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Finite dimensional representations of rational Cherednik algebras for Weyl groups when $t \neq 0$ $(t=1)$.

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I. Gordon math. RT/0208126

W: Weyl group $W \subset \mathfrak{h}$

Let h be the Coxeter number of W, $h = \dfrac{2|R_+|}{\dim \mathfrak{h}}$

↳ See the definition on the back

Goal: Study $\mathcal{O}(H_{t=1,c}(W))$ when $c = \frac{1}{h} + m$, $m \in \mathbb{Z}_{\geq 0}$, in particular finite dimensional modules.

(so c is constant, $c_{long} = c_{short}$)

Easy case: When $c = \frac{1}{h}$, $H_c(W)$ has a one-dimensional module.

Main ideas: KZ functor + $\mathcal{H}_q$..., use the (anti-) spherical subalgebra and shift functors. The Koszul resolution is not necessary, although it exists.

Lemma: Suppose $c = \frac{1}{h} + m$, $m \in \mathbb{Z}_{\geq 1}$. Let $\tau, \mu \in \mathrm{Irr}(W)$, $\tau \neq \mu$ and $\mathrm{Hom}_{H_{t=1,c}(W)}(\Delta(\tau), \Delta(\mu)) \neq 0$; then $\tau \cong \Lambda^i \mathfrak{h}$ and $\mu \cong \Lambda^j \mathfrak{h}$ for some $0 \leq i \neq j \leq \dim \mathfrak{h}$.

Proof: It is similar to the one in Type A. The representation theory of $\mathcal{H}_q(W)$ for $q = e^{2\pi i c}$ implies, via the KZ functor, that $\Delta(\rho)$ is irreducible (and projective) if $\rho \neq \Lambda^i \mathfrak{h}$ $\forall 0 \leq i \leq \dim \mathfrak{h}$. Furthermore, the $\mathcal{H}_q(W)$-modules $KZ(\Delta(\Lambda^i \mathfrak{h}))$ are all in the same block.

Coxeter elements: If $s_1, \dots, s_n$ are simple reflections in W,
then $s_1 s_2 \cdots s_n$ is called a Coxeter element. (These are all conjugate.)
For S_n, the cycle $(1, 2, \dots, n)$ is a Coxeter element.

Def.: The Coxeter number h of W is the order of a Coxeter element.

The Coxeter number of S_n is n.

Lemma: $h = \dfrac{2|R_+|}{\dim \mathfrak{h}}$ $|R_+|$ = number of positive roots