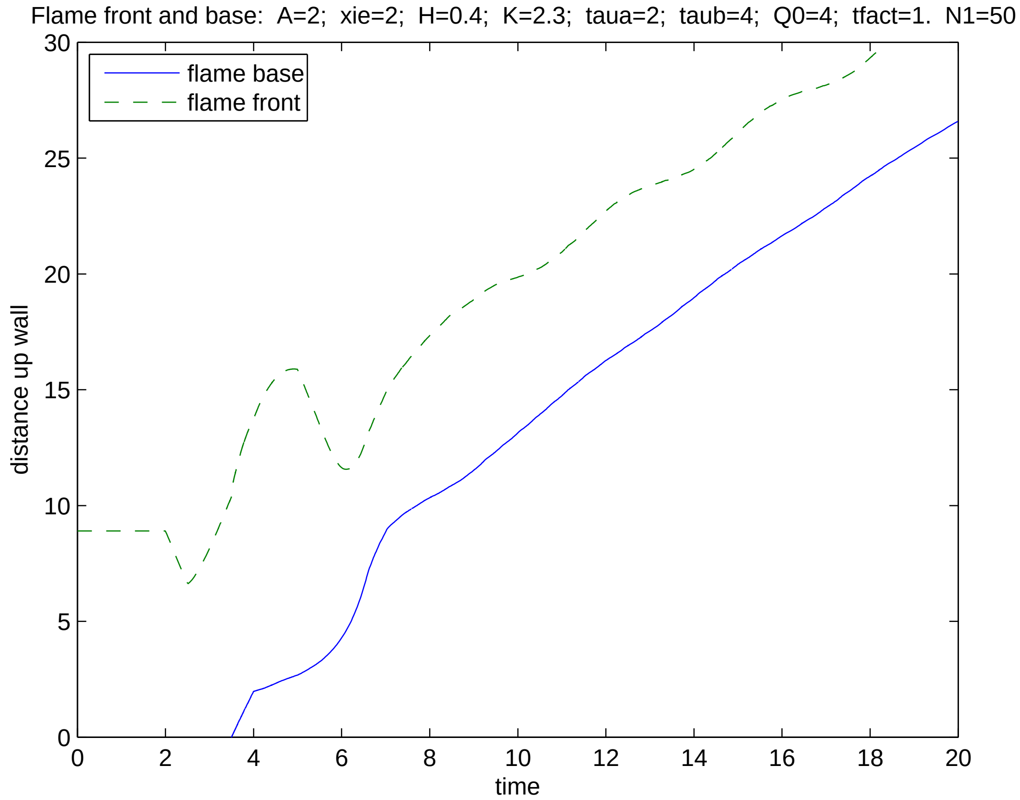
$xie = \xi_c =$ heat needed for total pyrolysis.

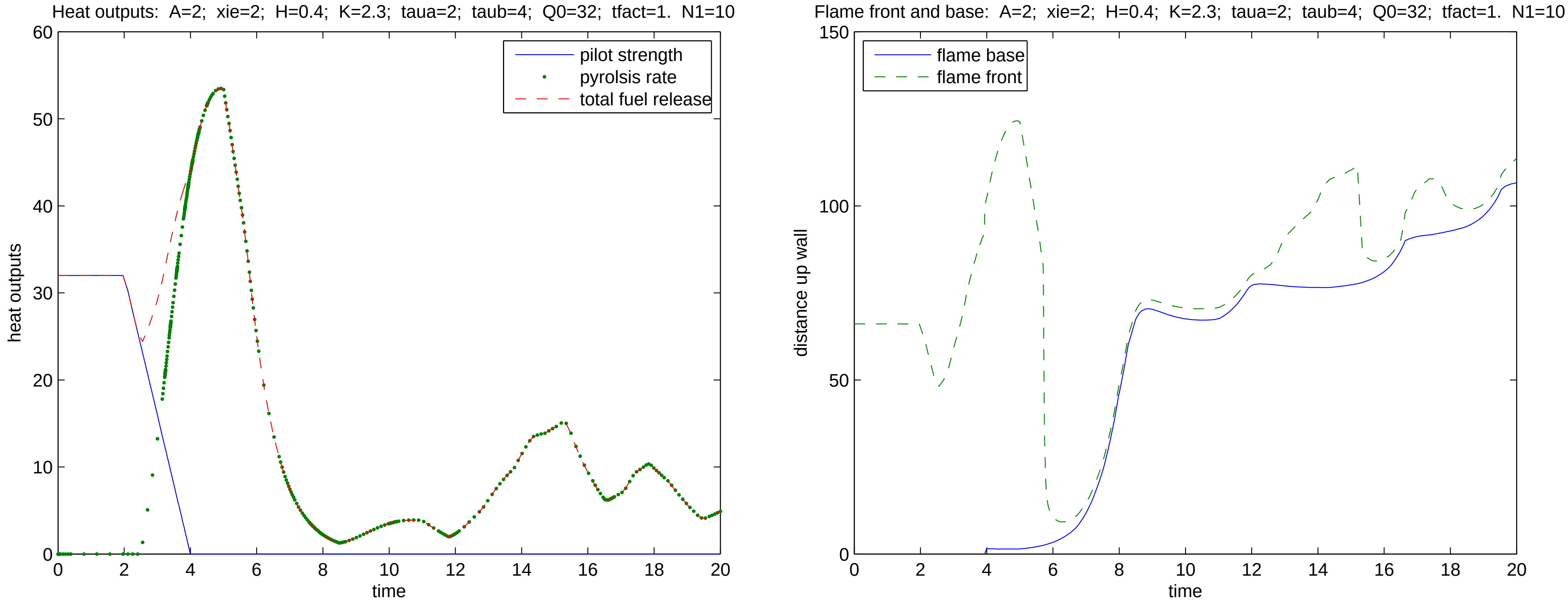
$H = H^* =$ power flux from flame between s_f and s_b.

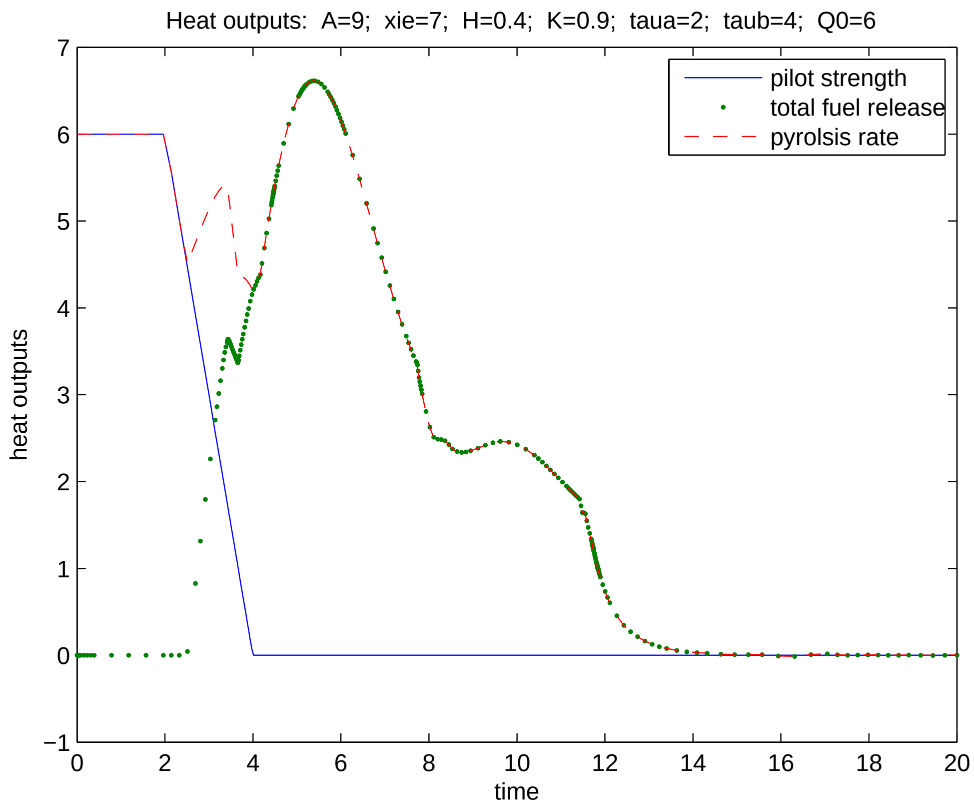
$\mathcal{F} = KQ = s_f - s_b =$ flame extent.

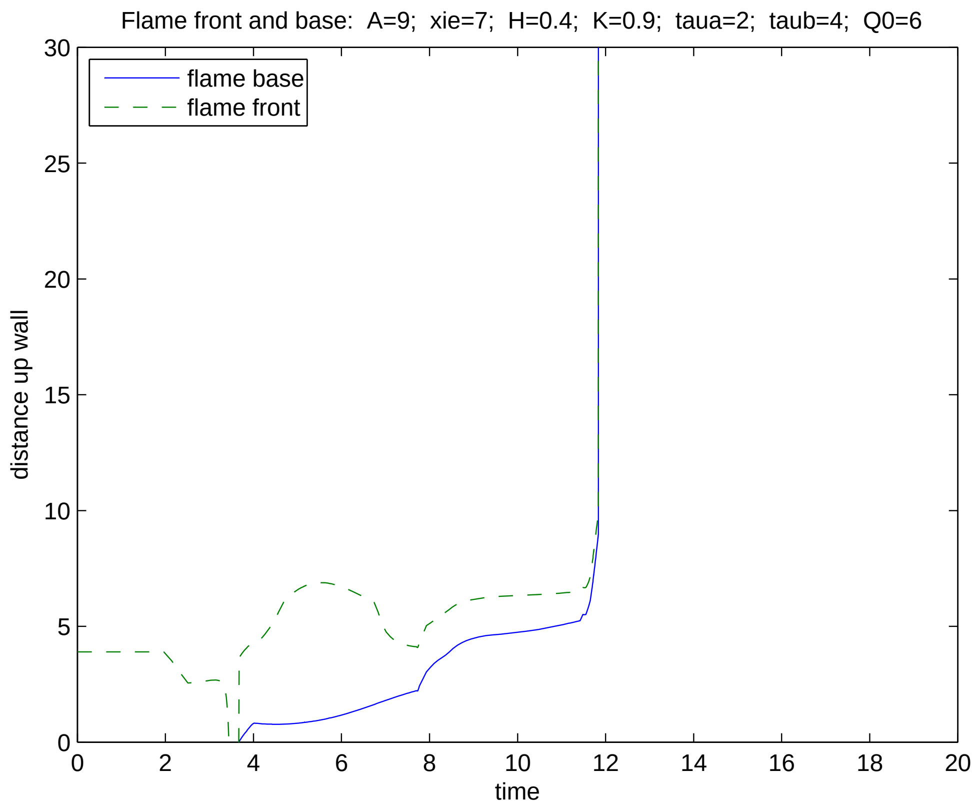
Flame base $x = s_b(t)$ determined by $\int_0^{s_b} P \, \mathrm{d}x = 1$.


Heat outputs: A=2; xie=2; H=0.4; K=2.3; taua=2; taub=4; Q0=4; tfact=1. N1=50
heat outputs
time
pilot strength
total fuel release
pyrolsis rate
0
1
2
3
4
5
6
0
2
4
6
8
10
12
14
16
18
20


Flame front and base: A=2; xie=2; H=0.4; K=2.3; taua=2; taub=4; Q0=4; tfact=1. N1=50
distance up wall
time
flame base
flame front
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14
16
18
20


Heat outputs: A=2; xie=2; H=0.4; K=2.3; taua=2; taub=4; Q0=32; tfact=1. N1=10
heat outputs
time
pilot strength
pyrolsis rate
total fuel release
Flame front and base: A=2; xie=2; H=0.4; K=2.3; taua=2; taub=4; Q0=32; tfact=1. N1=10
distance up wall
time
flame base
flame front


Heat outputs: A=9; xie=7; H=0.4; K=0.9; taua=2; taub=4; Q0=6
pilot strength
total fuel release
pyrolsis rate
heat outputs
time


Flame front and base: A=9; xie=7; H=0.4; K=0.9; taua=2; taub=4; Q0=6
flame base
flame front
distance up wall
time

Comment on extinction.

This model. Low pyrolysis $\Rightarrow$ rapid motion of base to increase fuel flux at base:

$$\frac{\mathrm{d}s_b}{\mathrm{d}t} \to \infty \quad \text{and} \quad s_b \to s_f \quad \text{as} \quad t \to t^*.$$

TT, PF, SP model. $\frac{\mathrm{d}s_b}{\mathrm{d}t}$ bounded but $s_f \to s_b$ as $t \to t^*$.

Conclusions

1. Better flame modelling needed!

2. Possible qualitative behaviour indicated by toy model.

3. Need for simulations with other models and comparison.

4. Scope for mathematical results on:
   (a) stability/instability;
   (b) existence of periodic *etc.* behaviour;
   (c) occurrence of extinction; ...