I was not informed before this meeting that my notes would be scanned and put on a CD. Had I known in advance, I would have prepared them with greater care. I apologize for their poor quality.

Mini-course
Cherednik algebras
CIRM, Luminy
April 2007

Category O

Reference: V. Ginzburg, N. Guay, E. Opdam, R. Rouquier
math.RT/0212036

W: Weyl group (= crystallographic finite real reflection group = Coxeter group) $\mathfrak{h}$: reflection representation of W (over $\mathbb{C}$).

$\mathfrak{h}$: Cartan subalgebra of a simple Lie algebra $\mathfrak{g}$ with associated Weyl group W.

ex: $\mathfrak{h} \cong \mathbb{C}^{n-1}$ = $n \times n$ diagonal matrices of trace 0, $W = S_n$ (type A).

Semi-direct product (smash): A: ring, G: a finite group acting on A by ring automorphisms $A \# G = \{ag \mid a \in A, g \in G\}$ $(a,g_1)\cdot(a_2,g_2) = (a, g_1(a_2))(g_1,g_2)$.

$T(\mathfrak{h} \oplus \mathfrak{h}^*)$ = tensor algebra of $\mathfrak{h} \oplus \mathfrak{h}^* = \bigoplus_{k \geq 0} (\mathfrak{h} \oplus \mathfrak{h}^*)^{\otimes k}$

$$\left\{ \begin{array}{c} \{\text{reflecting hyperplanes in } \mathfrak{h}\} \\ \{v \in \mathfrak{h} \mid s(v) = v\} \end{array} \right. \underset{\longleftarrow}{\overset{\text{bijection}}{\longleftrightarrow}} \left. \begin{array}{c} \{\text{reflections in } W\} \\ s \end{array} \right\}$$

$$V \longmapsto \text{pointwise stabilizer of } V \text{ in } W = \{1, s\}.$$

Ex: $W = S_n$, $\mathfrak{h} \cong \mathbb{C}^{n-1} = \{(d_1, \dots, d_n) \mid \sum_{i=1}^n d_i = 0,\ d_j = d_k\} = \mathfrak{h}_{s_{jk}}$

$(\ , \ )$: Killing form, non-degenerate W-invariant bilinear form on $\mathfrak{h}$
Ex: $A, B \in \mathfrak{sl}_n$. $(A,B) = \frac{1}{n} \mathrm{Tr}(AB)$.