

# Self-Organisation in Arid Vegetation

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*This talk can be downloaded from my web site*

`www.ma.hw.ac.uk/~jas`

# Outline

- 1 Vegetation Pattern Formation
- 2 History-Dependence in Vegetation Patterns
- 3 Inferring the Historical Origin of Patterned Vegetation

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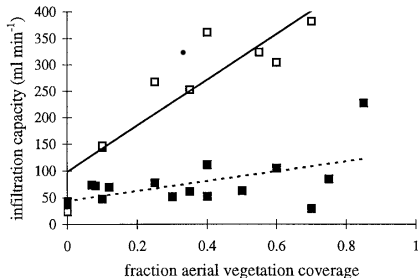


W National Park, Niger

Average patch width is 50 m

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Data from Burkina Faso

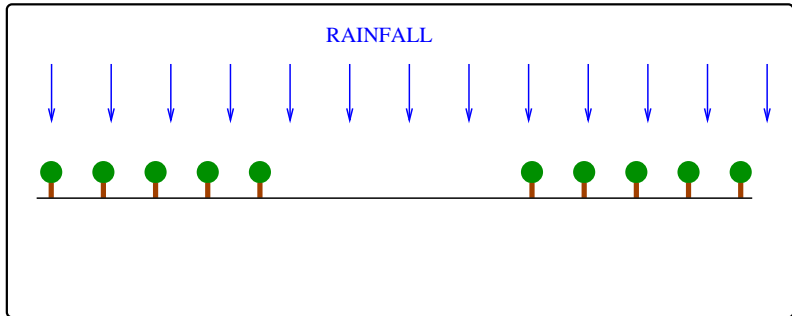
Rietkerk et al

Plant Ecology 148: 207-224, 2000

More plants  $\Rightarrow$  more roots and organic matter in soil  
 $\Rightarrow$  more infiltration of rainwater

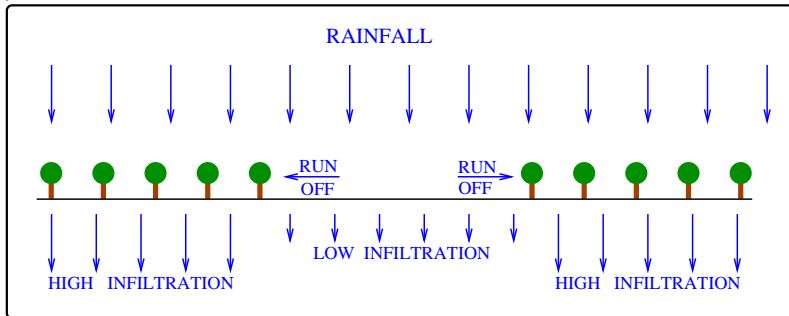
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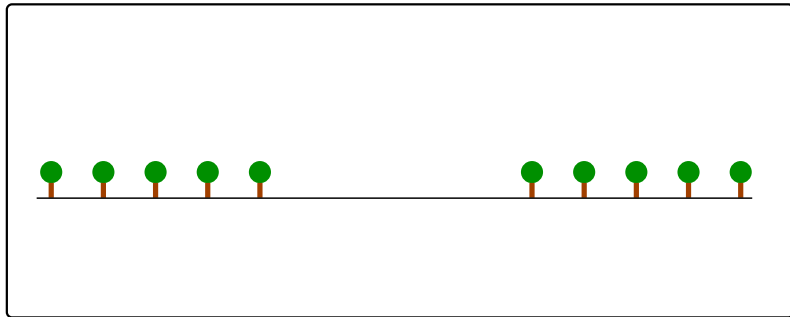
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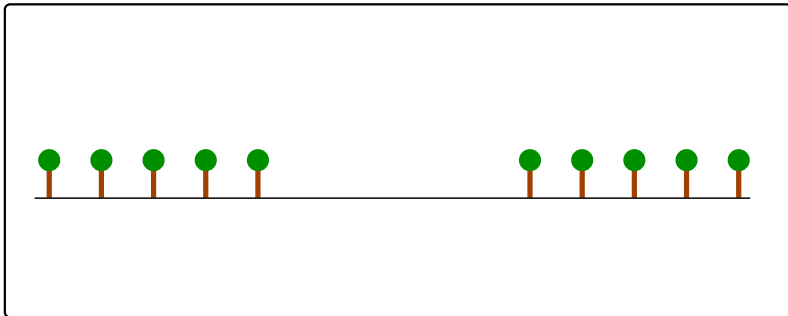
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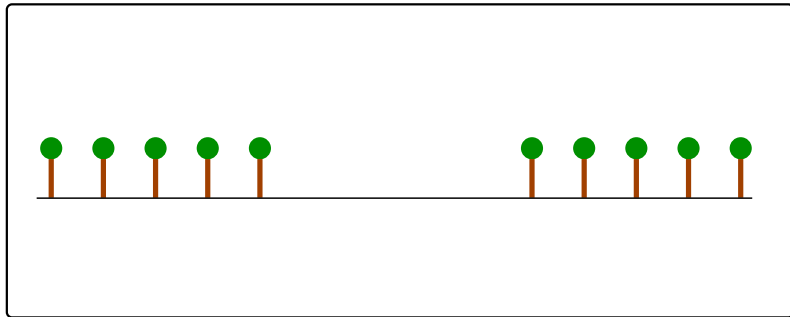
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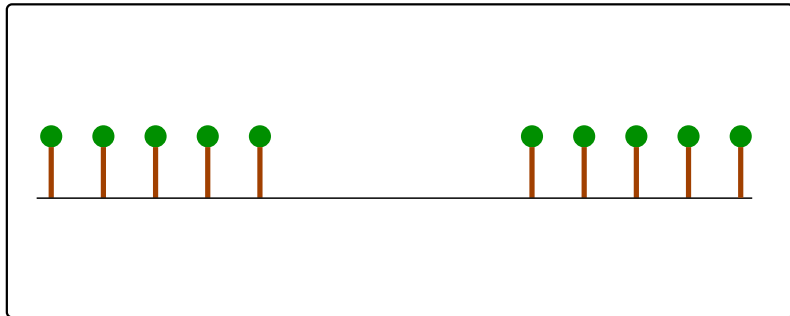
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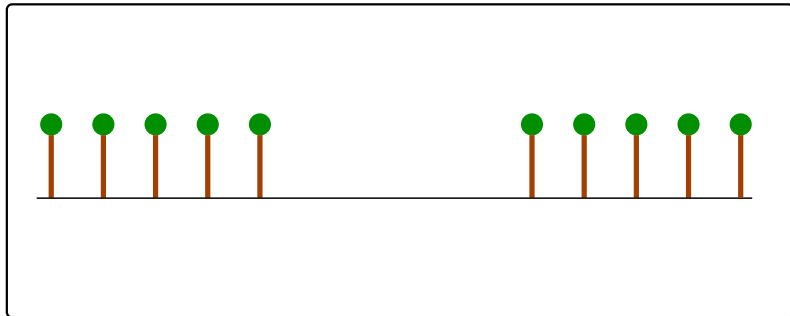
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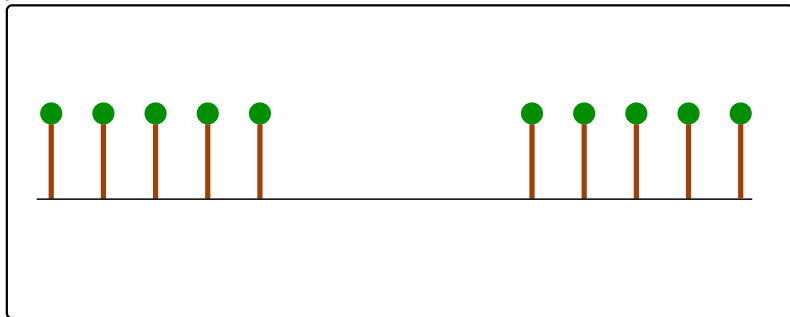
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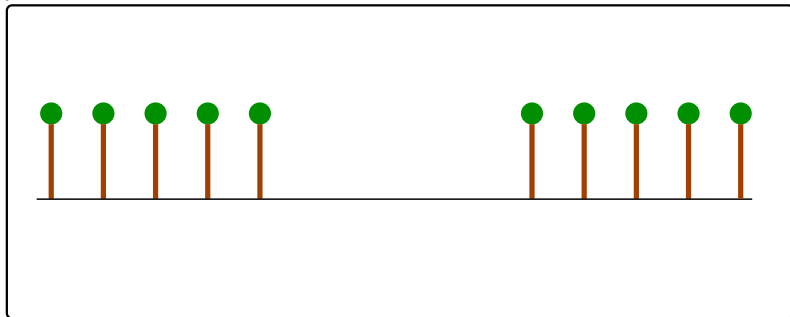
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# Banded Vegetation on Slopes

On slopes, run-off occurs in one direction only, giving striped patterns parallel to the contours.



Bushy vegetation in Niger



Mitchell grass in Australia

(Western New South Wales)

Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia, Mexico and S-W USA.

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# Mathematical Model of Klausmeier

$$\begin{aligned}
 \partial u / \partial t &= \underbrace{wu^2}_{\text{plant growth}} - \underbrace{Bu}_{\text{plant loss}} + \underbrace{\partial^2 u / \partial x^2}_{\text{plant dispersal}} \\
 \partial w / \partial t &= \underbrace{A}_{\text{average rainfall}} - \underbrace{w}_{\text{evaporation \& drainage}} - \underbrace{wu^2}_{\text{uptake by plants}} + \underbrace{\nu \partial w / \partial x}_{\text{flow downhill}} + \underbrace{D \partial^2 w / \partial x^2}_{\text{diffusion of water}}
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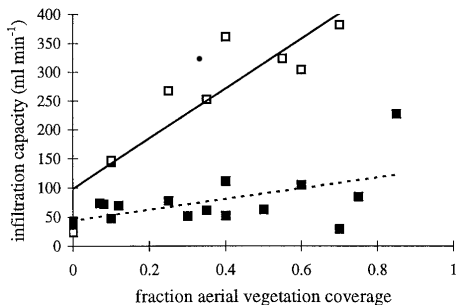
(Klausmeier, Science 284: 1826-8, 1999)

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The nonlinearity in water uptake occurs because the presence of plants increases water infiltration into the soil.

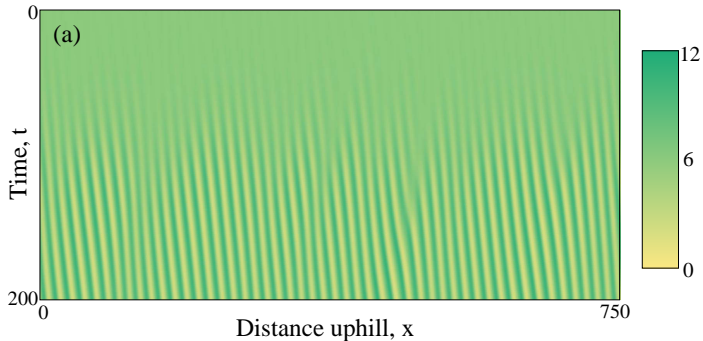
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$$\begin{aligned} \text{Water uptake} = & \\ & \text{Water density} \\ & \times \text{Plant density} \\ & \times \left( \text{infiltration rate} \right) \end{aligned}$$

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# Typical Solution of the Model



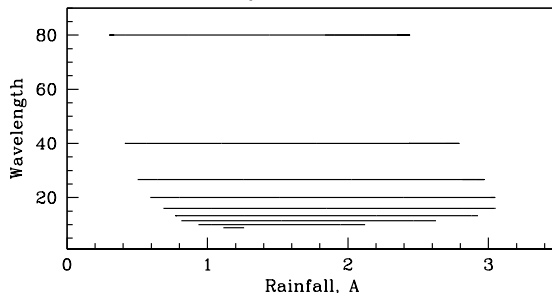
Note: pattern moves slowly uphill

# Pattern Existence and Stability

High rainfall  $\Rightarrow$  uniform vegetation

Low rainfall  $\Rightarrow$  no vegetation

Medium rainfall  $\Rightarrow$  patterns

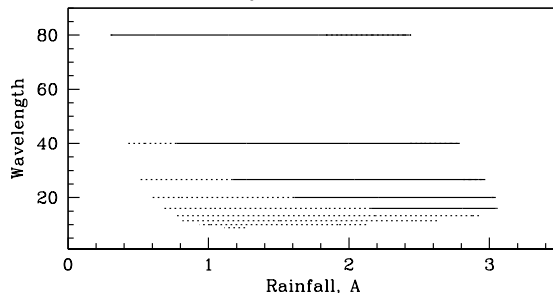


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# Pattern Stability: The Key Result

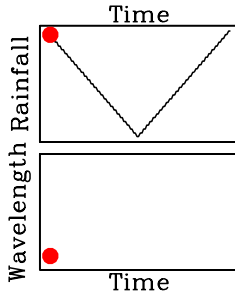
## Key Result

Many of the possible patterns are unstable and thus will never be seen.

However, for a wide range of rainfall levels, there are multiple stable patterns.

# Variations in Rainfall: Hysteresis

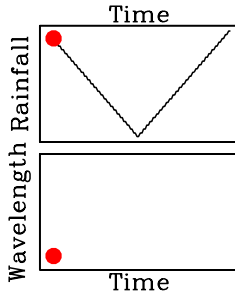
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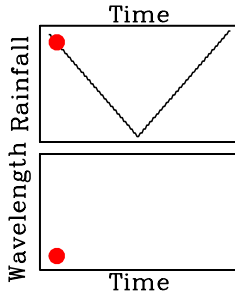
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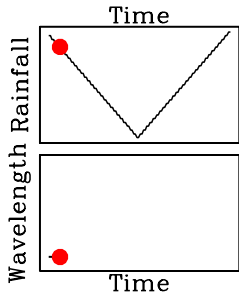
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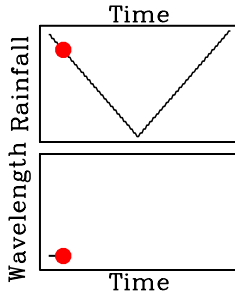
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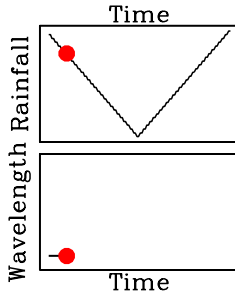
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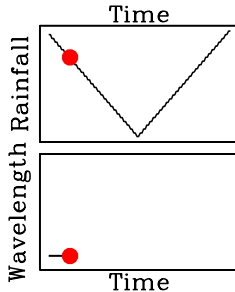
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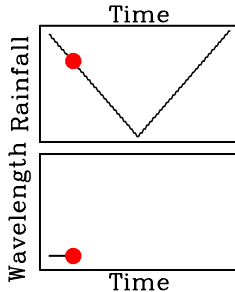
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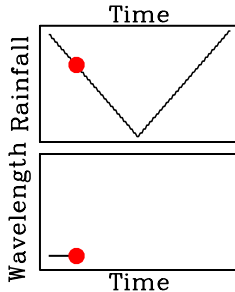
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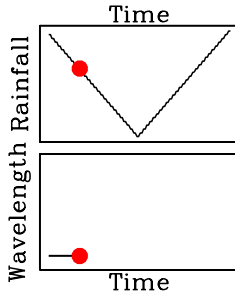
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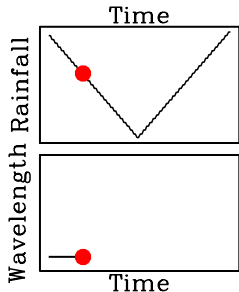
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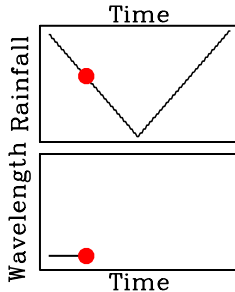
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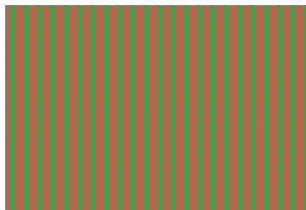
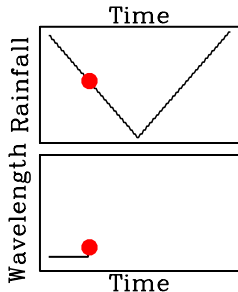
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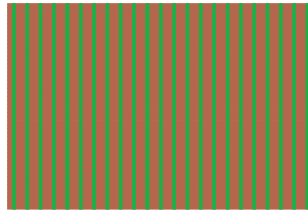
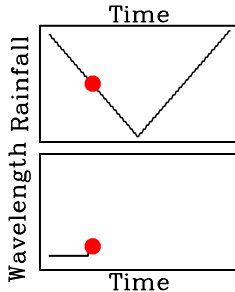
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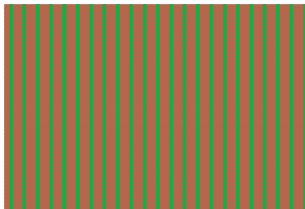
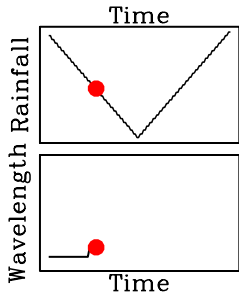
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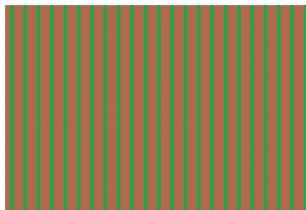
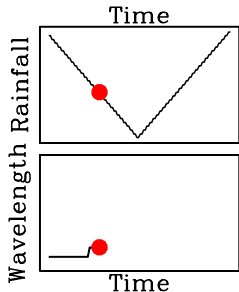
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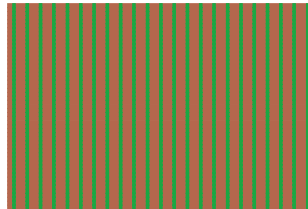
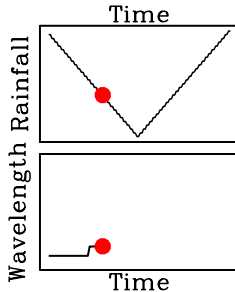
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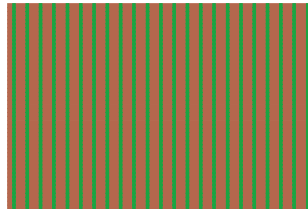
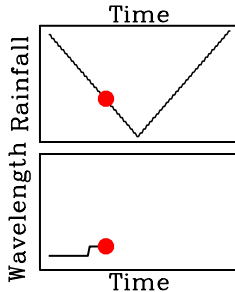
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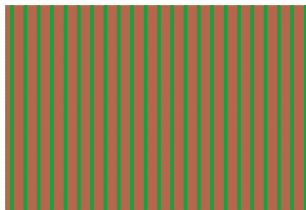
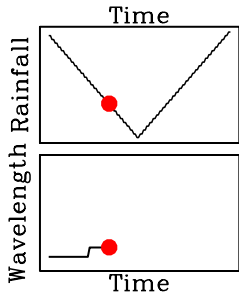
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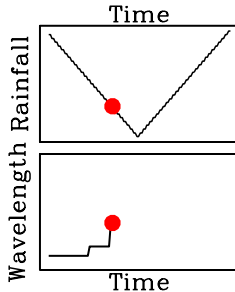
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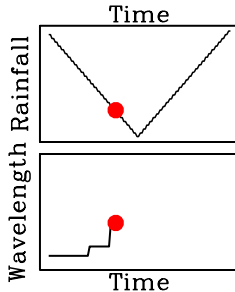
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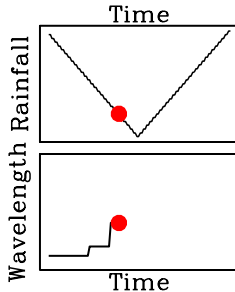
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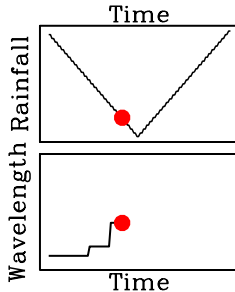
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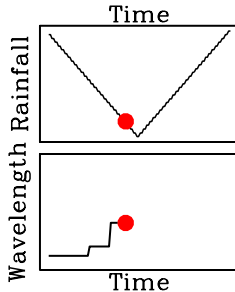
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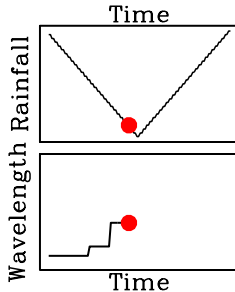
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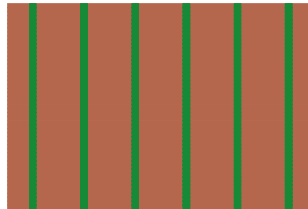
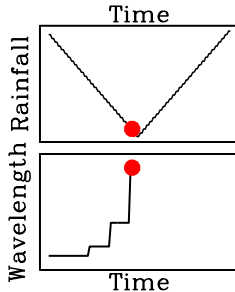
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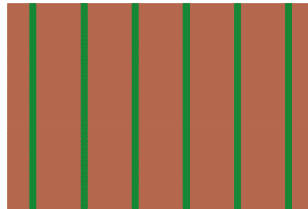
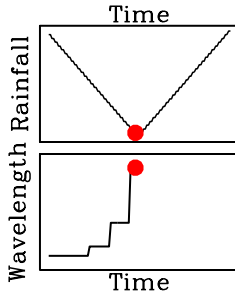
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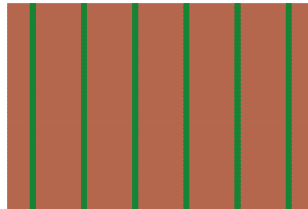
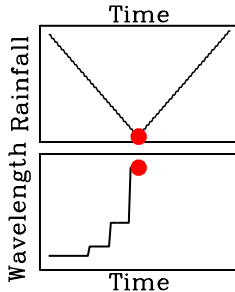
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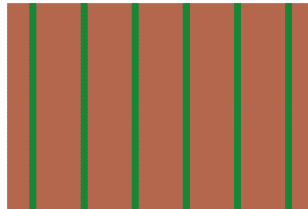
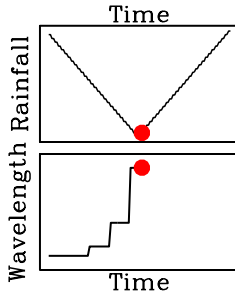
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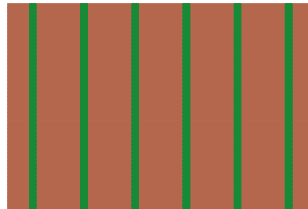
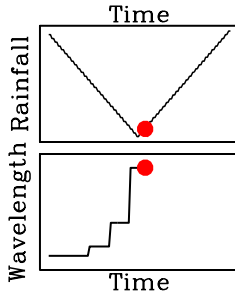
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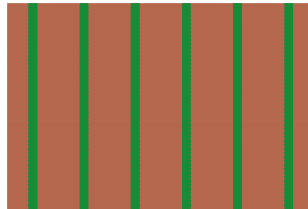
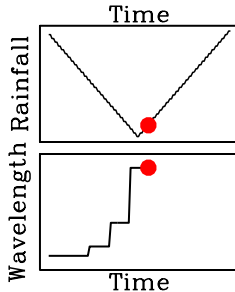
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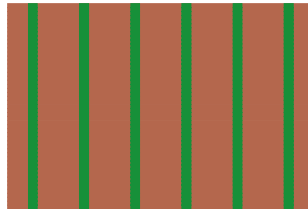
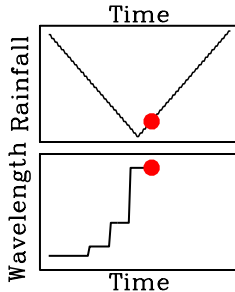
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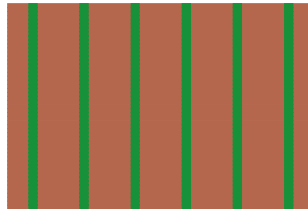
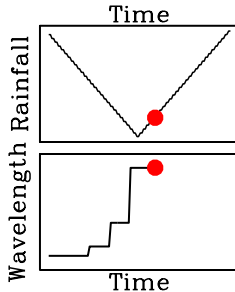
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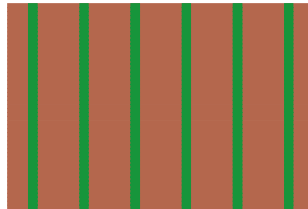
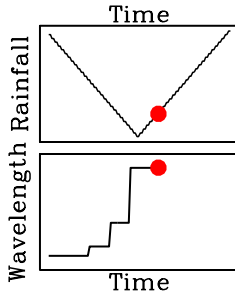
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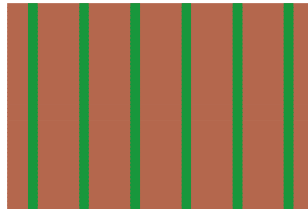
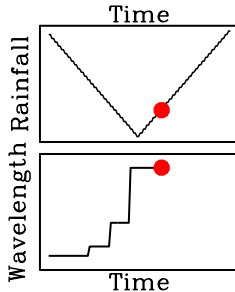
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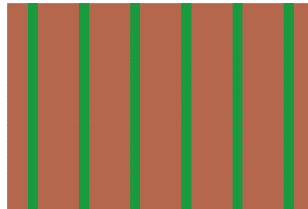
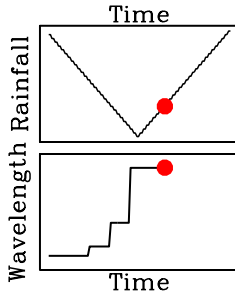
# Variations in Rainfall: Hysteresis

Model prediction: as rainfall is varied within the range giving patterns, abrupt changes in pattern wavelength occur.



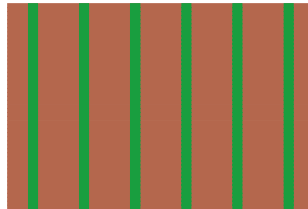
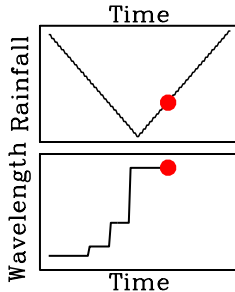
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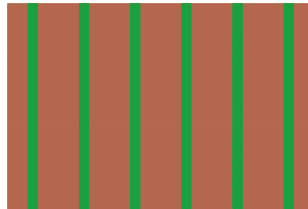
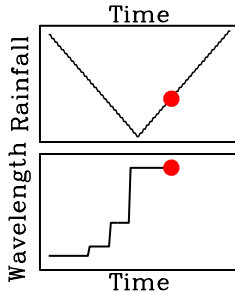
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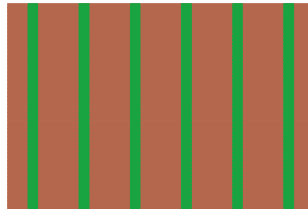
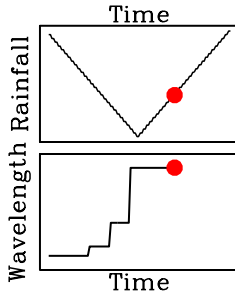
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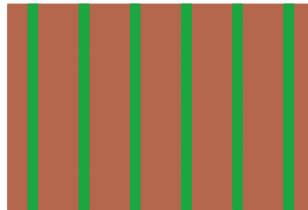
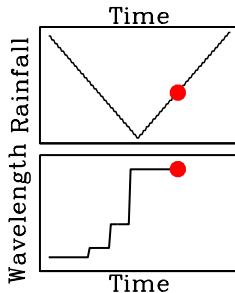
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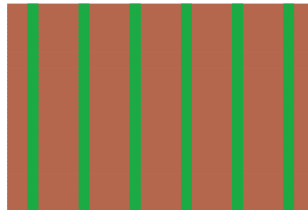
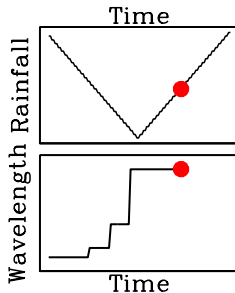
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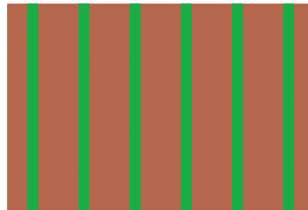
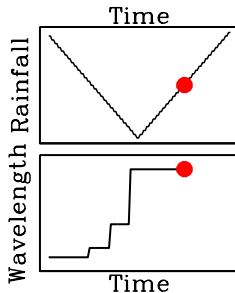
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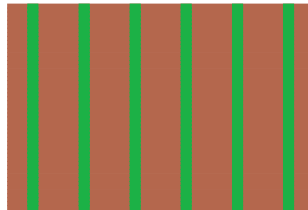
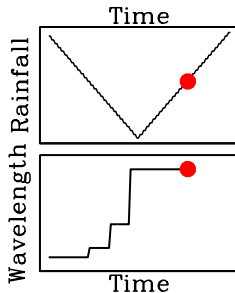
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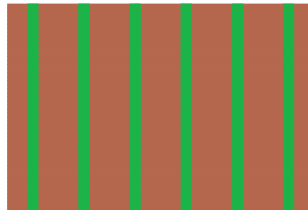
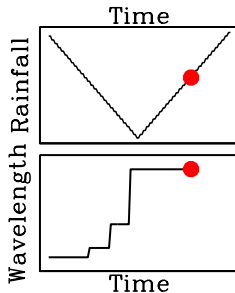
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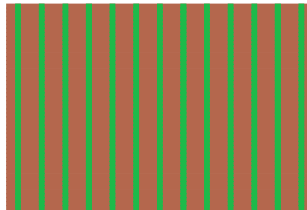
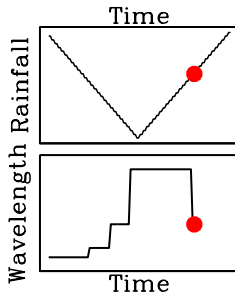
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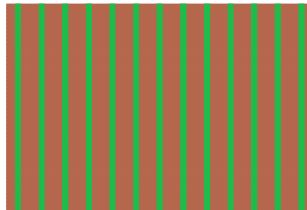
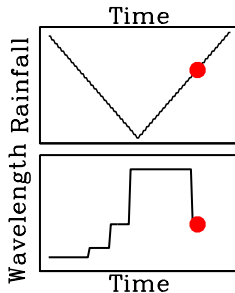
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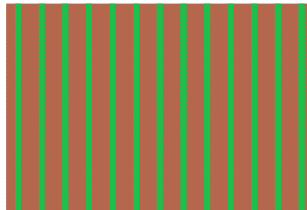
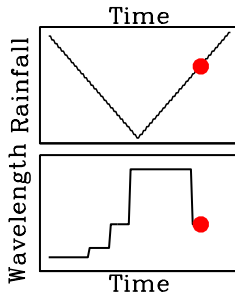
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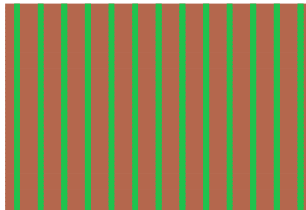
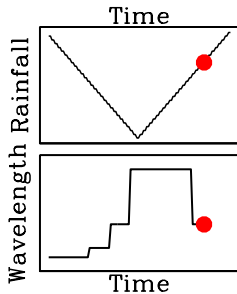
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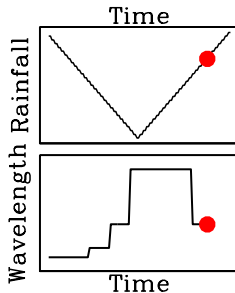
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# Variations in Rainfall: Hysteresis

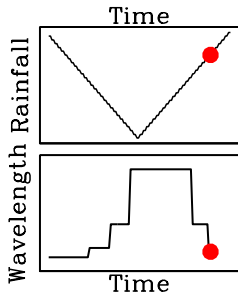
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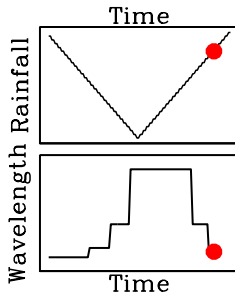
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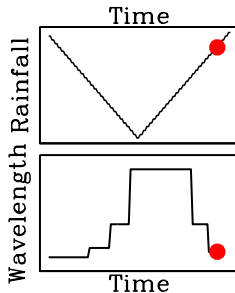
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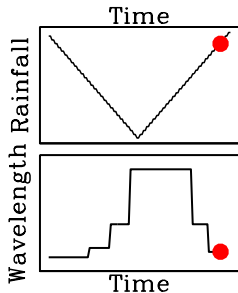
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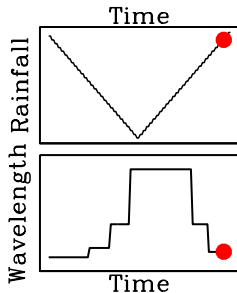
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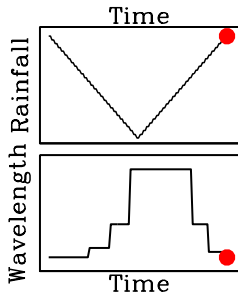
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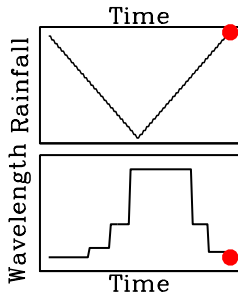
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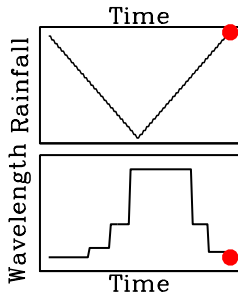
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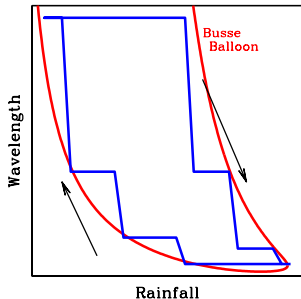
Model prediction: as rainfall is varied within the range giving patterns, abrupt changes in pattern wavelength occur.





# Variations in Rainfall: Hysteresis

Wavelength changes abruptly at the edge of the Busse Balloon.



(work of JAS, Koen Siteur, Eric Siero, Arjen Doelman, Max Rietkerk)

## Calculation of the Busse Balloon

The patterns move at constant shape and speed

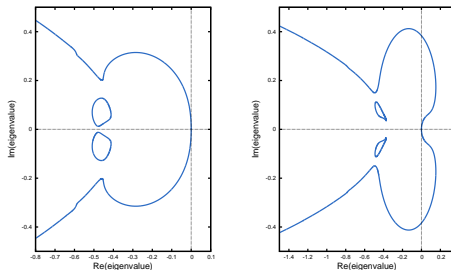
$$\Rightarrow u(x, t) = U(z), w(x, t) = W(z), z = x - ct$$

$$\begin{aligned}d^2 U/dz^2 + c dU/dz + WU^2 - BU &= 0 \\ D d^2 W/dz^2 + (\nu + c)dW/dz + A - W - WU^2 &= 0\end{aligned}$$

The patterns are periodic (limit cycle) solutions of these equations

## Pattern Stability: Numerical Approach

The boundary between stable and unstable patterns can be calculated by numerical continuation of the essential spectrum.



Calculations of this type can be performed using the software package WAVETRAN ([www.ma.hw.ac.uk/wavetrain](http://www.ma.hw.ac.uk/wavetrain)).

# Outline

- 1 Vegetation Pattern Formation
- 2 History-Dependence in Vegetation Patterns
- 3 Inferring the Historical Origin of Patterned Vegetation

## Banded Vegetation on Slopes

On slopes, run-off occurs in one direction only, giving striped patterns parallel to the contours.



Bushy vegetation in Niger



Mitchell grass in Australia  
(Western New South Wales)

Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia, Mexico and S-W USA.

## Banded Vegetation on Slopes

On slopes, run-off occurs in one direction only, giving striped patterns parallel to the contours.



Bushy vegetation in Niger

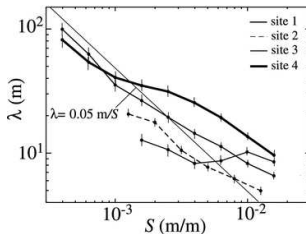


Mitchell grass in Australia  
(Western New South Wales)

Wavelength can be measured via remote sensing.

## Data on Wavelength vs Slope

I will show that the relationship between pattern wavelength and slope provides valuable historical insights.



Data from Nevada, USA (Pelletier et al, J. Geophys. Res. 117: F04026, 2012)

# The Origin of Vegetation Patterns

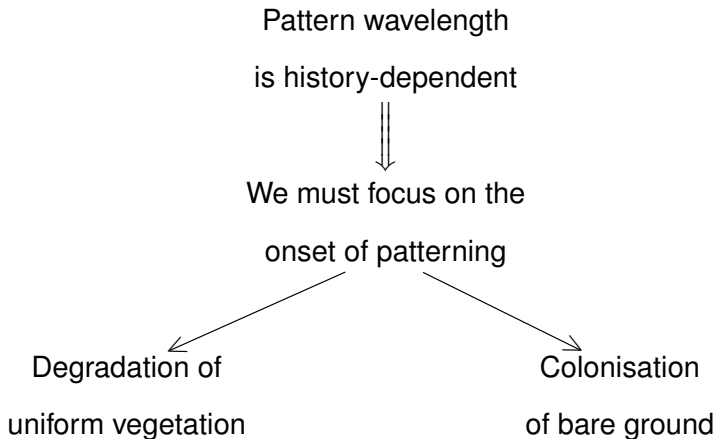
Vegetation patterns develop via  
either **degradation of uniform vegetation**  
or **colonisation of bare ground**



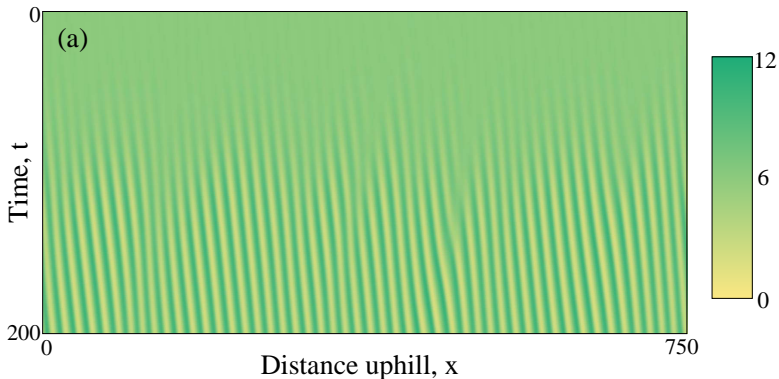
# Mathematical Model on a Slope

$$\begin{aligned}\frac{\partial u}{\partial t} &= \overbrace{wu^2}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\partial^2 u / \partial x^2}^{\text{plant dispersal}} \\ \frac{\partial w}{\partial t} &= \underbrace{A}_{\text{average rainfall}} - \underbrace{w}_{\text{evaporation \& drainage}} - \underbrace{wu^2}_{\text{uptake by plants}} + \underbrace{\nu \frac{\partial w}{\partial x}}_{\text{flow downhill}} + \underbrace{D \frac{\partial^2 w}{\partial x^2}}_{\text{diffusion of water}}\end{aligned}$$

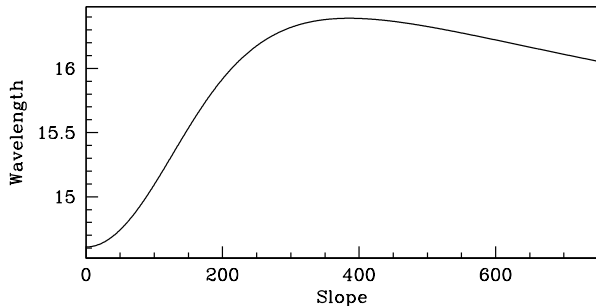
# How to Predict Pattern Wavelength



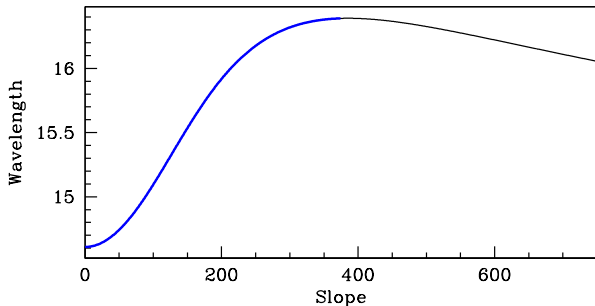
# Wavelength vs Slope for Degradation of Uniform Vegetation



# Wavelength vs Slope for Degradation of Uniform Vegetation

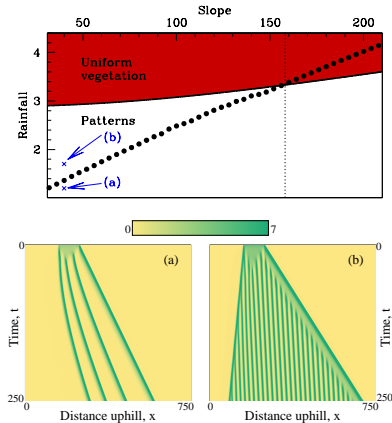


# Wavelength vs Slope for Degradation of Uniform Vegetation

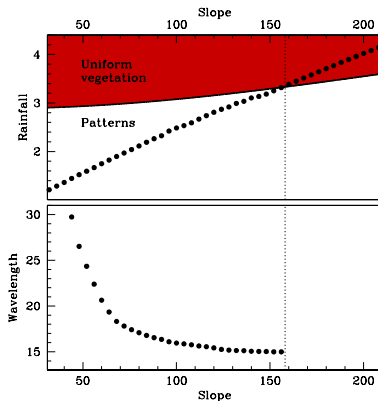


For realistic parameters, wavelength increases with slope

# Wavelength vs Slope for Colonisation



# Wavelength vs Slope for Colonisation



Wavelength decreases with slope

# Conclusions

Wavelength is positively correlated with slope  $\Rightarrow$  vegetation pattern originated by degradation of uniform vegetation

Wavelength is negatively correlated with slope  $\Rightarrow$  vegetation pattern originated by colonisation of bare ground

**Main message:** combined wavelength–slope data is much more valuable than wavelength data alone.



## Example: The African Sahel



- Patterned vegetation is widespread in the Sahel
- Several studies of banded vegetation show wavelength  $\downarrow$  as slope  $\uparrow$

# Rainfall History in the Sahel

- The Sahara and Sahel have been arid for about 5000 years, but the level of aridity has varied significantly.
- The Sahel was relatively humid in the 16th and 17th centuries.
- Reasonable assumption: areas with vegetation patterns today had uniform vegetation at the end of the 17th century.
- Since wavelength decreases with slope, my results imply that vegetation must have died out and then recolonised since the end of the 17th century.
- The most severe drought since 1700 was c. 1738-1756. So today's vegetation patterns result from recolonisation since 1760.

## References

- J.A. Sherratt:** History-dependent patterns of whole ecosystems. *Ecological Complexity* 14, 8-20 (2013).
- A.S. Dagbovie, J.A. Sherratt:** Pattern selection and hysteresis in the Rietkerk model for banded vegetation in semi-arid environments. *J. R. Soc. Interface* 11: 20140465 (2014).
- J.A. Sherratt:** Using wavelength and slope to infer the historical origin of semi-arid vegetation bands. *PNAS USA* 112: 4202-4207 (2015).
- J.A. Sherratt:** When does colonisation of a semi-arid hillslope generate vegetation patterns? *J. Math. Biol.* 73: 199-226 (2016).
- J.A. Sherratt:** Using History to Predict the Future of Vegetation in the African Sahel. Submitted.

# List of Frames

- 1 **Vegetation Pattern Formation**
  - Vegetation Patterns
  - Banded Vegetation on Slopes
- 2 **History-Dependence in Vegetation Patterns**
  - Mathematical Model of Klausmeier
  - Typical Solution of the Model
  - Variations in Rainfall: Hysteresis
  - Calculation of the Busse Balloon
- 3 **Inferring the Historical Origin of Patterned Vegetation**
  - Banded Vegetation on Slopes
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  - Wavelength vs Slope for Degradation of Uniform Vegetation
  - Conclusions