Minicours
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Fundamental results about $H_{t=0,c}(W)$

P. Etingof, V. Ginzburg: Symplectic reflection algebras, ... "$H_c(W)$"

Let Z_c be the center of $H_{t=0,c}(W)$. The center of $H_{t,c}(W)$ is trivial when $t \neq 0$ whereas $\mathrm{Spec}(Z_c)$ is a complicated algebraic variety. In type A, it is the Calogero-Moser space:

When $W = \underbrace{S_n,}_{\text{and } c \neq 0}$ $\mathrm{Spec}(Z_c) \cong \{(X,Y) \in \mathfrak{gl}_n \times \mathfrak{gl}_n \mid \mathrm{rank}([X,Y]+Id) = 1\} /\!/ PGL_n$

This is one particular example of a quiver variety.

For M a simple $H_c(W)$-module, Schur's lemma says that $z \in Z_c$ acts on M by a scalar, so to M we can associate a character $\chi : Z_c \to \mathbb{C}$. Given such a χ, we would like to construct a simple H_c-module. $\updownarrow$ (closed) point on $\mathrm{Spec} Z_c$

Satake isomorphism: The map $Z_c \to eH_ce$, $z \mapsto ze$, is an algebra isomorphism. (In particular, eH_ce is commutative!)

Classical invariant theory $\Rightarrow \mathbb{C}[\mathfrak{h} \oplus \mathfrak{h}^*]$ is a finitely generated module over $\mathbb{C}[\mathfrak{h} \oplus \mathfrak{h}^*]^W$. Since $\mathrm{gr}(H_ce) \cong \mathbb{C}[\mathfrak{h} \oplus \mathfrak{h}^*]$ and $\mathrm{gr}(eH_ce) \cong \mathbb{C}[\mathfrak{h} \oplus \mathfrak{h}^*]^W$ (PBW property), H_ce is a finitely generated (right) eH_ce-module.

If $\chi : Z_c \to \mathbb{C}$, we can view χ as a one-dimensional eH_ce-module via the Satake isomorphism. $H_ce \otimes_{eH_ce} \chi$ is a left H_c-module and it is finite dimensional.